

## Group A: Microeconomics

A.1 Let  $X$  be a finite set of alternatives and  $\Omega$  be all its non-empty subsets:  $\Omega := \{S \subseteq X : S \neq \emptyset\}$ . A choice function is a map  $Ch : \Omega \rightarrow X$  such that for all  $S \in \Omega$ , we have  $Ch(S) \in S$ . An agent uses such a choice function to choose an alternative from any menu of alternatives in  $\Omega$ .

Consider the following axioms on choice functions.

A choice function satisfies **always chosen (AC)** if for every  $S \in \Omega$ , if there exists  $x \in S$  such that  $Ch(\{x, y\}) = x$  for all  $y \in S \setminus \{x\}$ , then  $Ch(S) = x$ .

A choice function satisfies **no binary cycles (NBC)** if for every sequence of alternatives  $x_1, \dots, x_n$  such that  $Ch(\{x_i, x_{i+1}\}) = x_i$  for  $i \in \{1, \dots, n-1\}$ , we have  $Ch(\{x_1, x_n\}) = x_1$ .

Answer the following questions. [6 + 7 + 12 = 25]

A.1.1. Consider the following **rational** choice function: the agent has a strict ordering  $P$  over  $X$ . For every  $S \in \Omega$ , the choice  $Ch^r(S)$  is the top ranked alternative in  $S$  according to  $P$ . Verify whether  $Ch^r$  satisfies (i) NBC; (ii) AC (either provide a proof or a counterexample).

A.1.2. Consider the following choice function  $Ch^*$ . There is a pair of strict orderings  $P_1$  and  $P_2$  over  $X$ . For every  $S \in \Omega$ , the agent chooses the *top two* alternatives according to  $P_1$  in  $S$  and then chooses the best alternative among these two according to  $P_2$ . As an example, if  $S = \{a, b, x, y\}$  and  $a P_1 x P_1 b P_1 y$ ,  $b P_2 a P_2 y P_2 x$ , then  $Ch^*(S)$  will choose the best of  $\{a, x\}$  according to  $P_2$ , which is  $a$ . Verify whether  $Ch^*$  satisfies (i) NBC; (ii) AC (either provide a proof or a counterexample).

A.1.3. Suppose an arbitrary choice function  $Ch$  satisfies NBC and AC. Is it true that  $Ch = Ch^r$ ? Either provide a proof or a counterexample.

A.2. Consider a scenario with two firms, 1 and 2. The market demand function is  $p = 100 - q$ , where  $p$  is the market price and  $q$  is the aggregate output. Let  $q_i$  be the output level of firm  $i$ ,  $i = 1, 2$ . The cost function of both firms takes the following form:

$$C(q_i) = \begin{cases} 10q_i & \text{if } q_i \in [0, 10] \\ \infty & \text{if } q_i > 10 \end{cases}$$

Answer the following questions. [3 + 5 + 4 + 13 = 25]

A.2.1. Suppose the firms compete over quantities.

- (i) Are the strategic variables  $q_1$  and  $q_2$  strategic substitutes, or complements?
- (ii) Derive the reaction functions of the two firms.
- (iii) What is the Nash equilibrium of this game?

A.2.2. Suppose the firms compete over prices. In case  $p_1 = p_2$ , the demand to both firms is  $\frac{100-p_1}{2}$ . Whereas if  $p_1 > p_2$ , then the demand coming to firm 2 is  $100 - p_2$ , and that coming to firm 1 is  $\max\{0, 100 - p_1 - q_2\}$ , where  $q_2$  is the amount supplied by firm 2. Characterise one Nash equilibrium of this game.

## Group B: Macroeconomics

B.1. Consider an economy with a continuum  $[0,1]$  of agents. All agents derive utility from consumption and leisure with identical preference structure. However, they differ in their acquired education level  $e$ . Assume that the distribution of  $e$  across agents is represented by  $F(e)$ . An agent with exogenous education level  $e$  earns a total wage of  $e \times l$  if she chooses to work for  $l$  hours. The total numbers of available hours to each agent is  $a \geq l$ . Therefore,  $a - l$  represents leisure. The government runs a balanced budget program where a proportional tax at the rate  $t \in (0, 1)$  is imposed (on income of each agent) and it is distributed to agents in a lump-sum way (say, the amount for each agent is  $Z$ ). Utility of each agent is represented by  $U = c^\beta(a - l)^{1-\beta}$ , where  $c$  represents consumption and parameter  $\beta \in (0, 1)$ .

Answer the following questions. [9 + 4 + 4 + 8 = 25]

B.1.1. Write down the optimization problem of an agent.

B.1.2. Suppose there is no government. Therefore, there is no tax or lump-sum redistribution.

- i. Show that the optimal choice of working hours ( $l$ ) varies with exogenous level of education  $e$ .
- ii. Does individual utility (welfare) increase with the level of education  $e$ ?

B.1.3. Suppose the government is present. Therefore, there are taxes and redistributive lump-sum payments. If  $Z > 0$ , will an increase in  $e$  result in higher optimum labor supply  $l$ ?

B.2. Consider an economy with infinitely lived individuals who seek to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}); 0 < \beta < 1,$$

where  $E_t$  is the mathematical expectation conditional on information available at time  $t$  and  $c_{t+j}$  is her consumption in time period  $t + j$ . Utility function  $u$  is strictly increasing, concave and twice continuously differentiable. Agents can transfer their wealth over time through equity and one-period bonds. To be precise, a risk free bond that gives  $B_t$  for holding it between period  $t$  and  $t + 1$  has a present value  $\frac{B_t}{R_t}$  at time  $t$ , where  $R_t$  is the gross rate of interest on the bond. Agent's equity holding between period  $t$  and  $t + 1$  is given by  $S_t$ . Let  $p_t$  be the price of the equity in period  $t$  and the stochastic dividend is denoted by  $q_t$ . Given this, we can write down the budget constraint as

$$c_t + \frac{B_t}{R_t} + p_t S_t \leq Z_t,$$

where  $Z_t > 0$  represents wealth level. Answer the following questions. [9 + 8 + 8 = 25]

- B.2.1. Express  $Z_{t+1}$  in terms of  $B_t$  and  $S_t$  adjusted to its returns/prices.
- B.2.2. Write down the utility maximization problem. Clearly derive the first order conditions with respect to  $B_t$  and  $S_t$  for any  $t$ .
- B.2.3. Suppose utility function is logarithmic, i.e.,  $u(\cdot) = \log(\cdot)$ . Using part (B.2.2) above, express the path of  $p_t$  in terms of  $p_{t+1}$ , one period consumption growth  $\frac{c_{t+1}}{c_t}$ , period  $t + 1$  dividend  $q_{t+1}$  and  $\beta$ .

### Group C: Econometrics

C.1. A project is suggested that will study the effect of education of the household head on the household's per capita consumption. Two students Pooja and Srinivas start with different datasets. Pooja gets access to the full sample - a large cross-sectional dataset that has data on the years of education of the household head ( $edu$ ) and the log of per capita consumption ( $lny$ ) of a household  $i$ . Srinivas, on the other hand, gets access to a smaller data set - it has the same data as Pooja except that it contains data only for households whose per capita consumption is below (or equal to) the poverty line  $ypoverity$  (which is common knowledge).

The population model of interest is:

$$lny_i = \beta edu_i + \epsilon_i$$

Answer the following questions. [2 + 5 + 8 + 10 = 25]

- C.1.1. Srinivas and Pooja run Ordinary Least Squares (OLS) regressions using their respective data-sets. Assume standard assumptions used for OLS apply to the population model stated above. Derive mathematically the marginal effects (in terms of the population model) Pooja and Srinivas are estimating.
- C.1.2. Is the estimator  $\beta$  from OLS estimation run by Srinivas expected to be consistent? Explain your answer.
- C.1.3. Now suppose Pooja makes an accidental deletion in her data. All observations where the head has more than 10 years of education are deleted. The data is still large enough and has enough variation for her to run an OLS on whatever data that remains. Comment on the properties of the OLS estimator of  $\beta$  if she uses this data set.

C.1.4. Now assume that Pooja has the full data set again and this data set also includes data on the number of members in the household  $hhnum$ . She can run a multivariate regression—regressing log of per capita consumption on the education of the household head, now “controlling” for  $hhnum$ . But there is a new problem. Pooja realises that her statistical software is such that she can only run bivariate regressions. She cannot get another statistical software. Srinivas however suggests that she can estimate the coefficients of her multivariate regression model using multiple bivariate regressions. What procedure would she have to follow?

C.2. Consider a time series  $\{X_t\}$  which follows a  $MA(1)$  model as:

$$X_t = \eta_t + \theta\eta_{t-1}; t = 1, 2, \dots (\eta_0 = 0)$$

where  $\{\eta_t\}$  is White noise,  $WN(0, \sigma^2)$ . Answer the following questions. [4 + 5 + 3 + 5 + 5 + 3 = 25]

C.2.1 Derive the autocorrelation function (ACF) of  $\{X_t\}$ .

Unless otherwise indicated, consider two values of  $\theta$ :  
 $\theta = 0.5$  and  $\theta = 2.0$

C.2.2 For each value of  $\theta$ , find the first two autocorrelations.

C.2.3 For each value of  $\theta$ , determine if the time series  $\{X_t\}$  is stationary.

C.2.4 For each value of  $\theta$ , determine if the time series  $\{X_t\}$  is invertible.

C.2.5 If the time series is invertible for the given values of  $\theta$ , derive the infinite order autoregressive process for  $\{X_t\}$ .

C.2.6 If  $\theta = 1$ , determine if the time series is invertible.

### Group D: Indian Economy

- D.1. It is well known that the labour force participation of females is very low in India. Answer the following questions. [5 + 10 + 5 + 10 = 25]
- D.1.1. What is the female labour participation in India (indicate a reliable source and year)? How is labour force measured in India and world over? Are there reasons to believe that we are mis-measuring female labour in India?
- D.1.2. Let us assume that there are no serious issues regarding measurement of female labour. What are the factors that typically explain labour supply for females in India?
- D.1.3. What has been the trend in female labour force participation over the last four decades? Can the trends in any of the factors you point out above explain some part of the changes (if any) in female labour force participation?
- D.1.4. There is a rise in education outcomes (for example, enrollment) for girls in India. Assuming that human capital has a component of investment, why, in your opinion are households investing in the human capital of the girls in their household? Is there any published evidence supporting your argument?

D.2. The New Education Policy (NEP) for India has suggested many important changes that have implications for the human capital of India. Answer the following questions. [10 + 10 + 5 = 25]

D.2.1. NEP envisages an important role for the private sector in education. What is the proportion of children who study in private schools in India (indicate a reliable source and year)? Given the large outreach of public primary schools across villages of India, what explains the choice of private primary schools among households in rural India? What is the evidence on the efficiency and equity aspects of private schooling in parts of India where such studies have been conducted. Cite references, where relevant, for arguments made.

D.2.2. “English is a language, it is not a test of your intelligence” (Nawazuddin Siddiqui, 2012). Evaluate the statement in relation to the literature on economic micro returns to English language for India. What are the potential sources of bias in estimating such returns in general? Do you think these bias concerns are relevant for India?

D.2.3. Apart from language and private schooling, elaborate on one more aspect of the NEP that you feel has important implications for the human capital of India? Also elaborate why economists should ponder over the issue you raise, instead of just educationists.