JRF (Quality, Reliability and Operations Research): 2010
INDIAN STATISTICAL INSTITUTE

OUTLINE OF THE SYLLABUS

The syllabus for JRF (QROR) will include the following subject areas: 1) Statistics , 2) Statistical Quality Control, 3) Quality Management and Quality Assurance Systems, 4) Operations Research , 5) Reliability, and 6) Mathematics. A broad coverage for each of the above subject areas is given below.

1. **Statistics (STAT)**: Elementary probability and distribution theory , Linear models, Estimation and test of hypothesis , Design of experiments(block design, full and fractional factorial designs), Bivariate distributions, Markov chain

2. **Statistical Quality Control(SQC)**: Attribute and variable control charts, CUSUM and EWMA charts, Process capability analysis, Acceptance sampling

3. **Quality Management and Quality Assurance Systems (QMAS)**: Total Quality management, Six Sigma, Quality assurance systems like ISO 9000 etc., Environmental management systems

4. **Operations Research (OR)**: Linear programming (basic theory, simplex algorithms and its variants, duality theory, transportation and assignment problem), Non-linear programming (basic theory, convex function and its generalization, unconstrained and constrained optimization), Inventory theory (EOQ models, dynamic demand model, deterministic models with price-breaks, concept of probabilistic models), Queuing Theory ((M/M/S) and (M/G/1)), Game theory (two persons zero-sum game, bimatrix game)

5. **Reliability (REL)**: Coherent Systems and System reliability, Failure time modelling, Estimation and Testing in reliability

6. **Mathematics(MATH)**: Permutations and combinations, Set Theory, Calculus, Linear and matrix algebra, Difference equations - all at B.Sc level.

INSTRUCTIONS

The test will be divided into two sessions (i) Forenoon session and (ii) Afternoon session. Each session is for two hours. The forenoon session question paper will be identified by test code $RQRI$, whereas the afternoon session will have test code as $RQRII$. Candidates appearing for JRF (QROR) should verify and ensure that they are answering the right question paper.

The test **RQRI** will be of multiple choice type. Each question will have four alternatives, only one of which is correct/best possible answer. There will be thirty such questions, out of which 20 questions will be from Mathematics, and 2 questions each from five other areas, as given in the syllabus above. All questions have to be answered.

The test **RQRII** will have altogether 18 questions, 3 from each of the six areas(including Mathematics), as given in the syllabus above. A candidate has to choose three areas (out of six) and answer two questions from each area.

Some sample questions from each of the above mentioned six areas are given below.

Multiple Choice

1. The numerical values of the covariance and the correlation coefficient between the length and the weight of items made by a process are determined to be 1.50 and 0.27, respectively. If the process were adjusted to double the length and increase the weight of each item by 0.5 units, then the covariance and the correlation coefficient between the length and the weight of the items made by the adjusted process would, respectively, be

   (A) (1.50,0.27) (B) (3.00, 0.27) (C) (3.00,0.54) (D) (1.55,0.54)

2. In cricket, a bowler pitches the ball within the first 15 yards (short-pitch) or within 15-18 yards (good-length), or within 18-22 yards (over-pitch), or beyond 22 yards with probabilities 1/4, 1/2, 1/5 and 1/20, respectively. Given that a ball is good-length, the exact pitch follows a uniform distribution within 15-18 yards. The conditional probability that a ball pitches within 17-18 yards, given that it pitches within the 22 yards, is

   (A) 10/57 (B) 1/3 (C) 1/6 (D) 19/60

Short Answer Type

1. In an experiment, suppose the block-size (k) is less than v, the number of equally important treatments. Suppose the goal is to construct a block design in which all v treatments can be allocated in a balanced manner in certain number of blocks.

   (a) Can you balance the number of treatments in fewer than $\binom{v}{k}$ number of blocks? If so, state the necessary conditions.

   (b) When the number of blocks equals the number of treatments, will there be any additional necessary condition(s) for the purpose mentioned in (a)? If so, state it/them.

   (c) How do you adjust the treatment sum of squares to separate the treatment effect from the block effect? Give details.

2. Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. $U[0,1]$. Define $X_{(1)} = \min(X_1, X_2, \ldots, X_n)$ and $X_{(n)} = \max(X_1, X_2, \ldots, X_n)$.

   (a) Calculate $P(n[1 - X_{(n)}] > x, nX_{(1)} > y)$, for all positive real numbers x, y.

   (b) Show that as $n \to \infty$, the above probability tends to $e^{-(x+y)}$.

   (c) Do you think that $X_{(1)}$ and $X_{(n)}$ are independent in the limit? Justify.

3. There is a biased coin with probability of Head being 1/6. A person, unaware of this biasedness, is asked to toss it 80 times. He is instructed to call Heads for the first 40 tosses and Tails for the remaining tosses, and to report only the number of correct calls. After the 80 tosses, as per instructions, he reports 53 correct calls. Carry out a suitable statistical analysis to support or contest his report.

Sample Questions for Statistical Quality Control

Multiple Choice

1. Which of the following claims about the operating-characteristic curve of a fraction nonconforming control chart are true?
   I) It is a graphical display of the probability of incorrectly accepting the hypothesis of statistical control
   II) It provides a measure of the sensitivity of the control chart
   III) It is obtained by plotting $(1-\beta)$ against the process fraction nonconforming, where β is the probability of type-II error
   IV) It can be obtained from the binomial distribution

   $(A)\ I, II,$ and III only $(B)\ I, II,$ and IV only $(C)\ I, III,$ and IV only

   $(D)\ II, III,$ and IV only

2. The sampling plans of given strength $(p_1, p_2, \alpha, \beta)$ or stronger, satisfy the two inequalities $G(c, np_1) \geq (1-\alpha)$ and $G(c, np_2) \leq \beta, p_2 > p_1$, where $G(c, np)$ denotes the Poisson probability of acceptance. The smallest sample size that satisfies these two requirements is the one which makes
   (A) consumer risk equals to β and the producer's risk as small as possible
   (B) producer's risk equals to α and the consumer risk as small as possible
   (C) $\frac{G(c,np_2)}{1-G(c,np_1)} = \frac{\beta}{\alpha}$
   (D) none of the above

Short-Answer Type

1. (a) Show that the binomial OC of a single sampling plan is a decreasing function of p. Show that for $0 < c < n-1$, it is concave for $p < c/(n-1)$ and convex for $p > c/(n-1)$. Show also that for $c = 0$ it is convex and for $c = n-1$ concave. [Note that p denotes process average defectives, n denotes the sample size and c denotes the acceptance number.]

(b) A characteristic is being controlled using an $\bar{X} - R$ chart. Suppose the characteristic has a target μ and variance σ. Show that p_n, the probability of the mean of a random sample of size n exceeding the UCL of $\mu + 3\sigma/\sqrt{n}$ when the population mean has shifted to $\mu + k\sigma$ is $1 - \Phi(3 - k\sqrt{n})$, where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt$.

2. Let λ be the weight assigned to the current sample mean for the exponentially weighted moving average (EWMA) control chart. Let w be the span (period) used for a moving average control chart. Let $\lambda = 2/(w+1)$. Show that the average age of data used in computing the statistics to be plotted for the two charts are identical.

3. The density of a plastic part used in a pocket calculator, should be at least $0.70 gm/cm^2$. The parts are supplied in large lots and a variable sampling plan is to be used to screen the lots. It is desired to have $p_1 = 0.02, p_2 = 0.10, \alpha = 0.10$ and $\beta = 0.05$. The standard deviation of the manufacturing process is known to be $1.05 \times 10^{-2} gm/cm^2$. Given $Z_{0.02} = 2.05, Z_{0.05} = 1.65, Z_{0.10} = 2.33$, where $Z_p = Prob(x \geq Z_p)$, what is the chance of acceptance of a lot at 5% defective level?

Sample Questions for Quality Management and Quality Assurance Systems

Multiple Choice

1. Most quality problems in a manufacturing organization

   (A) originate in the Quality department where the ultimate responsibility for quality rests.
   (B) originate in the shop floor because of waste and product rework.
   (C) are the results of management's lack of attention to potential improvement ideas.
   (D) originate because of lack of monitoring by the shop supervisors.

2. While auditing a giant automobile company it was observed that from every department a few supervisors were sent for training based on a training calendar prepared by the training department. The training department prepared its yearly training calendar purely based on the training provided in the earlier years and in some cases based on its acquaintances with some of the training providers. Based on this information choose one of the following action plans, you think best, as an auditor.

   (A) Process is perfectly alright and no action is needed.
   (B) A non-conformance report of minor type should be generated.
   (C) A non-conformance report of major type should be generated.
   (D) The need for further improvement in the process should be reported.

Short-Answer Type

1. In a large organisation implementing Six Sigma initiatives, define the roles and responsibilities of

   (a) a champion,

(b) a blackbelt,

(c) a greenbelt.

2. It is commonplace that accuracy is paramount in a newspaper's reputation. The new head of the business section of a major newspaper, who assumed charge only about a week back, was shocked to find large number of errors being caught at the final editing stages, right before the paper went to press. Such errors were costly to fix, made it tough to meet deadlines, and made everyone nervous that some would slip through.

   Management Intervention: The Chief pressed into service one of the junior editors and instructed him to prepare a time plot of daily errors during a span of last three months. The time plot showed that, on an average, there were about 21 errors per day and this number varied from 10 to as high as 60 per day with a standard deviation of 13 errors. The new department chief wanted this average error figure (21) to be cut by half within a time frame of next fortnight.
   The data collected by the junior editor also indicated that, there were broadly five types of errors viz, spelling error, grammatical errors, error in facts, error in names, and error in numbers. Spelling errors perhaps originated from (i) copy editor busy tracking down facts,(ii) reporter/editor ignoring/not using spell-check, (iii) reporter not having requisite typing skill, (iv) reporter not possessing proper knowledge of spelling of words used by him, (v) reporter being lethargic to use of dictionary and so on. Some of the possible reasons for 'Errors in facts, numbers, and names' were perhaps: (i) presence of hard to detect error,(ii) results of accuracy checks being not clear, (iii) accuracy checks skipped/not done due to tearing hurry etc. Similarly, 'grammatical errors' perhaps originated from (i) reporters being unaware of correct grammar, (ii) reporter and editor not reviewing grammar and so on.
   The new head raised a dedicated project team of 5 staff members to see through the entire process with a view to reduce the average number of errors per day by half of the present average within a time-frame of fifteen days. After first 5 days since the dedicated team started working

on the project, the Head checked the data on errors. He was visibly enthused to find that the average number of errors came down sharply to 13 from the level of 21. However, to his utter dismay, the Head, who kept on tracking the number of errors creeping in daily, found that, from the very sixth day onwards, the average number of errors again touched the earlier average of 21 and hovered around this figure with almost same standard deviation.

(a) Can you please prepare a Cause and Effect diagram for errors creeping into the newspaper from different sources. Use your imagination to enumerate as exhaustibly as possible, all the causes responsible for errors in the final printed version of the newspaper.

(b) If you are assigned to work for this project as the leader, give the steps in detail as to how would you proceed to crack this problem which was badly mauling the brand image of the newspaper. You might consider the following while giving your answer.

   i. What all aspects of the team (such as, composition, dynamics, management etc.) would you look for to ensure success ?

   ii. How would you structure your problem solving approach right from the start ?

   iii. How would you define and formulate the problem and what measurements would you need to formulate the problem correctly? [Incidentally you might also comment on the target figure given by your chief and the prudence of his setting such target].

   iv. At what stages would you collect data/information and how would you collect the data?

   v. What techniques/tools would you use to analyse the data at various stages of the problem solving approach?

   vi. What would be your final recommendations to the Head?

   vii. What type of results would constitute 'Success' for the project and for you?

3. (a) Explain the use of SIPOC Diagram.

(b) Consider the following elements of the product realization process in a job type organization engaged in manufacturing and delivery of excavator. The marketing department receives order from customer and accordingly a job order is prepared and issued to the production plant. Based on the bill of materials the manufacturing department sends requisition for bought out items. The procurement department checks with the stores the availability of these items. In case the items are not in the stock, purchase orders are placed with the approved vendors and materials are procured accordingly. The materials are thereafter received by the stores and issued to the production. Thereafter the process of production and delivery of the product follow. Identify the steps of the process of procurement of the bought out items. Draw a flow diagram of the procurement process indicating clearly the Start and End Points. Hence make a SIPOC diagram of the process of procurement of bought out items.

Sample Questions for Operations Research

Multiple Choice

1. Find out the statement which is false about a transportation problem:

   (A) A transportation problem can always be represented by a balanced model
   (B) A balanced transportation problem always has a feasible solution
   (C) If a constant value is added to every cost element c_{ij} in the transportation tableau, the optimal values of the variable x_{ij} will change
   (D) The transportation technique essentially uses the same steps as that of the simplex method

2. Which of the following triplets (x_1, x_2, x_3) is the solution to the LP problem?

   Minimize $2x_1 + 3x_2 + 4x_3$,
   subject to

$$\begin{aligned} x_1 + 2x_2 + x_3 &\geq 3 \\ 2x_1 - x_2 + 3x_3 &\geq 4 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

$(A)\ (\frac{13}{5}, \frac{2}{5}, 0)$ $(B)\ (\frac{11}{5}, 0, \frac{6}{5})$ $(C)\ (0, \frac{5}{7}, \frac{11}{7})$ (D) None of these.

Short Answer Type

1. (a) Consider the problem of minimizing $c^t x$ subject to $Ax \geq b, x \geq 0$, where $c \in R^n, A \in R^{m\times n}$ and $b \in R^m$. Let x_0 and y_0 be feasible solutions of the primal and the dual problems, respectively. Then show that a necessary and sufficient condition for x_0 and y_0 to be optimal solutions for the corresponding problems, is

$$\begin{aligned} y_0^t(Ax_0 - b) &= 0, \text{ and} \\ x_0^t(c - A^t y_0) &= 0. \end{aligned}$$

(b) Let S be a non-empty open convex set in R^n and $f : S \to R$ be differentiable on S. Then show that f is convex if and only if, for each $x_1, x_2 \in S$,

$$[\nabla f(x_2) - \nabla f(x_1)]^t (x_2 - x_1) \geq 0$$

where $\nabla f(x)$ is the gradient vector at x.

2. (a) Let S be a non-empty convex set in R^n and $f : S \to R$ be convex on S. Consider the problem of minimizing $f(x)$ subject to $x \in S$. Suppose that $\bar{x} \in S$ is a local optimal solution to the problem, then show that $\bar{x}$ is the global optimal solution. Under what condition $\bar{x}$ is a unique global optimal solution?

   (b) Suppose A is an $m \times n$ matrix. Then show that exactly one of the following systems has a solution:

   System 1: $Ax < 0$ for some $x \in R^n$
   System 2: $A^t y = 0, y \geq 0$, for some $y \in R^m$

3. (a) Show that the set of optimal strategies for each player in an $m \times n$ two-person-zero-sum game is a convex set.

   (b) Consider a queuing model $(M \mid M \mid 1) : (\infty \mid FCFS)$. In the steady state, find the conditional expectation of queue length, given that the queue is non-empty, and obtain the corresponding variance.

Multiple Choice

1. Consider the following two modifications of a coherent system ϕ: (a) another identical and independent copy of each component of ϕ is attached with the corresponding original component in parallel, and (b) another identical and independent copy of ϕ is attached with the original ϕ in parallel. Then the reliabilities r_a and r_b of the two modified systems in (a) and (b), respectively, satisfy

$$(A)\ r_a = 2r_b \quad (B)\ r_a \geq r_b, \quad (C)\ r_b = 2r_a, \quad (D)\ r_a \leq r_b.$$

2. Consider n independent items under test, each having *exponential* lifetime distribution with mean $1/\lambda$, subject to random censoring with censoring time having *exponential* distribution with mean $1/\mu$ $(\mu < \lambda)$. The expected number of censored items during the test is

$$(A)\ n\mu/(\lambda+\mu) \quad (B)\ n\lambda/(\lambda+\mu), \quad (C)\ n\mu/\lambda, \quad (D)\ n\lambda\mu/(\lambda+\mu)^2.$$

Short-Answer Type

1. Consider a two-component series system. The joint probability distribution of life of the components is given by :

$$f(x, y) = \lambda^2 \exp^{-\lambda y} \quad \text{for} \quad 0 \leq x < y \quad \text{and} \quad \lambda > 0.$$

   (a) Find the expression for the reliability function of the system.

   (b) Consider the following two data configurations:

      i. Life time of both the components from n such independent systems, that is $\{(x_i, y_i), i = 1, 2, \ldots, n\}$.

      ii. Life time of n such independent systems, that is

$$\{t_i = min(x_i, y_i), i = 1, 2, \ldots, n\}.$$

Find MLE of reliability at time t for both the cases (i) and (ii). Which one of these two estimates is more precise? Justify.

2. (a) Construct a coherent system in which the minimal path sets and the minimal cut sets are identical. Construct two coherent systems so that the minimal path sets of one are the minimal cut sets of the other and vice versa.

   (b) A simplified automatic alarm system for gas leakage functions when at least one of two alarm bells rings. If at least two of the three gas-detectors can detect gas leakage, a signal is sent to a signaling unit which activates the power source which then, through a relay control, triggers off the alarm bells. Write this as a coherent system giving a diagram and the structure function. Find the minimal path sets and the minimal cut sets. Also find a non-trivial modular decomposition of the system.

3. (a) Purchase of every unit of certain product is accompanied by the following (renewing) warranty of length t_0 days. Warranty expires if it survives till t_0 days. Otherwise, seller replaces it on failure instantaneously by a new unit (free-of-charge) whose warranty lenth is again t_0 days. Likewise, this process continues until a replaced unit has survived t_0 days. Thus, failure of a sold unit within its warranty period generates a sequence of free replacements. If the cumulative distribution of unit life is denoted by $F(t)$, find the distribution of number of replacements, and its expectation, corresponding to a sale.

   (b) Consider a device subjected to shocks arriving according to a Poisson process with parameter λ. The effects of the successive shocks on the device are independent and each shock either kills the device with probability p or not. Obtain the reliability of the device at time t.

Multiple Choice

1. Number of roots of the equation

   $x^2 + sin^2 x = 1$, is

   (A) 0 (B) 1 (C) 2 (D) 4.

2. Let $f(x) =\mid sin^3 x \mid$, $g(x) = sin^3 x$, $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$. Then

   (A) $f'(x) = g'(x)$, $\forall x$
   (B) $f'(x) = -g'(x)$, $\forall x$
   (C) $f'(x) =\mid g'(x) \mid$, $\forall x$
   (D) $\mid f'(x) \mid = g'(x)$, $\forall x$

3. If k is a positive integer such that

$$\lim_{n\to\infty}[(cos\frac{k\pi}{4})^n - (cos\frac{k\pi}{6})^n] = 0, \quad \text{then}$$

   (A) k is divisible neither by 4, nor by 6.
   (B) k is divisible by 12, but not necessarily by 24.
   (C) k is divisible by 24
   (D) Either k is divisible by 24; or k is divisible neither by 4, nor by 6.

4. Consider the following system of equations:

$$\begin{aligned} x + 2y + z &= 5 \\ 2x + 3y + 3z &= 12 \\ x + 4y - z &= c, \quad \text{for some real } c \end{aligned}$$

   Then which value of c does not lead to an inconsistent system of equations?

   (A) 1 (B) 0 (C) -1 (D) 2

5. The value of the expression

$$\binom{100}{48} + 2\binom{100}{49} + \binom{100}{50} \quad \text{is}$$

(A) $\binom{100}{52}$ (B) $\binom{101}{51}$ (C) $\binom{102}{48}$ (D) $\binom{102}{50}$

6. The number of ways of placing 5 similar balls in 3 boxes is

(A) 15 (B) 21 (C) 10 (D) None of these

7. The sequence $\{a_n\}, n \geq 1$, satisfies the following equation
$a_n - 2a_{n-1} - 1 = 0$, with $a_0 = 1$. The general expression for a_n is

(A) $2^n - 1$ (B) $2^{n+1} - 1$ (C) $\frac{n(n+1)}{2}$ (D) $\frac{n(n-1)}{2}$

8. The area enclosed by the curve $|\, x \,| + |\, y \,| = 1$ is

(A) 1 (B) 2 (C) $\sqrt{2}$ (D) 4

9. The value of $\int_{\alpha}^{\alpha+1} [x]\, dx$, for real non-integer α, where $[x]$ denotes the largest integer less than or equal to x, is

(A) α (B) $[\alpha]$ (C) 1 (D) $\frac{[\alpha]+[\alpha+1]}{2}$

10. The function $f(x)$ is defined as

$$\begin{aligned} f(x) &= x, \text{ if } x \text{ is rational,} \\ &= 0, \text{ if } x \text{ is irrational} \end{aligned}$$

Then $f(x)$ is

(A) discontinuous only at $x = 0$
(B) discontinuous at every point except at $x = 0$
(C) continuous at every point
(D) discontinuous at every point

11. The value of the integral $\int_{-c}^{c} log(\frac{x+\sqrt{x^2+a^2}}{a})\ dx$, where a and c are positive real numbers, is

(A) $\frac{a}{c}$ (B) $\frac{c}{a}$ (C) $\frac{2c}{a}$ (D) 0

12. Let $a_i > 0$ for $i = 1, 2, \cdots, n$ and $a_1.a_2.\cdots.a_n = 1$. The minimum value of $(1 + a_1)(1 + a_2)\cdots(1 + a_n)$ is

(A) 1 (B) n (C) $n + 1$ (D) None of these

13. The value of $\sum_{k=0}^{\infty} \frac{2}{(k+1)(k+2)(k+3)}$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$

14. The value of $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{2^n-1}$ lies between

(A) 1 and 2 (B) $\frac{n}{2}$ and n (C) 1 and $\frac{n}{2}$ (D) 2 and $\frac{n}{2}$

15. Let $P\Delta Q$ be the set of elements each of which belongs to exactly one of the two sets P and Q. Let P and Q are such that if an element does not belong to Q, it also does not belong to P. Then $P\Delta Q$ is equal to

(A) $P \bigcap Q$ (B) Q (C) $P^c \bigcup Q$ (D) $P^c \bigcap Q$

16. A password consists of three characters of which the first has to be a letter from a to z, and the other two can be letters from a to z or digits from 0 to 9. Letters are all in lowercase. Further, the password can have at most one digit and no repetition of characters. The number of passwords is

(A) $26 \times 20 \times 25$ (B) $26 \times 35 \times 25$ (C) $26 \times 35 \times 34$ (D) $26 \times 44 \times 25$

17. The following data were obtained from the fast food restaurants of a town:
13 served hamburgers, 8 served sandwiches, 10 served pizzas, 5 served hamburgers and sandwiches, 3 served hamburgers and pizzas, 2 served sandwiches and pizzas, 1 served hamburgers, sandwiches and pizzas,

and 5 served items other than these three. The number of fast food restaurants in the town is

(A) 22 (B) 25 (C) 27 (D) 29

18. $A = \begin{pmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{pmatrix}$. The determinant of A is

(A) a^3 (B) abc (C) a^2b (D) None of these

19. Let f be differentiable at $x = a$ and $f(a) \neq 0$. Then

$$\lim_{n\to\infty} \left[\frac{f(a+1/n)}{f(a)}\right]^n =$$

(A) $\frac{f^{'}(a)}{f(a)}$ (B) $e^{\frac{f^{'}(a)}{f(a)}}$ (C) $e^{f^{'}(a)}$ (D) $f^{'}(a)$

20. Consider $f(x) = x^3 - x^2, -1 \leq x < 1$. If x_{min} and x_{max} are the values of x corresponding to the global minimum and the global maximum value of f within the given domain respectively, then (x_{min}, x_{max}) is

(A) $(2/3, 0)$ (B) $(-1, 0)$ (C) $(-1, 2/3)$ (D) $(0, 2/3)$

Short-Answer Type

1. (a) Let

$$\begin{aligned} f(x) &= x^3 cos(\frac{1}{x}), x \neq 0 \\ &= 0, \qquad\qquad x = 0. \end{aligned}$$

Find the value of $f^{'}(0)$ and $f^{''}(0)$.

(b) Find $\int e^{2x} cos 3x \, dx$.

2. Let $x = (x_1, x_2, \cdots, x_n)$ and $y = (y_1, y_2, \cdots, y_n)$ be two vectors. Define the $n \times n$ matrix A with the $(i, j)^{th}$ entry being $a_{ij} = 1 + x_i y_j$. Find $\mid A \mid$ for $n \geq 1$.

3. Consider the equation $x^5 + x = 10$. Show that

   (a) it has only one real root which is between 1 and 2.

   (b) the real root is not rational.