POST-GRADUATE DIPLOMA IN STATISTICAL METHODS WITH APPLICATIONS

TEST CODE: MPD (Objective type) 2012

SYLLABUS

Algebra—Arithmetic, Geometric and Power series, sequences. Permutations and combinations. Binomial theorem. Theory of quadratic equations. Inequalities. Elementary set theory. Vectors and matrices. Determinant, rank and inverse of a matrix. Solutions of linear equations. Eigenvalues and eigenvectors of matrices.

Coordinate geometry — Straight lines, circles, parabolas, ellipses and hyperbolas. Elements of three dimensional coordinate geometry.

Calculus—Taylor and Maclaurin series. Limits and continuity of functions of one real variable. Differentiation and integration of functions of one real variable with applications. Definite integrals. Areas using integrals. Maxima and minima and their applications. Ordinary linear differential equations.

SAMPLE QUESTIONS

<u>Note:</u> *For each question there are four suggested answers of which only one is correct.*

(1) If a, b are positive real variables whose sum is constant λ, then the minimum value of $\sqrt{(1+1/a)(1+1/b)}$ is

(A) $\lambda - 1/\lambda$

(B) $\lambda + 2/\lambda$

(C) $\lambda + 1/\lambda$

(D) none of the above.

(2) Suppose in a competition 11 matches are to be played, each having one of 3 distinct outcomes as possibilities. The number of ways one can predict the outcomes of all 11 matches such that exactly 6 of the predictions turn out to be correct is

(A) $\binom{11}{6} \times 2^5$ (B) $\binom{11}{6}$ (C) 3^6 (D) none of these.

(3) Let A be a set of n elements. The number of ways we can choose an ordered pair (B, C), where B, C are disjoint subsets of A, equals

(A) n^2 (B) n^3 (C) 2^n (D) 3^n.

(4) Let $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, n being a positive integer. The value of

$$\left(1+\frac{C_0}{C_1}\right)\left(1+\frac{C_1}{C_2}\right)\cdots\left(1+\frac{C_{n-1}}{C_n}\right)$$

is

(A) $\left(\frac{n+1}{n+2}\right)^n$ (B) $\frac{n^n}{n!}$ (C) $\left(\frac{n}{n+1}\right)^n$ (D) $\frac{(n+1)^n}{n!}$.

(5) The number of positive integers which are less than or equal to 1000 and are divisible by none of 17, 19 and 23 equals

(A) 854 (B) 153 (C) 160 (D) none of these.

(6) Let $a_n = \left(1-\frac{1}{\sqrt{2}}\right)\cdots\left(1-\frac{1}{\sqrt{n+1}}\right)$, $n \geq 1$. Then $\lim_{n\to\infty} a_n$

(A) equals 1 (B) does not exist (C) equals $\dfrac{1}{\sqrt{\pi}}$ (D) equals 0.

(7) $\displaystyle\lim_{x\to\infty}\left(\frac{3x-1}{3x+1}\right)^{4x}$ equals

(A) 1 (B) 0 (C) $e^{-8/3}$ (D) $e^{4/9}$.

(8) $\displaystyle\lim_{n\to\infty}\frac{1}{n}\left(\frac{n}{n+1}+\frac{n}{n+2}+\cdots+\frac{n}{2n}\right)$ is equal to

(A) ∞ (B) 0 (C) $\log_e 2$ (D) 1.

(9) The set $\{x : \left|x+\frac{1}{x}\right| > 6\}$ equals the set

(A) $(0, 3-2\sqrt{2}) \cup (3+2\sqrt{2}, \infty)$
(B) $(-\infty, -3-2\sqrt{2}) \cup (-3+2\sqrt{2}, \infty)$
(C) $(-\infty, 3-2\sqrt{2}) \cup (3+2\sqrt{2}, \infty)$
(D) $(-\infty, -3-2\sqrt{2}) \cup (-3+2\sqrt{2}, 3-2\sqrt{2}) \cup (3+2\sqrt{2}, \infty)$.

(10) Consider the sets defined by the real solutions of the inequalities

$$A = \{(x,y) : x^2 + y^4 \leq 1\} \qquad B = \{(x,y) : x^4 + y^6 \leq 1\}.$$

Then

(A) $B \subseteq A$
(B) $A \subseteq B$
(C) Each of the sets $A-B$, $B-A$ and $A \cap B$ is non-empty

(D) none of the above.

(11) If $f(x)$ is a real valued function such that

$$2f(x) + 3f(-x) = 15 - 4x,$$

for every $x \in \mathbb{R}$, then $f(2)$ is

(A) -15 (B) 22 (C) 11 (D) 0.

(12) If $f(x) = \dfrac{\sqrt{3}\sin x}{2 + \cos x}$, then the range of $f(x)$ is

(A) the interval $[-1, \sqrt{3}/2]$ (B) the interval $[-\sqrt{3}/2, 1]$

(C) the interval $[-1, 1]$ (D) none of these.

(13) The eigenvalues of the matrix $X = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ are

(A) 1, 1, 4 (B) 1, 4, 4 (C) 0, 1, 4 (D) 0, 4, 4.

(14) If M is a 3×3 matrix such that

$$[0 \quad 1 \quad 2]M = [1 \quad 0 \quad 0] \quad \text{and} \quad [3 \quad 4 \quad 5]M = [0 \quad 1 \quad 0]$$

then $[6 \quad 7 \quad 8]M$ is equal to

(A) $[2 \quad 1 \quad -2]$ (B) $[0 \quad 0 \quad 1]$ (C) $[-1 \quad 2 \quad 0]$ (D) $[9 \quad 10 \quad 8]$.

(15) The values of η for which the following system of equations

$$\begin{array}{ccccccc} x & + & y & + & z & = & 1 \\ x & + & 2y & + & 4z & = & \eta \\ x & + & 4y & + & 10z & = & \eta^2 \end{array}$$

has a solution are

(A) $\eta = 1, -2$ (B) $\eta = -1, -2$ (C) $\eta = 3, -3$ (D) $\eta = 1, 2$.

(16) Suppose the circle with equation $x^2 + y^2 + 2fx + 2gy + c = 0$ cuts the parabola $y^2 = 4ax$, $(a > 0)$ at four distinct points. If d denotes the sum of ordinates of these four points, then the set of possible values of d is

(A) $\{0\}$ (B) $(-4a, 4a)$ (C) $(-a, a)$ (D) $(-\infty, \infty)$.

(17) If a sphere of radius r passes through the origin and cuts the three co-ordinate axes at points A, B, C respectively, then the centroid of the triangle ABC lies on a sphere of radius

(A) r (B) $\frac{r}{\sqrt{3}}$ (C) $\sqrt{\frac{2}{3}}\, r$ (D) $\frac{2r}{3}$.

(18) If $0 < x < 1$, then the sum of the infinite series $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \cdots$ is

(A) $\log \frac{1+x}{1-x}$ (B) $\frac{x}{1-x} + \log(1+x)$
(C) $\frac{1}{1-x} + \log(1-x)$ (D) $\frac{x}{1-x} + \log(1-x)$.

(19) In the Taylor series expansion of the function $f(x) = e^{x/2}$ about $x = 3$, the coefficient of $(x-3)^5$ is

(A) $e^{3/2}\frac{1}{5!}$ (B) $e^{3/2}\frac{1}{2^5 5!}$ (C) $e^{-3/2}\frac{1}{2^5 5!}$ (D) none of these.

(20) The number of divisors of 6000, where 1 and 6000 are also considered as divisors of 6000 is

(A) 40 (B) 50 (C) 60 (D) 30.

(21) Let x_1 and x_2 be the roots of the quadratic equation $x^2 - 3x + a = 0$, and x_3 and x_4 be the roots of the quadratic equation $x^2 - 12x + b = 0$. If x_1, x_2, x_3 and x_4 $(0 < x_1 < x_2 < x_3 < x_4)$ are in G.P., then ab equals

(A) 64 (B) 5184 (C) -64 (D) -5184.

(22) The integral

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{50} x}{\sin^{50} x + \cos^{50} x}\, dx$$

equals

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) none of these.

(23) Let the function $f(x)$ be defined as $f(x) = |x-1| + |x-2|$. Then which of the following statements is true?

(A) $f(x)$ is differentiable at $x = 1$

(B) $f(x)$ is differentiable at $x = 2$

(C) $f(x)$ is differentiable at $x = 1$ but not at $x = 2$

(D) none of the above.

(24) $x^4 - 3x^2 + 2x^2y^2 - 3y^2 + y^4 + 2 = 0$ represents

(A) A pair of circles having the same radius

(B) A circle and an ellipse

(C) A pair of circles having different radii

(D) none of the above.

(25) If $|a| < 1$ and $|b| < 1$, then the series

$a(a+b) + a^2(a^2+b^2) + a^3(a^3+b^3) + \ldots$

converges to

(A) $\frac{a^2}{1-a^2} + \frac{b^2}{1-b^2}$ (B) $\frac{a^2}{1-a^2} + \frac{ab}{1-ab}$ (C) $\frac{a(a+b)}{1-a(a+b)}$ (D) $\frac{a^2}{1-a^2} - \frac{ab}{1-ab}$.

(26) Let $\mathbb{N} = \{1, 2, 3, \ldots\}$ be the set of natural numbers. For each $n \in \mathbb{N}$, define $A_n = \{(n+1)k, k \in \mathbb{N}\}$. Then $A_1 \cap A_2$ equals

(A) A_3 (B) A_4 (C) A_5 (D) A_6.

(27) The sum of the series

$$\frac{1}{1.2} + \frac{1}{2.3} + \ldots + \frac{1}{n(n+1)} + \ldots\ldots$$

is

(A) 1 (B) 1/2 (C) 0 (D) non-existent.

(28) Let $f(x)$ and $g(x)$ be two functions defined on the same domain D such that $f(x) \geq 0$ and $g(x) \leq 0$ for all $x \in D$. Further, $f(x)$ is strictly decreasing and $g(x)$ is strictly increasing. Then what can you say about the monotonic property of the product function $h(x)$, where $h(x)$ is defined as $h(x) = f(x)g(x)$?

(A) non-increasing (B) non-decreasing

(C) strictly increasing (D) strictly decreasing.

(29) $\lim_{x \to 2} \frac{1}{1 + e^{\frac{1}{x-2}}}$

is

(A) 0 (B) 1/2 (C) 1 (D) non-existent.

(30) ${}^nC_0 + 2\ {}^nC_1 + 3\ {}^nC_2 + \ldots + (n+1)\ {}^nC_n$

equals

(A) $2^n + n2^{n-1}$ (B) $2^n - n2^{n-1}$ (C) 2^n (D) none of these.