

Test code: ME I/ME II, 2008

Syllabus for ME I, 2008

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Linear Programming: Formulations, statements of Primal and Dual problems, Graphical solutions.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2008

Syllabus for ME II (Economics), 2008

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2008

- (a) Find the labour demand function.
- (b) Equate labour demand with labour supply to eliminate W . You will get an expression involving P , P^e and N . Derive the Lucas supply function in the form given above and find the expressions for λ and Y^* .
- (c) How is this type of model referred to in the literature? Explain

6. Consider an IS – LM model given by the following equations

$$C = 200 + .5 Y_D$$

$$I = 150 - 1000 r$$

$$T = 200$$

$$G = 250$$

$$\left(\frac{M}{P}\right)^d = 2Y - 4000i$$

$$\left(\frac{M}{P}\right)^s = 1600$$

$$i = r - \Pi^e$$

where C is consumption, Y_D is disposable income, I is investment, r is real rate of interest, i is nominal rate of interest, T is tax, G is government expenditure,

$\left(\frac{M}{P}\right)^d$ and $\left(\frac{M}{P}\right)^s$ are real money demand and real money supply respectively and

and Π^e is the expected rate of inflation. The current price level P remains always rigid.

- (a) Assuming that $\Pi^e = 0$, i.e., the price level is expected to remain unchanged in future, determine the equilibrium levels of income and the rates of interest.
- (b) Suppose there is a *temporary* increase in nominal money supply by 2%. Find the new equilibrium income and the rates of interest.
- (c) Now assume that the 2% increase in nominal money supply is *permanent* leading to a 2% increase in the expected future price level. Work out the new equilibrium income and the rates of interest.

7. A firm is contemplating to hire a salesman who would be entrusted with the task of selling a washing machine. The hired salesman is efficient with probability 0.25 and inefficient with probability 0.75 and there is no way to tell, by looking at the salesman, if he is efficient or not. An efficient salesman can sell the washing machine with probability 0.8 and an inefficient salesman can sell the machine with probability 0.4. The firm makes

a profit of Rs. 1000 if the machine is sold and gets nothing if it is not sold. In either case, however, the salesman has to be paid a wage of Rs. 100.

- (a) Calculate the expected profit of the firm.
(b) Suppose instead of a fixed payment, the firm pays a commission of t % on its profit to the salesman (i.e., if the good is sold the salesman gets Rs. $1000 \times \frac{t}{100}$ and nothing if the good remains unsold). A salesman, irrespective of whether he is efficient or inefficient, has an alternative option of working for Rs. 80. A salesman knows whether he is efficient or not and cares only about the expected value of his income: find the value of t that will maximize the expected profit of the firm.

8. (a) On a tropical island there are 100 boat builders, numbered 1 through 100. Each builder can build up to 12 boats a year and each builder maximizes profit given the market price. Let y denote the number of boats built per year by a particular builder, and for each i , from 1 to 100, boat builder has a cost function $C_i(y) = 11 + iy$. Assume that in the cost function the fixed cost, 11, is a quasi-fixed cost, that is, it is only paid if the firm produces a positive level of output. If the price of a boat is 25, how many builders will choose to produce a positive amount of output and how many boats will be built per year in total?

(b) Consider the market for a particular good. There are two types of customers: those of type 1 are the low demand customers, each with a demand function of the form $p = 10 - q_1$, and those of type 2, who are the high demand customers, each with a demand function of the form $p = 2(10 - q_2)$. The firm producing the product is a monopolist in this market and has a cost function $C(q) = 4q^2$ where $q = q_1 + q_2$.

- (i) Suppose the firm is unable to prevent the customers from selling the good to one another, so that the monopolist cannot charge different customers different prices. What prices per unit will the monopolist charge to maximize its total profit and what will be the equilibrium quantities to be supplied to the two groups in equilibrium?
- (ii) Suppose the firm realizes that by asking for IDs it can identify the types of the customers (for instance, type 1's are students who can be identified using their student IDs). It can thus charge different per unit prices to the two groups, if it is optimal to do so. Find the profit maximizing prices to be charged to the two groups.

9. Consider the following box with 16 squares:

| | | | |
|----|----|----|----|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 |

There are two players 1 and 2, and the game begins with player 1 selecting one of the boxes marked 1 to 16. Following such a selection, the selected box, as well as all boxes in the square of which the selected box constitutes the *leftmost and lowest* corner, will be deleted. For example, if he selects box 7, then all the boxes, 3, 4, 7 and 8 are deleted. Similarly, if he selects box 9, then all boxes 1 to 12 are deleted. Next it is player 2's turn to select a box from the remaining boxes. The same deletion rule applies in this case. It is then player 1's turn again, and so on. Whoever deletes the last box loses the game? What is a winning strategy for player 1 in this game? [20]

10. (i) Mr. A's yearly budget for his car is Rs. 100,000, which he spends completely on petrol (P) and on all other expenses for his car (M). All other expenses for car (M) is measured in Rupees, so you can consider that price of M is Re. 1. When price of petrol is Rs. 40 per liter, Mr. A buys 1,000 liters per year. The petrol price rises to Rs. 50 per liter, and to offset the harm to Mr. A, the government gives him a cash transfer of Rs. 10,000 per year.

- Write down Mr. A's yearly budget equation under the 'price rise plus transfer' situation.
- What will happen to his petrol consumption – increase, decrease, or remain the same?
- Will he be better or worse off after the price rise plus transfer than he was before?

[Do not refer to any utility function or indifference curves to answer (b) and (c)]

(iii) Mr. B earns Rs. 500 today and Rs. 500 tomorrow. He can save for future by investing today in bonds that return tomorrow the principal plus the interest. He can also borrow from his bank paying an interest. When the interest rates on both bank loans and bonds are 15% Mr. B chooses neither to save nor to borrow.

- Suppose the interest rate on bank loans goes up to 30% and the interest rate on bonds fall to 5%. Write down the equation of the new budget constraint and draw his budget line.
- Will he lend or borrow? By how much?