Test Code : RP : (Short Answer type) 2012

Junior Research Fellowship in Theoretical Physics and Applied Mathematics

The candidates for Junior Research Fellowships in Applied Mathematics and Theoretical Physics will have to write two papers −Test MIII (objective type) in the forenoon session and Test RP (short answer type) in the afternoon session.

The RP test booklet will consist of two parts. The candidates are required to answer Part I and only one of the remaining parts II & III.

The syllabi and sample questions for the test are as follows.

PART-I

Mathematical and logical reasoning

Syllabus

B.Sc. Pass Mathematics syllabus of Indian Universities.

Sample Questions

1. If
$$S = \left\{\frac{p}{q}\right\} + \left\{\frac{2p}{q}\right\} + \left\{\frac{(q-1)p}{q}\right\}$$
where p and q are relatively prime positive integers, then show that $2S$ is divisible by $q-1$. [$\{x\}$= fractional part of x]

2. Let f be a real valued function defined on the interval $[-2, 2]$ as:
$$f(x) = \begin{cases} (x+1)2^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & \text{for} \quad x \neq 0 \\ 0 & \text{for} \quad x = 0 \end{cases}$$
   i) Find the range of the function.
   ii) Is f continuous at every point in $(-2, 2)$? Justify your answer.

3. Let $A = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \omega_1 \\ \omega_2 & 0 \end{pmatrix}$, where ω_1 and ω_2 are roots of the equation $x^2 - 2x + 2 = 0$. If $B = A^{99} - A^{97} + A^{95}$, then show that
$$Tr\ (A) = Tr\ (B).$$

4. The position of a particle moving in a plane is given by $x = \sin \omega t$, $y = \cos \alpha\omega t$. Show that the trajectory repeats itself periodically, only if α is a rational number.

5. Evaluate $\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$

6. X is a uniformly distributed random variable with probability density function

$$f(x) = \begin{cases} \dfrac{5}{a} & \text{for} \quad -\frac{a}{10} \leq x \leq \frac{a}{10} \\ 0 & \text{for} \quad \text{otherwise} \end{cases}$$

where a is a non-negative constant. If $P(|x| < 2) = 2P(|x| > 2)$, then find a.

7. Find the roots of the equation $z^5 = -i$, and indicate their locations in the complex plane.

8. A ball falling with zero initial velocity on a smooth inclined plane forming an angle α with the horizontal, traverses a distance h before it strikes the plane. The ball rebounds elastically off the inclined plane. At what distance from the impact point the ball will rebound for the second time?

9. Displacement of a particle executing periodic motion is given by $y = 4\cos^2(t)\sin(5t)$. How many harmonic waves need to be superposed to get the above displacement?

10. Evaluate $\displaystyle\int_0^{\pi/2} \frac{dx}{1 + (tan\ x)^{100}}$.

11. Let p, q be two prime numbers each greater than or equal to 5 and $p > q$. Show that $p^2 - q^2$ is divisible by 24.

12. If $f(x, f(y)) = x^p y^q$ for all x, y, then show that $p^2 = q$ and find $f(x)$.

13. Show that the area of the triangle formed by z, iz, and $z + iz$ is $\frac{r^2}{2}$, where $r = |z|$ and $z = a + ib$, with a, b being real nonzero numbers.

14. At time $t = 0$ a particle of unit mass is at rest at the origin. If it is acted upon by a force $\vec{F} = 100t\, e^{-2t}\, \hat{i}$, find the change in momentum of the particle in going from $t = 1$ to $t = 2$.

15. Given any polynomial $A(x)$ with coefficients in R, show that there exists a polynomial $B(x)$ such that $A(x).B(x) = C(x^2)$, where $C(y)$ is some polynomial in y with coefficients in R.

16. Find the maximum possible value of xy^2z^3 subject to the conditions x, y, $z \geq 0$ and $x + y + z = 3$.

17. A point moves in the (x, y)-plane such that at time $t(> 0)$ it has the coordinates $(1/t, (1+t)/\sqrt{2})$. Find the time when it comes closest to the origin.

PART-II

Applied Mathematics

Syllabus

1. *Abstract algebra* : Groups, rings, fields.

2. *Real analysis* : Functions of single and several variables, metric space, normed linear space, Riemann Integral, Fourier series, Integral Transform.

3. *Differential equations* : ODE − Existence of solution, fundamental system of integrals, elementary notions, special functions. PDE upto second order, equations of parabolic, hyperbolic and elliptic type.

4. *Dynamics of particles and rigid bodies* : Motion of a particle in a plane and on a smooth curve under different laws of resistance, kinematics of a rigid body, motion of a solid body on an inclined smooth or rough plane.

5. *Functions of complex variables* : Analytic function, Cauchy's theorem Taylor and Laurent series, singularities, branch-point, contour integration, analytic continuation.

6. *Fluid Mechanics* : Kinematics of fluid, equation of continuity, irrotational motion, velocity potential, dynamics of ideal fluid, Eulerian and Lagrangian equations of motion, stream function, sources, sinks and doublets, vortex, surface waves, group velocity, Viscous flow − Navier Stokes equation, boundary layer theory, simple problems.

7. *Probability and statistics* : Probability axioms, conditional probability, probability distribution, mathematical expectations, characteristic functions, covariance, correlation coefficient. Law of large numbers, central limit theorem. Random samples, sample characteristics, estimation, statistical hypothesis, Neyman Pearson theorem, likelihood ratio testing.

Sample Questions

1. Let m and n be positive integers and F is a field. Let f_i $(i = 1, 2, \ldots, m)$ be linear functionals on F^n. For α in F^n, let $T\alpha = (f_1(\alpha), f_2(\alpha), \ldots \ldots, f_m(\alpha))$. Show that T is a linear transformation from F^n into F^m. Also show that every linear transformation from F^n into F^m is of the above form for some f_i $(i = 1, 2, \ldots, m)$.

2. a) Let $X_1 = [1, 2]$ and $X_2 = [0, 1]$. Let d_1 denote the Euclidean metric in X_1 and let $d_2(x, y) = 2|x - y|$ in X_2. Show that (X_1, d_1) and (X_2, d_2) are equivalent metric spaces.

b) Two different metrics on the space $X = x \in R : 0 < x \leq 1$ are defined by $d_1(x,y) = |x-y|$ and $d_2(x,y) = |\frac{1}{x} - \frac{1}{y}|$. Are the spaces (X, d_1) and (X, d_2) equivalent? Give reasons for your answer.

3. a) If G is a group of even order, then prove that it has an element $a \neq e$, satisfying $a^2 = e$.

   b) Let $A = \{a \;:\; a = 9x + 15y, \;\; x\;, y$ are integers and $|a| \leq 1000\ \}$. Find the cardinality of A?

4. a) Does there exist a hexagon with sides of lengths $2, 2, 3, 3, 4, 4$ (with certain order) and with each angle equal? Justify your answer.

   b) Let a, b be positive integers with a odd. Define the sequence $\{u_n\}$,

$$\begin{aligned} u_{n+1} &= \frac{1}{2}u_n, && \text{if } u_n \text{ is even} \\ &= u_n + a && \text{otherwise} \end{aligned}$$

   Show that $u_n \leq a$ for some $n \in N$.

5. a) A uniform ball of radius r rolls without slipping down from the top of a sphere of radius R. Find the angular velocity of the ball at the moment it breaks off the sphere. The initial velocity of the ball can be neglected

   b) An ice cube is melting, becoming smaller but still cubical.The rate of melting is proportional to the surface area of the cube. Let $v(t)$ be the volume of the cube at time t. Explain why $\frac{dv}{dt} = \alpha v^{\frac{2}{3}}$ for some constant α. Is α positive or negative ?

6. Show that if the solution of the ODE

$$2xy'' + (3 - 2x)y' + 2y = 0$$

is expressed in the form $y = \sum_{n=0}^{\infty} a_n\, x^{n+\sigma}$, σ can take two possible values. Find the relation between a_n and a_{n+1}, and show that one solution reduces to a polynomial.

7. a) Show that $x_n^3 + y_n^3 \;\to\; 0$ implies $x_n + y_n \;\to\; 0$. Is the reverse implication true?

   b) A function is defined as follows:

$$\begin{aligned} f(x) &= 0, && \text{where } x \text{ is irrational} \\ &= \frac{1}{q}, && \text{where } x = \frac{p}{q}, \end{aligned}$$

   where p, q are two positive integers prime to each other. Show that $f(x)$ is continuous at $x = a$, if a is irrational and $f(x)$ is discontinuous at $x = a$, if a is rational.

8. The function $u(x,t)$ satisfies the differential equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \qquad -\infty < x < \infty, \quad t > 0$$

subject to the boundary conditions $u(x,t) \to 0$ as $|x| \to \infty$ and the initial conditions

$$\begin{aligned} u(x,0) &= -e^{x} \quad & x < 0 \\ &= 0 \quad & x = 0 \\ &= e^{-x} \quad & x > 0 \end{aligned}$$

Show by Fourier transform that for $t > 0$,

$$u(x,t) = \frac{2}{\pi}\int_0^\infty \frac{ke^{-k^2 t} sinkx}{k^2+1} dk.$$

9. Show that

$$\frac{\cos 2nx}{\sqrt{\pi/2}}, \frac{\sin 2nx}{\sqrt{\pi/2}}, \quad (n = 1, 2, ...)$$

are orthogonal functions on $[0,\pi]$. Hence or otherwise show that the Fourier series expansion of $\cos x$, when $0 < x < \pi$, is given by

$$\cos x = \frac{8}{\pi}\sum_{n=1}^{\infty} \frac{n \sin 2nx}{4n^2-1}.$$

10. Show that

$$\frac{\sin x}{x} = \prod_{r=1}^{\infty}\left(1 - \frac{x^2}{r^2\pi^2}\right)$$

(Hint: if $n =$ even integer and $n = 2m$, $x^n - 1 = 0$ has two real roots and $m-1$ complex conjugate pairs)

11. a) Let $f(x)$ be defined on $[1,\infty)$. Also $f(x)$ is continuous and differentiable on that interval and further the derivative $f'(x)$ is given by $f'(x) = \dfrac{1}{x^2 + (f(x))^2}$. Show that for all $x \geq 1$, $f(x) \leq 1 + \dfrac{\pi}{4}$.

    b) If $f(z)$ is an analytic function, prove that $\log|f(z)|$ is harmonic where $(z = x + iy)$.

12. Calculate the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$. Show that the eigenvectors are orthogonal and form a basis in C^2. Find also a unitary matrix U.

13. Find the integral surface of the equation

$$(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$$

passing through the curve $xz=a^3$, $y=0$, where $p=\dfrac{\partial z}{\partial x}$, $q=\dfrac{\partial z}{\partial y}$.

14. Evaluate the integral $\int\limits_c \dfrac{e^{1/z^2}}{z^2+1}dz$, where c is the curve given by $|z-i|=\dfrac{7}{2}$, the integration being taken counterclockwise.

15. a) Evaluate $\int\int\sqrt{4x^2-y}\ dxdy$ over the triangle formed by the straight lines $y=0, x=1, y=x$.

    b) Use Laplace transform to solve the following differential equation

$$Y''(t)-Y(t)=1\ +e^{3t},$$

    given $Y(0)=-\frac{7}{8}$, $Y'(0)=0$. [Here, $Y'(t)=\frac{dY}{dt}, Y''(t)=\frac{d^2Y}{dt^2}$]

16. a) Suppose a flow passes through a channel. One plate of the channel is porous and other one is impermeable; and the plates are separated by a distance b. The x-component of the velocity u is a function of x only and its value at the inlet is u_0. There is a uniform inflow v_0 through the porous plate, so that the velocity component v in the y-direction is a function of y only. Considering the flow to be incompressible, find the expressions for the velocities u and v respectively as function of x and y. Find also the stream function ψ.

    b) Water flows through a circular pipe. At one section, diameter of the pipe is $0.3m$, static pressure is 260 KPa gauge, velocity is $3m/sec$ and the elevation is $10m$. The pipe diameter at the other section is 0.15 m with zero elevation. Find the pressure at the downstream section neglecting the frictional effect. Density of water may be assumed as 999 Kg/m^3.

17. a) Let a number be drawn at random from $\{1,2,.....n\}$. Call it X. A number is drawn at random from $\{1,2,.....x\}$. Call it Z. Find $E(z)$ and Var (z).

    b) Let $X\sim Exponential(\lambda)$ with $\lambda>0$. Show that for all $t\ >0$, the value of $E(X/X>t)-t$ does not depend on t.

PART-III

Theoretical Physics

Syllabus

1. **Classical Mechanics**

   Mechanics of a particle and system of particles−conservation laws− scattering in a central field, Lagrange's equation and their applications, Hamilton's equation, Canonical transformation, Special theory of relativity, Small oscillation, Vibration & acoustics.

2. **Electromagnetic theory**

   Electrostatics, Magnetostatics, Classical electrodynamics, Maxwell's equations, Gauge transformation, Poynting's theorem, Wave equation and plane waves, Radiating system and scattering.

3. **Statistical Physics & Condensed Matter Physics**

   Thermodynamic equilibrium, Partition functions, Density matrix, Phase transitions, Spin systems, Statistical fluctuations, Band theory of electrons, Semiconductor Physics.

4. **Quantum Mechanics and Quantum Field Theory**

   Inadequacy of classical physics, Schroedinger wave equation, General formalism of wave mechanics, Exactly soluble eigenvalue problems, Approximation methods, Scattering theory, Time dependent perturbation theory, Symmetries and Conservation Laws, Relativistic equations, Klein-Gordon/Dirac equations, Lagrangian field theory, Examples of quantum field theory − ϕ^4, Quantum electrodynamics.

5. **Elementary Particles**

   Elementary particles, Weak and strong interactions, Selection rules, CPT theorem, Symmetry Principles in Particle Physics.

Sample questions for Theoretical Physics

1. Consider a particle of mass m constrained to move on a frictionless circular loop of radius R. The loop is rotated with angular frquency ω about a verticle axis passing through its center. [Assume that at any instant of time the mass is at a position $\theta(t)$].

   i) Set up the Lagrangian for this system (upto a constant).

   ii) Write down the equation of motion for the particle.

iii) Find out the equilibrium positions where the mass would settle down when ω changes.

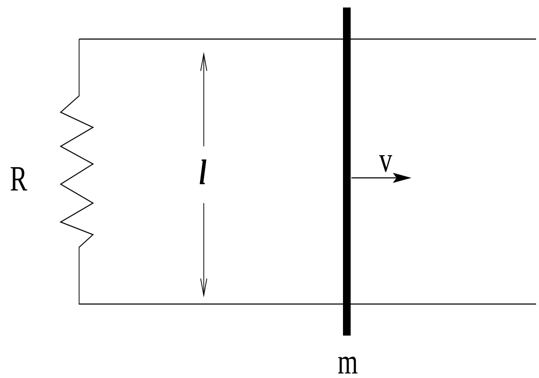
2. a) Let an observer on Earth see two particles born from a source and moving in opposite directions. The particle moving in the left has a velocity $\frac{1}{3}c$ km/sec with respect to the observer, where c is the velocity of light in vacuum . The observer sees the rate of increase of the distance between the particles as c km/sec. Find the relative velocity of one particle with respect to the other.

   b) Show that the transformation

$$\begin{aligned} q &= \sqrt{2P}\sin\theta \\ p &= \sqrt{2P}\cos\theta \end{aligned}$$

   is a canonical transformation.

3. A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart. A resistor R is connected across the rails and a uniform magnetic field $\mathbf{B}$, pointing into the page, fills the entire region.



   i) If the bar moves to the right at speed v, what is the current in the resistor? In what direction does it flow?

   ii) What is the magnetic force on the bar? In what direction?

   iii) If the bar starts out with speed v_0 at time $t = 0$, and is left to slide, what is its speed at a later time t?

   iv) Check that the energy delivered to the resistor is exactly $\frac{1}{2}mv_0^2$ which is the initial kinetic energy of the bar.

4. A particle of charge q_1 is at rest at the origin. A second particle of charge q_2 moves along the positive z-axis at constant velocity v.

i) Find the force $F_{12}(t)$ of q_1 on q_2, at time t.

ii) Find the force $F_{21}(t)$ of q_2 on q_1, at time t.

iii) Does Newton's third law hold in this case?

5. A uniform flat disc of mass M and radius r rotates about a horizontal axis through its center with angular speed ω_0. A chip of mass m breaks off the edge of the disc at an instant such that the chip rises vertically above the point at which it broke off. How high does the chip rise above the point before it starts to fall off? What is the final angular momentum of the disc?

6. a) It is known from special theory of relativity that the Doppler shift is given $\lambda' = \lambda\sqrt{(1+V/c)\ (1-V/c)}$ where λ is the emitted wavelength as seen in a reference frame at rest with respect to the source, and λ' is the wavelength measured in a frame moving with velocity V away from the source along the line of sight. Show that the Doppler shift in wavelength is $z \approx V/c$ for $V < c$ with c being the velocity of light.

   b) A rod of proper length L_0 is at rest in a reference frame S'. It lies in the (x', y') plane and makes an angle of $sin^{-1}\frac{3}{5}$ with the x' axis. If S' moves with constant velocity v parallel to the x axis of another frame S:
   (a) What is the value of v if, as measured in S, the rod is at 45^0 to the x axis? (b) What is the length of the rod as measured in S under these conditions?

7. A wire bent as a parabola $y = ax^2$ is located in a uniform magnetic field of induction $\vec{B}$, the vector $\vec{B}$ being perpendicular to the plane (x, y). At time $t = 0$, a connector starts sliding translation wise from the vertex of the parabola with a constant acceleration f. Find the e.m.f. of electromagnetic induction in the loop thus formed as a function of y.

8. Consider a possible solution to Maxwell's equation given by

$$\vec{A}(\vec{x}, t) \ = \ A_0 \ e^{i(\vec{k}.\vec{x}-\omega t)}, \ \phi(\vec{x}, t) = 0$$

where $\vec{A}$ is the vector potential and ϕ is the scalar potential. Further, suppose $\vec{A}_0, \vec{k}$ and ω are constants in space-time. Give, and interpret the constraints on $\vec{A}_0, \vec{k}$ and ω imposed by each of the Maxwell's equations given below

a) $\vec{\nabla}.\vec{B} \ = \ 0,$ b) $\vec{\nabla} \times \vec{\mathrm{E}} + \frac{1}{\mathrm{c}}\frac{\partial \vec{\mathrm{B}}}{\partial \mathrm{t}} \ = \ 0,$

c) $\vec{\nabla}.\vec{E} \ = \ 0,$ d) $\vec{\nabla} \times \vec{\mathrm{B}} - \frac{1}{\mathrm{c}}\frac{\partial \vec{\mathrm{E}}}{\partial \mathrm{t}} \ = \ 0,$

9. a) Consider a gas of N non-interacting atoms in a d -dimensional box of volume V with a kinetic energy $H = \sum_{i=1}^{N} A|\mathbf{p}_i|^s$, where $\mathbf{p}_i$ is the momentum of the i^{th} particle. Assume that A and s are both real and positive. Show that the classical partition function $Z(N,T,V)$ is given by

$$Z(N,T,V) = \frac{1}{N!}\left(\frac{VS_d}{h^d S}\right)^N \left(\frac{A}{k_B T}\right)^{-dN/s} \left[\Gamma\left(\frac{d}{s}\right)\right]^N$$

where h is Planck's constant and S_d denotes the surface area of a unit sphere in d dimensions.
(Hint: $\int d^d\mathbf{p} = S_d \int dp\; p^{d-1}$)

b) Consider a gas in a container obeying Van der Waals gas equation

$$(P + \frac{a}{v^2})(V - b) = nRT$$

where a and b are constants. The initial volume is V and then isothermally it is compressed to one half of its volume. Find the work done by the gas.

10. a) If the effective density of states in valence band is eight times that in conduction band in a pure semiconductor at $27^o C$, find the shift of Fermi level from the middle of the energy gap assuming low concentration of electrons and holes in the semiconductor. (Boltzmann's constant $K_B = 1.33 \times 10^{-23} J/$ mole^o K, $\ln 2 = .693$)

b) Consider a line of $2N$ ions of alternating charges $\pm q$ with a repulsive potential A/R^n between nearest neighbours in addition to the usual Coulomb potential. Neglecting surface effects find the equilibrium separation R_0 for such a system. Let the crystal be compressed so that R_0 becomes $R_0(1-\delta)$. Calculate work done in compressing a unit length of the crystal to order δ^2.

11. An assembly of N particles with spin $\frac{1}{2}$ and magnetic moment μ_0 is in a uniform applied magnetic field. The spins interact with the applied field (otherwise free).

a) Express the energy of the system as a function of its total magnetic moment and the applied field.

b) Find the total magnetic moment and the energy, assuming the system is in thermal equilibrium at temperature T.

12. a) Two levels in an atom whose nuclear spin is $I = 3$, have the designations $^2D_{3/2}$ and $^2P_{1/2}$. Find the expected number of components in the hyperfine structure of the corresponding spectral line.

b) A beam of electrons with kinetic energy 1 keV is diffracted as it passes through a polycrystalline metal foil. The metal has a cubic crystal structure with a spacing of $1A^0$. Calculate the wave length of electron and the Bragg angle for the first order diffraction. Take m, h, c the mass of the electron, Planck's constant and speed of light respectively as follows: $mc^2 = .5$Mev, $c = 3 \times 10^8$m/s, $h = 6.6 \times 10^{-34}$Js. Also take $1eV = 1.6 \times 10^{-19}$J.

13. a) Consider the Dirac Hamiltonian $H = c\vec{\alpha}.\vec{p} + \beta mc^2 + V(r)$ where the symbols have their usual meaning. Show that $[H, \vec{L}] = -i\hbar c(\vec{\alpha} \times \vec{p})$.

b) Derive the continuity equation for the Klein-Gordon equation and hence derive the densities ρ and $\vec{J}$. Explain how the continuity equation implies conservation of $\int d^3x\rho$. Can ρ be interpreted as a probability density?

14. Consider the Lagrangian of a scalar field theory with the self interaction term $\lambda\phi^5$.
i) Find out the equation of motion and the Hamiltonian.
ii) Consider a process where three ϕ-particles collide and two ϕ-particles come out after the collision. Draw possible Feynman diagrams for the above process having amplitude proportional to λ and λ^3.
iii) If one tries to quantize the theory in $3+1$ (physical) spacetime what are the problems one will face?

15. Consider the real free Klein-Gordon Lagrangian density,

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x_\mu} - m^2\phi^2\right).$$

Using the expansion,

$$\phi(x) = \sum_k \sqrt{\frac{\hbar c^2}{2V\omega_k}}\left(a(\vec{k})e^{-ik^\mu x_\mu} + a^\dagger(\vec{k})e^{ik^\mu x_\mu}\right)$$

show that the Hamiltonian is given by

$$H = \sum_k \hbar\omega_k\left[a^\dagger(\vec{k})a(\vec{k}) + \frac{1}{2}\right].$$

Note: $k^\mu x_\mu = \omega_k t - \vec{k}.\vec{x}$.

16. a) A particle is initially in its ground state in a one-dimensional harmonic oscillator potential, $V(x) = \frac{1}{2}\omega x^2$. If the coupling constant ω is suddenly doubled, calculate the probability of finding the particle in the ground state of the new potential.

b) Consider a particle of mass m inside a one-dimensional box of length a. Let at $t = 0$, the state of the particle is given by the following:

$$\begin{aligned}\psi(x,0) &= A\cos(\frac{2\pi x}{a})\sin(\frac{3\pi x}{a}), \qquad 0 \leq x \leq a \\ &= 0, \qquad \text{everywhere else.}\end{aligned}$$

i) Calculate the value of A.

ii) What is the average energy of the system at $t = 0$?

iii) What is the smallest positive time t_0 for which $\psi(x, t_0)$ would be orthogonal to $\psi(x, 0)$?

iv) If a measurement for energy is performed at $t = 0$, what is the probability of getting the value of energy as $\frac{h^2}{2ma^2}$?

c) Let $S_{\pm} = S_x \pm iS_y$ where S_x, S_y and S_z are Pauli spin matrices. If $|\pm, \frac{1}{2}>$ are eigenvectors of S_z, then find $S_{\pm}|\pm, \frac{1}{2}>$.

17. Which of the following reactions violate a conservation law? State the law that is violated in case(s) of forbiddenness.

i) $\pi^+ \rightarrow \mu^+ + \nu_\mu$

ii) $\mu^+ \rightarrow e^+ + \gamma$

iii) $e^- \rightarrow \nu_e + \gamma$

iv) $p + p \rightarrow K^+ + \Sigma^+$

v) $p + n \rightarrow \Xi^0 + p$

vi) $\Xi^0 \rightarrow \Sigma^0 + \Lambda^0$

vii) $n \rightarrow p + e^- + \bar{\nu}_e$

b) Obtain the exact ratio of the energy released when 1 gm of Uranium undergoes fission to the energy released when 1 gm of TNT explodes.