

Test Code: CS (Short answer type) 2005

M.Tech. in Computer Science

The candidates for M.Tech. in Computer Science will have to take two tests – Test MIII (objective type) in the forenoon session and Test CS (short answer type) in the afternoon session. The CS test booklet will have two groups as follows.

GROUP A

A test for all candidates in analytical ability and mathematics at the B.Sc. (pass) level, carrying 30 marks.

GROUP B

A test, divided into several sections, carrying equal marks of 70 in mathematics, statistics, and physics at the B. Sc. (Hons.) level and in computer science, and engineering and technology at the B.Tech. level. A candidate has to answer questions from *only* one of these sections according to his/her choice.

The syllabus and sample questions of the CS test are given below.

Note: All questions in the sample set are not of equal difficulty. They may not carry equal marks in the test.

Syllabus

GROUP A

Logical Reasoning.

Elements of set theory. Permutations and combinations. Functions and relations. Theory of equations. Inequalities.

Limit, continuity, sequences and series, differentiation and integration with applications, maxima-minima, elements of ordinary differential equations, complex numbers and De Moivre's theorem.

Elementary Euclidean geometry and Trigonometry.

Elementary number theory, divisibility, congruences, primality.

Determinants, matrices, solutions of linear equations, vector spaces, linear independence, dimension, rank and inverse.

GROUP B

Mathematics (B.Sc. Hons. level)

In addition to the syllabus of Mathematics in Group A, the syllabus includes:

Calculus and real analysis – Real numbers, basic properties; convergence of sequences and series; limits, continuity, uniform continuity of functions; differentiability of functions of one or more variables and applications. Indefinite integral, fundamental theorem of Calculus, Riemann integration, improper integrals, double and multiple integrals and applications. Sequences and series of functions, uniform convergence.

Linear algebra - Vector spaces and linear transformations; matrices and systems of linear equations, characteristic roots and characteristic vectors, Cayley-Hamilton theorem, canonical forms, quadratic forms.

Graph Theory - Connectedness, trees, vertex colouring, planar graphs, Eulerian graphs, Hamiltonian graphs, digraphs and tournaments.

Abstract algebra – Groups, subgroups, cosets, Lagrange's theorem; normal subgroups and quotient groups; permutation groups; rings, subrings, ideals, integral domains, fields, characteristics of a field, polynomial rings, unique factorization domains, field extensions, finite fields.

Differential equations – Solutions of ordinary and partial differential equations and applications.

Linear programming including duality theory.

Statistics (B.Sc. Hons. level)

Notions of sample space and probability, combinatorial probability, conditional probability, Bayes theorem and independence, random variable and expectation, moments, standard univariate discrete and continuous distributions, sampling distribution of statistics based on normal samples, central limit theorem, approximation of binomial to normal. Poisson law, Multinomial, bivariate normal and multivariate normal distributions.

Descriptive statistical measures, product-moment correlation, partial and multiple correlation; regression (simple and multiple); elementary theory and methods of estimation (unbiasedness, minimum variance, sufficiency, maximum likelihood method, method of moments, least squares methods). Tests of hypotheses (basic concepts and simple applications of Neyman-Pearson Lemma). Confidence intervals. Tests of regression. Elements of non-parametric inference. Contingency Chi-square, ANOVA, basic designs (CRD/RBD/LSD) and their analyses. Elements of factorial designs. Conventional sampling techniques, ratio and regression methods of estimation.

Physics
(B.Sc. Hons. level)

Kinetic theory of gases. Laws of thermodynamics. Heat engines. Lagrangian and Hamiltonian formulation of classical mechanics. Simple harmonic motion. Conservation laws. Atomic physics and basic idea of nuclear physics. Non-relativistic quantum mechanics. Special theory of relativity. Semiconductor physics – transport phenomenon of electrons and holes, p-n junctions, transistors and diodes, oscillators, amplifiers, Fundamentals of electric circuits – LR, RC, LCR, Boolean algebra and logic circuits. Electricity and magnetism – Coulomb's Law, Gauss' theorem, Biot-Savart law, Ampere's law. Electro-magnetic induction – self and mutual induction. Electro-magnetic theory of light – reflection and refraction.

Computer Science
(B.Tech. level)

Data structures - stack, queue, linked list, binary tree, heap, AVL tree, B tree, design of algorithms, internal sorting, searching, programming fundamentals, switching theory and logic design, computer organization and architecture, operating systems, principles of compiler construction, formal languages and automata theory, database systems, computer networks.

Engineering and Technology
(B.Tech. level)

Moments of inertia, motion of a particle in two dimensions, elasticity, friction, strength of materials, surface tension, viscosity, gravitation.

Geometrical optics.

Laws of thermodynamics, and heat engines.

Electrostatics, magnetostatics, electromagnetic induction.

Magnetic properties of matter - dia, para and ferromagnetism.

Laws of electrical circuits - RC, RL and RLC circuits, measurement of currents, voltages and resistance.

D.C. generators, D.C. motors, induction motors, alternators, transformers.

p-n junction, bipolar & FET devices, transistor amplifiers, oscillator, multi-vibrators, operational amplifiers.

Digital circuits - Logic gates, multiplexers, de-multiplexers, counters, A/D and D/A converters.

Boolean algebra, minimization of switching functions, combinational and sequential circuits.

Sample Questions

GROUP A

Mathematics

A1. If $1, a_1, a_2, \dots, a_{n-1}$ are the n roots of unity, find the value of $(1 - a_1)(1 - a_2)\dots(1 - a_{n-1})$.

A2. Let $A = (a_{ij})$ be an $n \times n$ matrix, where

$$a_{ij} = \begin{cases} b & \text{if } i = j, \\ c & \text{if } i \neq j, \end{cases}$$

and b, c are real numbers such that $b \neq c$. When is the matrix A invertible?

A3. (a) Let

$$S = \{(a_1, a_2, a_3, a_4) : a_i \in \mathfrak{R}, i=1,2,3,4 \text{ and } a_1 + a_2 + a_3 + a_4 = 0\}$$

and

$$\Gamma = \{(a_1, a_2, a_3, a_4) : a_i \in \mathfrak{R}, i=1,2,3,4 \text{ and } a_1 - a_2 + a_3 - a_4 = 0\}.$$

Find a basis for $S \cap \Gamma$.

(b) Provide the inverse of the following matrix:

$$\begin{pmatrix} c_0 & c_1 & c_2 & c_3 \\ c_2 & c_3 & c_0 & c_1 \\ c_3 & -c_2 & c_1 & -c_0 \\ c_1 & -c_0 & c_3 & -c_2 \end{pmatrix}$$

$$\text{where } c_0 = \frac{1 + \sqrt{3}}{4\sqrt{2}}, c_1 = \frac{3 + \sqrt{3}}{4\sqrt{2}}, c_2 = \frac{3 - \sqrt{3}}{4\sqrt{2}}, \text{ and } c_3 = \frac{1 - \sqrt{3}}{4\sqrt{2}}.$$

(Hint: What is $c_0^2 + c_1^2 + c_2^2 + c_3^2$?)

A4. For any real number x and for any positive integer n show that

$$\left[x \right] + \left[x + \frac{1}{n} \right] + \left[x + \frac{2}{n} \right] + \dots + \left[x + \frac{n-1}{n} \right] = [nx]$$

where $[a]$ denotes the largest integer less than or equal to a .

A5. Let $b_q b_{q-1} \dots b_1 b_0$ be the binary representation of an integer b , i.e.,

$$b = \sum_{j=0}^q 2^j b_j, \quad b_j = 0 \text{ or } 1, \text{ for } j = 0, 1, \dots, q.$$

Show that b is divisible by 3 if $b_0 - b_1 + b_2 - \dots + (-1)^q b_q = 0$.

A6. A sequence $\{x_n\}$ is defined by $x_1 = \sqrt{2}$, $x_{n+1} = \sqrt{2+x_n}$, $n=1,2, \dots$
Show that the sequence converges and find its limit.

A7. Is $\sin(x|x|)$ differentiable for all real x ? Justify your answer.

A8. Find the total number of English words (all of which may not have proper English meaning) of length 10, where all ten letters in a word are not distinct.

A9. Let $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \dots + \frac{a_n}{n+1} = 0$, where a_i 's are some real constants. Prove that the equation $a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n = 0$ has at least one solution in the interval $(0, 1)$.

A10. Let $\phi(n)$ be the number of positive integers less than n and having no common factor with n . For example, for $n = 8$, the numbers 1, 3, 5, 7 have no common factors with 8, and hence $\phi(8) = 4$. Show that

(i) $\phi(p) = p - 1$,

(ii) $\phi(pq) = \phi(p)\phi(q)$, where p and q are prime numbers.

A11. A set S contains integers 1 and 2. S also contains all integers of the form $3x + y$ where x and y are distinct elements of S , and every element of S other than 1 and 2 can be obtained as above. What is S ? Justify your answer.

A12. Let f be a real-valued function such that $f(x+y) = f(x) + f(y) \forall x, y \in \mathbf{R}$. Define a function ϕ by $\phi(x) = c + f(x)$, $x \in \mathbf{R}$, where c is a real constant. Show that for every positive integer n ,

$$\phi^n(x) = (c + f(c) + f^2(c) + \dots + f^{n-1}(c)) + f^n(x);$$

where, for a real-valued function g , $g^n(x)$ is defined by

$$g^0(x) = 0, g^1(x) = g(x), g^{k+1}(x) = g(g^k(x)).$$

- A13. Consider a square grazing field with each side of length 8 metres. There is a pillar at the centre of the field (i.e. at the intersection of the two diagonals). A cow is tied with the pillar using a rope of length $\frac{8}{\sqrt{3}}$ metres. Find the area of the part of the field that the cow is allowed to graze.
- A14. There are four geometrical objects in the form of square, rhombus, circle and triangle. Each one is made from one of the 4 different materials gold, copper, silver, and bronze and coloured differently using blue, red, green and yellow paints. The square is of green colour. The blue object is made of bronze. The circle is not red. The triangle is not made of gold. The square is not made of copper. The rhombus is not blue and is not made of silver. The circle is not made of bronze. The triangle is not yellow. The red object is not made of copper. Deduce logically the colour and material of the circle.
- A15. A milkmaid has a 4-litre can full of milk and two other empty cans of 2.5 litre and 1.5 litre sizes respectively. She is to divide the milk equally in two cans. Find out the procedure to do this in a minimum number of operations.

GROUP B

Mathematics

- M1. Let $0 < x_1 < 1$. If $x_{n+1} = \frac{x_n+3}{3x_n+1}$, $n = 1, 2, 3, \dots$
- (i) Show that $x_{n+2} = \frac{5x_n+3}{3x_n+5}$, $n = 1, 2, 3, \dots$
 - (ii) Hence or otherwise, show that $\lim_{n \rightarrow \infty} x_n$ exists.
 - (iii) Find $\lim_{n \rightarrow \infty} x_n$.
- M2. (a) A function f is defined over the real line as follows:

$$f(x) = \begin{cases} x \sin \frac{\pi}{x}, & x > 0 \\ 0, & x = 0. \end{cases}$$

Show that $f'(x)$ vanishes at infinitely many points in $(0,1)$.

(b) Let $f : [0,1] \rightarrow \mathfrak{R}$ be a continuous function with $f(0) = 0$. Assume that f' is finite and increasing on $(0,1)$.

Let $g(x) = \frac{f(x)}{x}$ $x \in (0,1)$. Show that g is increasing.

M3. (a) Prove the inequality $e^x > 1 + (1+x) \log(1+x)$, for $x > 0$.

(b) Show that the series $\sum \frac{x}{n(1+nx)^2}$ is uniformly convergent on $[0,1]$.

M4. Consider the function of two variables

$$F(x,y) = 21x - 12x^2 - 2y^2 + x^3 + xy^2.$$

(a) Find the points of local minima of F .

(b) Show that F does not have a global minimum.

M5. Find the volume of the solid given by $0 \leq y \leq 2x$, $x^2 + y^2 \leq 4$ and $0 \leq z \leq x$.

M6. (a) Let A , B and C be $1 \times n$, $n \times n$ and $n \times 1$ matrices respectively. Prove or disprove: $\text{Rank}(ABC) \leq \text{Rank}(AC)$.

(b) Let S be the subspace of \mathbf{R}^4 defined by $S = \{(a_1, a_2, a_3, a_4) : 5a_1 - 2a_3 - 3a_4 = 0\}$. Find a basis for S .

M7. Let A be a 3×3 matrix with characteristic equation $\lambda^3 - 5\lambda^2 = 0$.

(i) Show that the rank of A is either 1 or 2.

(ii) Provide examples of two matrices A_1 and A_2 such that the rank of A_1 is 1, rank of A_2 is 2 and A_i has characteristic equation $\lambda^3 - 5\lambda^2 = 0$ for $i = 1, 2$.

M8. Define B to be a multi-subset of a set A if every element of B is an element of A and elements of B need not be distinct. The ordering of elements in B is not important.

For example, if $A = \{1,2,3,4,5\}$ and $B = \{1,1,3\}$, B is a 3-element multi-subset of A . Also, multi-subset $\{1,1,3\}$ is the same as the multi-subset $\{1,3,1\}$.

- (a) How many 5-element multi-subsets of a 10-element set are possible?
- (b) Generalize your result to m -element multi-subsets of an n -element set ($m < n$).

M9. Let G be the group of all 2×2 non-singular matrices with matrix multiplication as the binary operation. Provide an example of a normal subgroup H of G such that $H \neq G$ and H is not a singleton.

M10. Let \mathbf{R} be the field of reals. Let $\mathbf{R}[x]$ be the ring of polynomials over \mathbf{R} , with the usual operations.

- (a) Let $I \subseteq \mathbf{R}[x]$ be the set of polynomials of the form $a_0 + a_1x + \dots + a_nx^n$ with $a_0 = a_1 = 0$. Show that I is an ideal.
- (b) Let \mathbf{P} be the set of polynomials over \mathbf{R} of degree ≤ 1 . Define \oplus and \ominus on \mathbf{P} by $(a_0 + a_1x) \oplus (b_0 + b_1x) = (a_0 + b_0) + (a_1 + b_1)x$ and $(a_0 + a_1x) \ominus (b_0 + b_1x) = a_0b_0 + (a_1b_0 + a_0b_1)x$. Show that $(\mathbf{P}, \oplus, \ominus)$ is a commutative ring. Is it an integral domain? Justify your answer.

M11. (a) If G is a group of order 24 and H is a subgroup of G of order 12, prove that H is a normal subgroup of G .

(b) Show that a field of order 81 cannot have a subfield of order 27.

M12. (a) Consider the differential equation:

$$\frac{d^2y}{dx^2} \cos x + \frac{dy}{dx} \sin x - 2y \cos^3 x = 2 \cos^5 x.$$

By a suitable transformation, reduce this equation to a second order linear differential equation with constant coefficients. Hence or otherwise solve the equation.

- (b) Find the surfaces whose tangent planes all pass through the origin.

M13. (a) Consider the following two linear programming problems:

P₁: Minimize x_1 subject to

$$x_1 + x_2 \geq 1$$

$$-x_1 - x_2 \geq 1$$

where both x_1 and x_2 are unrestricted.

P₂: Minimize x_1 subject to

$$x_1 + x_2 \geq 1$$

$$-x_1 - x_2 \geq 1$$

$$x_1 \geq 0, x_2 \geq 0.$$

Solve both the LPs. Write the duals of both the LPs and solve the duals.

(b) If an LP is infeasible, what can you say about the solution of its dual?

M14. Solve the following linear programming problem without using Simplex method:

$$\text{minimize} \quad 6w_1 + 8w_2 + 7w_3 + 15w_4 + w_5$$

$$\text{subject to} \quad w_1 + w_3 + 3w_4 \geq 4,$$

$$w_2 + w_3 + w_4 - w_5 \geq 3,$$

$$w_1, w_2, w_3, w_4, w_5 \geq 0.$$

M15. (a) Show that a tree on n vertices has at most $n-2$ vertices with degree > 1 .

(b) Show that in an Eulerian graph on 6 vertices, a subset of 5 vertices cannot form a complete subgraph.

M16. (a) Show that the edges of K_4 can be partitioned into 2 edge-disjoint spanning trees.

(b) Use (a) to show that the edges of K_6 can be partitioned into 3 edge-disjoint spanning trees.

(c) Let K_n denote the complete undirected graph with n vertices and let n be an even number. Prove that the edges of K_n can be partitioned into exactly $n/2$ edge-disjoint spanning trees.

Statistics

- S1. (a) X and Y are two independent and identically distributed random variables with $\text{Prob}[X = i] = p_i$, for $i = 0, 1, 2, \dots$. Find $\text{Prob}[X < Y]$ in terms of the p_i values.
- (b) Based on one random observation X from $N(0, \sigma^2)$, show that $\sqrt{\pi/2} |X|$ is an unbiased estimate of σ .
- S2. (a) Let X_0, X_1, X_2, \dots be independent and identically distributed random variables with common probability density function f . A random variable N is defined as
$$N = n \text{ if } X_1 \leq X_0, X_2 \leq X_0, \dots, X_{n-1} \leq X_0, X_n > X_0, n = 1, 2, 3, \dots$$
Find the probability of $N = n$.
- (b) Let X and Y be independent random variables distributed uniformly over the interval $[0, 1]$. What is the probability that the integer closest to $\frac{X}{Y}$ is 2?
- S3. If a die is rolled m times and you had to bet on a particular number of sixes occurring, which number would you choose? Is there always one best bet, or could there be more than one?
- S4. Let X_1, X_2 and X_3 be independent random variables with X_i following a uniform distribution over $(0, i\theta)$, for $i = 1, 2, 3$. Find the maximum likelihood estimate of θ based on observations x_1, x_2, x_3 on X_1, X_2, X_3 respectively. Is it unbiased? Find the variance of the estimate.
- S5. New laser altimeters can measure elevation to within a few inches, without bias. As a part of an experiment, 25 readings were made on the elevation of a mountain peak. These averaged out to be 73,631 inches with a standard deviation (SD) of 10 inches. Examine each of the following statements and ascertain whether the statement is true or false, giving reasons for your answer.
- (a) 73631 ± 4 inches is a 95% confidence interval for the elevation of the mountain peak.
- (b) About 95% of the readings are in the range 73631 ± 4 inches.

(c) There is about 95% chance that the next reading will be in the range of 73631 ± 4 inches.

S6. Consider a randomized block design with two blocks and two treatments A and B . The following table gives the yields:

	Treatment A	Treatment B
Block 1	a	b
Block 2	c	d

- How many orthogonal contrasts are possible with a, b, c and d ? Write down all of them.
- Identify the contrasts representing block effects, treatment effects and error.
- Show that their sum of squares equals the total sum of squares.

S7. Let X be a discrete random variable having the probability mass function

$$p(x) = \Lambda^x(1 - \Lambda)^{1-x}, x = 0, 1,$$

where Λ takes values ≥ 0.5 only. Find the most powerful test, based on 2 observations, for testing $H_0 : \Lambda = \frac{1}{2}$ against $H_1 : \Lambda = \frac{2}{3}$, with level of significance 0.05.

S8. Let X_1, X_2, \dots, X_n be n independent $N(\theta, 1)$ random variables where $-1 \leq \theta \leq 1$. Find the maximum likelihood estimate of θ and show that it has smaller mean square error than the sample mean \bar{X} .

S9. Let t_1, t_2, \dots, t_k be k independent and unbiased estimators of the same parameter θ with variances $\sigma_1^2, \sigma_2^2, \dots, \sigma_k^2$. Define \bar{t} as $\sum_{i=1}^k \frac{t_i}{k}$. Find

$E(\bar{t})$ and the variance of \bar{t} . Show that $\sum_{i=1}^k (t_i - \bar{t})^2 / \{k(k-1)\}$ is an

unbiased estimator of $\text{var}(\bar{t})$.

S10. Consider a simple random sample of n units, drawn without replacement from a population of N units. Suppose the value of Y_1 is unusually low whereas that of Y_n is very high. Consider the following estimator of \bar{Y} , the population mean.

$$\hat{Y} = \begin{cases} \bar{y} + c, & \text{if the sample contains unit 1 but not unit } N; \\ \bar{y} - c, & \text{if the sample contains unit } N \text{ but not unit 1;} \\ \bar{y}, & \text{for all other samples;} \end{cases}$$

where \bar{y} is sample mean and c is a constant. Show that \hat{Y} is unbiased. Given that

$$V(\hat{Y}) = (1-f) \left[\frac{S^2}{n} - \frac{2c}{N-1} (Y_N - Y_1 - nc) \right]$$

where $f = \frac{n}{N}$ and $S^2 = \frac{1}{N-1} \sum_{i=1}^N (Y_i - \bar{Y})^2$, comment on the choice of c .

S11. In order to compare the effects of four treatments A, B, C, D , a block design with 2 blocks each having 3 plots was used. Treatments A, B, C were given randomly to the plots of one block and treatments A, B, D were given randomly to the plots of the other block. Write down a set of 3 orthogonal contrasts with the 4 treatment effects and show that all of them are estimable from the above design.

S12. Let y_1, y_2 and y_3 be independent and identically distributed random variables with distribution $N(\mu, 1)$. Find a_1, a_2 and b_1, b_2, b_3 , such that $U_1 = a_1 y_1 + a_2 y_2$ and $U_2 = b_1 y_1 + b_2 y_2 + b_3 y_3$ are independent $N(0, 1)$. Hence express

$y_1^2 + y_2^2 + y_3^2 - \frac{(y_1^2 + y_2^2 + y_3^2)^2}{3}$ in terms of U_1 and U_2 and show that $y_1^2 + y_2^2 + y_3^2 - \frac{(y_1^2 + y_2^2 + y_3^2)^2}{3}$ follows the χ^2 distribution with two degrees of freedom.

S13. In a factory, the distribution of workers according to age-group and sex is given below.

Sex ↓	Age-group		Row total
	20-40 yrs.	40-60 yrs.	
Male	60	40	100
Female	40	10	50
Column Total	100	50	150

Give a scheme of drawing a random sample of size 5 so that both the sexes and both the age-groups are represented. Compute the first-order inclusion probabilities for your scheme.

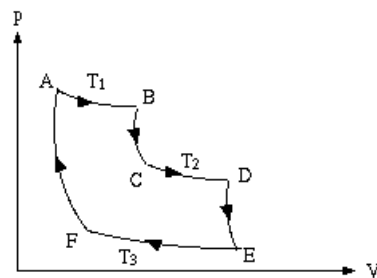
Physics

P1. A beam of X-rays of frequency ν falls upon a metal and gives rise to photoelectrons. These electrons in a magnetic field of intensity H describe a circle of radius γ . Show that

$$h(\nu - \nu_0) = m_0 c^2 \left[\left(\frac{1 + e^2 \gamma^2 H^2}{m_0^2 c^4} \right)^{\frac{1}{2}} - 1 \right]$$

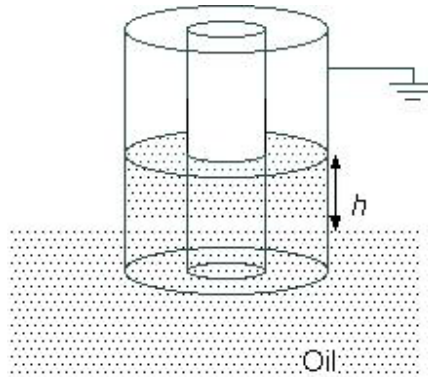
where ν_0 is the frequency at the absorption limit and m_0 is the rest mass of the electron, e being expressed in e.s.u.

P2. An ideal gas goes through a cycle consisting of alternate isothermal and adiabatic curves as shown in the figure.

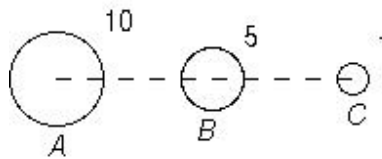


AB , CD , and EF are isothermal curves at temperatures T_1 , T_2 and T_3 respectively, while BC , DE , and FA are adiabatic curves. Find the efficiency of such a cycle, if in each isothermal expansion the gas volume increases by the same factor.

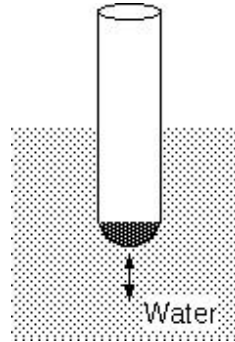
- P3. Two long coaxial cylindrical metal tubes of length L (the inner one of radius a , the outer one of radius b) stand vertically in a tank of dielectric oil having susceptibility χ_e and mass density ρ . The inner tube is maintained at potential V and the outer cylinder is grounded. To what height h above the oil level outside the two cylinders will the oil rise in the space between the tubes?



- P4. (a) Compute the rate at which our Sun is losing its mass, given that the mean radius R of Earth's orbit is 1.5×10^8 Km and the intensity of solar radiation at the Earth is $4/\pi \times 10^3$ Watt/m². If the present mass of our Sun is 2×10^{30} Kg, how long is it expected to last?
- (b) Find the proper length of a rod in the laboratory frame of reference if its velocity is $v = c/2$, its length is $l = 1$ metre, and the angle between the rod and its direction of motion is 45 deg.
- P5. (a) Three rigid spheres A , B , and C of masses 10, 5, and 1 unit respectively are arranged in a row as shown in the figure (the centers of the spheres are collinear). Initially, all spheres are at rest. Next, A moves towards B with some velocity to collide with B . As a result, B moves towards C and collides with C .



- (i) If all the collisions are direct (head-on) and perfectly elastic, what is the ratio of the final velocities of A , B , and C ?
- (ii) If the collisions are assumed to be perfectly inelastic, what is the percentage of energy absorbed in the whole inelastic process?

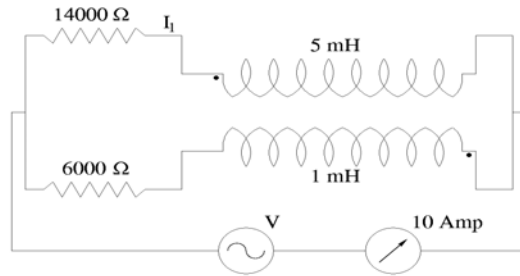


- (b) A test tube of mass 4 gm and diameter 1.5 cm floats vertically in a large tub of water. It is further depressed vertically by 2 cm from its equilibrium position and suddenly released, whereby the tube is seen to execute a damped, oscillatory motion in the vertical direction. If the resistive force due to viscous damping offered by water to the tube in motion is $\sqrt{\pi} Dv$, where v is the instantaneous velocity of the tube in water, and D is the diameter of the tube in cm, then find the time period of oscillation of the tube. (Assume that there is no ripple generated in the water of the tub.)

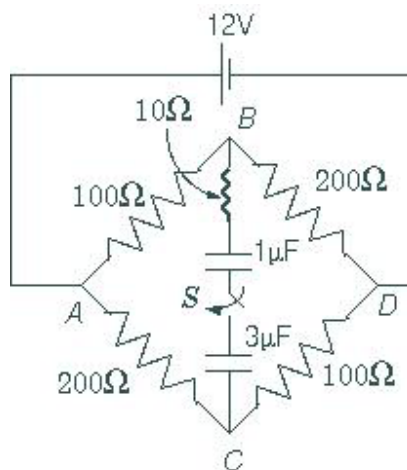
P6. An electron is confined to move within a linear interval of length L . Assuming the potential to be zero throughout the interval except for the two end points, where the potential is infinite, find the following:

- (a) probability of finding the electron in the region $0 < x < L/4$, when it is in the lowest (ground) state of energy;
- (b) taking the mass of the electron m_e to be 9×10^{-31} Kg, Planck's constant h to be 6.6×10^{-34} Joule-sec and $L = 1.1$ cm, determine the electron's quantum number when it is in the state having an energy equal to 5×10^{-32} Joule.

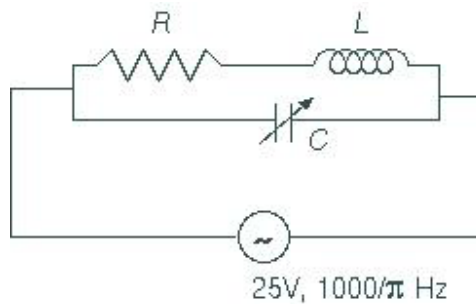
- P7. Consider the following circuit in which an a.c. source of V volts at a frequency of $10^6/\pi$ cycles/sec is applied across the combination of resistances and inductances. The total rms current flowing through the circuit as measured by an a.c. ammeter is 10 amp. Find the rms current I_1 flowing through the upper branch of impedances. The self inductance of the two coils are as shown in the figure. The mutual inductance between the coils is 2 mH and is such that the magnetization of the two coils are in opposition.



- P8. (a) Consider the circuit shown below.



- (i) Will there be any current flowing through the arm BC when the switch S is closed?
- (ii) Calculate the steady state voltage across each of the two capacitors in the figure.

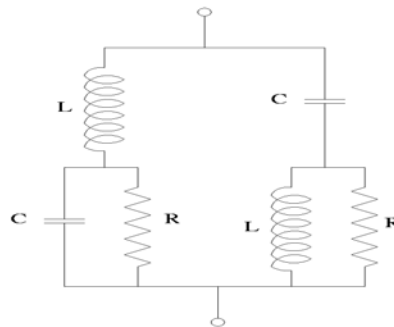


- (b) A coil of resistance 30Ω and inductance 20 mH is connected in parallel with a variable capacitor across an a.c. supply of 25V amplitude, and frequency $1000/\pi\text{ Hz}$. The capacitance of the capacitor is varied until the current taken from the supply is a minimum. For this condition, find
- the value of the capacitance;
 - the amplitude of the current.
- P9. (a) Calculate the donor concentration of an n-type Germanium specimen having a specific resistivity of 0.1 ohm-metre at 300K , if the electron mobility $\mu_e = 0.25\text{ metre}^2/\text{Volt-sec}$ at 300K , and the magnitude of the electronic charge is $1.6 \times 10^{-19}\text{ Coulomb}$.
- (b) An n-type Germanium specimen has a donor density of $1.5 \times 10^{15}\text{ cm}^{-3}$. It is arranged in a Hall effect experiment where the magnitude of the magnetic induction field B is 0.5 Weber/metre^2 and current density $J = 480\text{ amp/metre}^2$. What is the Hall voltage if the specimen is 3 mm thick?

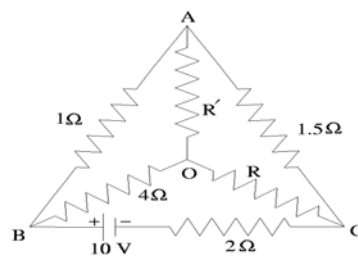
- P10. A conducting rod AB makes contact with metal rails AD and BC which are 50cm apart in a uniform magnetic field $\vec{B} = 1.0 \text{ wb/m}^2$ perpendicular to the plane $ABCD$. The total resistance (assumed constant) of the circuit $ADCB$ is 0.4Ω .



- (b) What is the direction and magnitude of e.m.f. induced in the rod when it is moved to the left with a velocity of 8m/s?
- (c) What force is required to keep the rod in motion?
- (d) Compare the rate at which mechanical work is done by the force \vec{F} with the rate of development of electric power in the circuit.
- P11. An elementary particle called Σ^- , at rest in laboratory frame, decays spontaneously into two other particles according to $\Sigma^- \rightarrow \pi^- + n$. The masses of Σ^- , π^- and n are M_1 , m_1 , and m_2 respectively.
- (a) How much kinetic energy is generated in the decay process?
- (b) What are the ratios of kinetic energies and momenta of π^- and n ?
- P12. (a) A set of binary operations is said to be functionally complete if and only if every switching function can be expressed entirely by means of operations from this set. Prove that the *nor* operation is functionally complete.
- (b) Given a switching function:
 $f(x,y,z) = x'y'z' + x'y'z + x'yz + xy'z + xyz$,
 find the canonical product-of-sums form for f .
- P13. (a) Find the relationship between L , C and R in the circuit shown in the figure such that the impedance of the circuit is independent of frequency. Find out the impedance.



(b)



Find the value of R and the current flowing through R shown in the figure when the current is zero through R' .

Computer Science

C1. (a) A grammar is said to be left recursive if it has a non-terminal A such that there is a derivation $A \Rightarrow^+ A\alpha$ for some sequence of symbols α . Is the following grammar left-recursive? If so, write an equivalent grammar that is not left-recursive.

$$\begin{array}{ll}
 A \rightarrow Bb & A \rightarrow a \\
 B \rightarrow Cc & B \rightarrow b \\
 C \rightarrow Aa & C \rightarrow c
 \end{array}$$

(b) An example of a function definition in C language is given below:

```
char fun (int a, float b, int c)
{ /* body */ ... }
```

Assuming that the only types allowed are char, int, float (no arrays, no pointers, etc.), write a grammar for function headers, i.e., the portion `char fun (int a, ...)` in the above example.

- C2. a) Construct a binary tree whose pre-order and in-order traversals are *CBAFDEHGJI* and *ABCDEFHGHIJ* respectively.
b) Convert it into an *AVL* tree with minimum number of rotations.
c) Draw the resultant *AVL* tree upon deletion of node *F*.
- C3. a) A relation $R(\underline{A}, B, C, D)$ has to be accessed under the query $\sigma_{B=10}(R)$. Out of the following possible file structures, which one should be chosen and why?
i) R is a heap file.
ii) R has a clustered hash index on B .
iii) R has an unclustered B^+ tree index on (A, B) .
- b) If the query is modified as $\pi_{A,B}(\sigma_{B=10}(R))$, which one of the three possible file structures given above should be chosen in this case and why?
- c) Let the relation have 5000 tuples with 10 tuples/page. In case of a hashed file, each bucket needs 10 pages. In case of B^+ tree, the index structure itself needs 2 pages. If the disk needs 25 msec. to read or write a disk page, what would be the disk access time for answering the above queries?
- C4. Let A and B be two arrays, each of size n . A and B contain numbers in sorted order. Give an $O(\log n)$ algorithm to find the median of the combined set of $2n$ numbers.
- C5. a) Consider a pipelined processor with m stages. The processing time at every stage is the same. What is the speed-up achieved by the pipelining?
b) In a certain computer system with cache memory, 750 ns (nanosec) is the access time for main memory for a cache miss and 50 ns is the access time for a cache hit. Find the percentage

decrease in the effective access time if the hit ratio is increased from 80% to 90%.

- C6. (a) A disk has 500 bytes/sector, 100 sectors/track, 20 heads and 1000 cylinders. The speed of rotation of the disk is 6000 rpm. The average seek time is 10 milliseconds. A file of size 50 MB is written from the beginning of a cylinder and a new cylinder will be allocated only after the first cylinder is totally occupied.
- Find the maximum transfer rate.
 - How much time will be required to transfer the file of 50 MB written on the disk? Ignore the rotational delay but not the seek time.
- (b) Following are the solutions for the two process (p_i and p_j) critical section problem. Find the errors (if any) in these solutions and rectify them. The notations have usual meanings and $i = 0, 1; j = 1-i$.

Solution 1

```
 $P_i$ : repeat
    while flag [j] do skip;
    flag [i] = true;
    critical section;
    flag [i] = false;
    exit section;
until false;
```

Solution 2

```
 $P_i$ : repeat
    flag[i] = true;
    while flag [j] do skip ;
    critical section:
    flag [i] = false ;
    exit section;
until false;
```

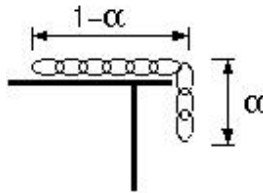
- C7. (a) A computer on a 6 Mbps network is regulated by a token bucket. The bucket is filled at a rate of 2 Mbps. It is initially filled to capacity with 8 Megabits. How long can the computer transmit at the full 6 Mbps?
- (b) Sketch the Manchester encoding for the bit stream 0001110101.

- (c) If delays are recorded in 8-bit numbers in a 50-router network, and delay vectors are exchanged twice a second, how much bandwidth per (full-duplex) line is consumed by the distributed routing algorithm? Assume that each router has 3 lines to other routers.
- C8. Consider a binary operation *shuffle* on two strings, that is just like shuffling a deck of cards. For example, the operation *shuffle* on strings ab and cd , denoted by $ab \parallel cd$, gives the set of strings $\{abcd, acbd, acdb, cabd, cadb, cdab\}$.
- (a) Define formally by induction the *shuffle* operation on any two strings $x, y \in \Sigma^*$.
- (b) Let the *shuffle* of two languages A and B , denoted by $A \parallel B$ be the set of all strings obtained by shuffling a string $x \in A$ with a string $y \in B$. Show that if A and B are regular, then so is $A \parallel B$.
- C9. (a) Minimize the switching function $w'xy'z + wx'y'z + w'xyz' + wx'yz'$.
- (b) A certain four-input gate G realizes the switching function $G(a, b, c, d) = abc + bcd$. Assuming that the input variables are available in both complemented and uncomplemented forms:
- (i) Show a realization of the function $f(u, v, w, x) = \Sigma(0, 1, 6, 9, 10, 11, 14, 15)$ with only three G gates and one OR gate.
- (ii) Can all switching functions be realized with $\{G, OR\}$ logic set?
- C10. Consider a set of n temperature readings stored in an array T . Assume that a temperature is represented by an integer. Design an $O(n + k \log n)$ algorithm for finding the k coldest temperatures.
- C11. Assume the following characteristics of instruction execution in a given computer:
- ALU/register transfer operations need 1 clock cycle each,
 - each of the load/store instructions needs 3 clock cycles, and
 - branch instructions need 2 clock cycles each.
- (a) Consider a program which consists of 40% ALU/register transfer instructions, 30% load/store instructions, and 30% branch instructions. If the total number of instructions in this program is 10 billion and the clock frequency is 1GHz, then

- compute the average cycles per instruction (CPI), total execution time for this program, and the corresponding MIPS rate.
- (b) If we now use an optimizing compiler which reduces the total number of ALU/register transfer instructions by a factor of 2, keeping the number of other instruction types unchanged, then compute the average CPI, total time of execution and the corresponding MIPS rate for this modified program.
- C12. A tape S contains n records, each representing a vote in an election. Each candidate for the election has a unique id . A vote for a candidate is recorded as his/her id .
- (i) Write an $O(n)$ time algorithm to find the candidate who wins the election. Comment on the main memory space required by your algorithm.
- (iii) If the number of candidates k is known *a priori*, can you improve your algorithm to reduce the time and/or space complexity?
- (iv) If the number of candidates k is unknown, modify your algorithm so that it uses only $O(k)$ space. What is the time complexity of your modified algorithm?
- C13. (a) The *order* of a regular language L is the smallest integer k for which $L^k = L^{k+1}$, if there exists such a k , and ∞ otherwise.
- (i) What is the order of the regular language $a + (aa)(aaa)^*$?
- (ii) Show that the order of L is finite if and only if there is an integer k such that $L^k = L^*$, and that in this case the order of L is the smallest k such that $L^k = L^*$.
- (b) Solve for $T(n)$ given by the following recurrence relations:
 $T(1) = 1$;
 $T(n) = 2T(n/2) + n \log n$, where n is a power of 2.
- C14. Let L_1 and L_2 be two arrays each with $n = 2^k$ elements sorted separately in ascending order. If the two arrays are placed side by side as a single array of $2n$ elements, it may not be found sorted. All the $2n$ elements are distinct. Considering the elements of both the arrays, write an algorithm with $k + 1$ comparisons to find the n -th smallest element among the entire set of $2n$ elements.

Engineering and Technology

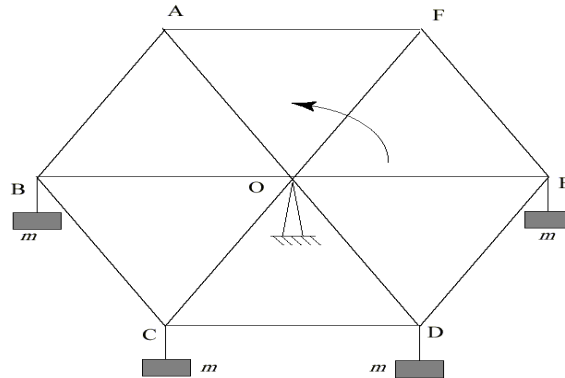
- E1. A rocket weighing 50,000 kg has been designed so as to eject gas at a constant velocity of 250 meters/sec. Find the minimum rate at which the rocket should lose its mass (through ejection of gas) so that the rocket can just take off.
- E2. A particle of mass m is attached to a fixed point by means of a string of length l and hangs freely. Show that if it is pushed horizontally with a velocity greater than $\sqrt{5gl}$, it will completely describe a vertical circle.
- E3. A chain of total length $L = 4$ metre rests on a table top, with a part of the chain hanging over the edge, as shown in the figure below. Let α be the ratio of the length of the overhanging part of the chain to L .



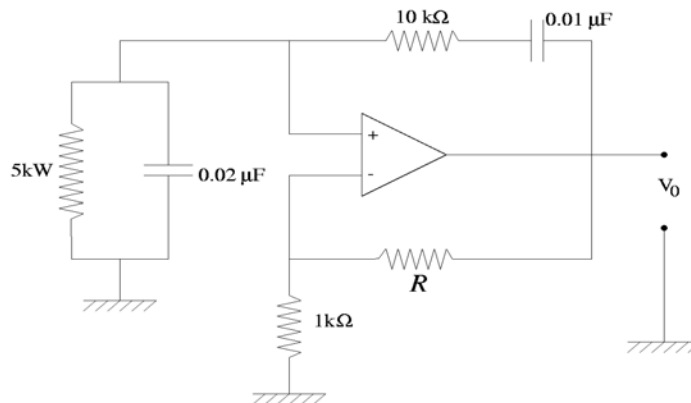
If the coefficient of friction between the chain and the table top is 0.5, find the values of α for which the chain remains stationary. If $\alpha = 0.5$, what is the velocity of the chain when the last link leaves the table?

- E4. A flywheel of mass 100 kg and radius of gyration 20 cm is mounted on a light horizontal axle of radius 2 cm, and is free to rotate on bearings whose friction may be neglected. A light string wound on the axle carries at its free end a mass of 5 kg. The system is released from rest with the 5 kg mass hanging freely. If the string slips off the axle after the weight has descended 2 m, prove that a couple of moment $10/\pi^2$ kg.wt.cm. must be applied in order to bring the flywheel to rest in 5 revolutions.
- E5. The truss shown in the figure rotates around the pivot O in a vertical plane at a constant angular speed ω . Four equal masses (m) hang from the points B, C, D and E. The members of the truss are rigid,

weightless and of equal length. Find a condition on the angular speed ω so that there is compression in the member OE.

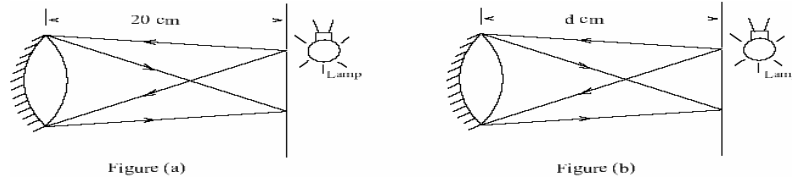


- E6. In the circuit shown below, the Op-Amp is an ideal one.
- Show that the conditions for free oscillation can be met in the circuit.
 - Find the ideal value of R to meet the conditions for oscillation.
 - Find the frequency of oscillation. (Assume $\pi = 3.14$).



- E7. Two bulbs of 500cc capacity are connected by a tube of length 20 cm and internal radius 0.15 cm. The whole system is filled with oxygen, the initial pressures in the bulbs before connection being 10 cm and 15 cm of Hg, respectively. Calculate the time taken for the pressures to become 12 cm and 13 cm of Hg, respectively. Assume that the coefficient of viscosity η of oxygen is 0.000199 cgs unit.

E8. Two identical watch glasses with negligible thickness are glued together.



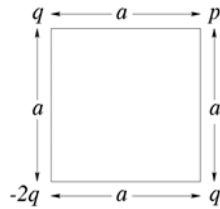
The rear one is silvered [see Figure(a)]. Sharp focus is obtained when both object and image distance are equal to 20 cm. Suppose the space between the glasses is filled with water (refractive index = $4/3$) [see Figure (b)]. Calculate d [Figure (b)] for which a sharp real image is formed.

- E9. (a) Two systems of equal mass m_1 and m_2 and heat capacity C are at temperatures T_1 and T_2 respectively ($T_1 > T_2$). If the first is used as source and the second as sink, find the maximum work obtainable from such an arrangement.
- (b) A Carnot engine A operates between temperatures T_1 and T_2 whose dissipated heat at T_2 is utilised by another Carnot engine B operating between T_2 and T_3 . What is the efficiency of a third engine that operates between T_1 and T_3 in terms of the efficiencies h_A and h_B of engines A and B respectively?
- E10. (a) A system receives 10 Kcal of heat from a reservoir to do 15 Kcal of work. How much work must the system do to reach the initial state by an adiabatic process?
- (b) A certain volume of Helium at 15°C is suddenly expanded to 8 times its volume. Calculate the change in temperature (assume that the ratio of specific heats is $5/3$).
- E11. A spherical charge distribution has a volume density ρ , which is a function of r , the radial distance from the center of the sphere, as given below.

$$\rho = \begin{cases} A/r, & A \text{ is constant for } 0 \leq r \leq R \\ 0, & \text{for } r > R \end{cases}$$

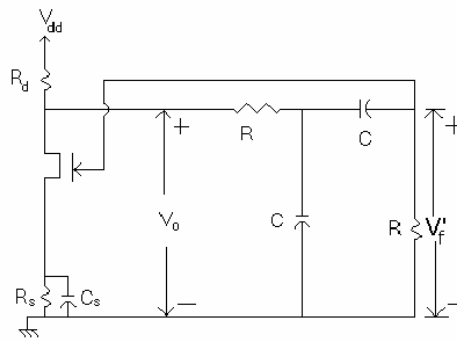
Determine the electric field as a function of r , for $r \geq R$. Also deduce the expression for the electrostatic potential energy $U(r)$, given that $U(\infty) = 0$ in the region $r \geq R$.

- E12. Consider the distribution of charges as shown in the figure below. Determine the potential and field at the point p .



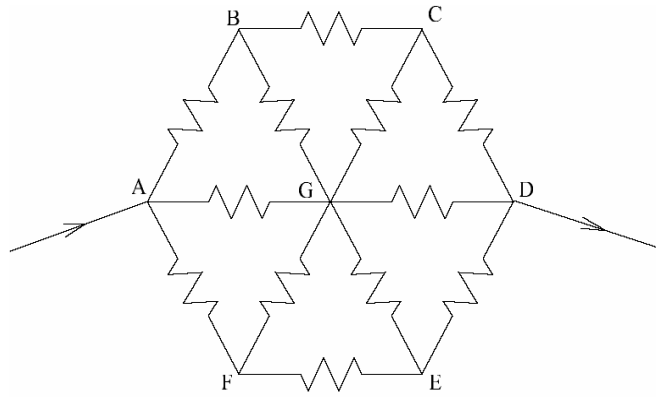
- E13. A proton of velocity 10^7 m/s is projected at right angles to a uniform magnetic induction field of 0.1 w/m². How much is the path of the particle deflected from a straight line after it has traversed a distance of 1 cm? How long does it take for the proton to traverse a 90° arc?

- E14. (a) State the two necessary conditions under which a feedback amplifier circuit becomes an oscillator.
 (b) A two-stage FET phase shift oscillator is shown in the diagram below.



- (i) Derive an expression for the feedback factor β .
 (ii) Find the frequency of oscillation.
 (iii) Establish that the gain A must exceed 3 .
- E15. A circular disc of radius 10 cm is rotated about its own axis in a uniform magnetic field of 100 weber/m², the magnetic field being perpendicular to the plane of the disc. Will there be any voltage developed across the disc? If so, then find the magnitude of this voltage when the speed of rotation of the disc is 1200 rpm.

- E16. A 3-phase, 50-Hz, 500-volt, 6-pole induction motor gives an output of 50 HP at 900 rpm. The frictional and windage losses total 4 HP and the stator losses amount to 5 HP. Determine the slip, rotor copper loss, and efficiency for this load.
- E17. A 20 KVA, 2000/200 V two-winding transformer is to be used as an auto-transformer with a constant source voltage of 2000 V. At full load with unity power factor, calculate the power output, power transformed and power conducted. If the efficiency of the two-winding transformer at 0.7 power factor is 90%, find the efficiency of the auto-transformer.
- E18. An alternator on open-circuit generates 360 V at 60 Hz when the field current is 3.6 A. Neglecting saturation, determine the open-circuit e.m.f. when the frequency is 40 Hz and the field-current is 24 A.
- E19. A 150 KVA, 4400/440 volt single phase transformer has primary and secondary resistance and leakage reactance values as follows:
 $R_p = 2.4 \Omega$, $R_s = 0.026 \Omega$, $X_p = 5.8 \Omega$, and $X_s = 0.062 \Omega$.
 This transformer is connected with a 290 KVA transformer in parallel to deliver a total load of 330 KVA at a lagging power factor of 0.8. If the first transformer alone delivers 132 KVA, calculate the equivalent resistance, leakage reactance and percentage regulation of the second transformer at this load. Assume that both the transformers have the same ratio of the respective equivalent resistance to equivalent reactance.
- E20. The hybrid parameters of a $p-n-p$ junction transistor used as an amplifier in the common-emitter configuration are: $h_{ie} = 800 \Omega$, $h_{fe} = 46$, $h_{oe} = 8 \times 10^{-5} \text{ mho}$, $h_{re} = 55.4 \times 10^{-4}$. If the load resistance is $5 \text{ k}\Omega$ and the effective source resistance is 500Ω , calculate the voltage and current gains and the output resistance.
- E21. Find the equivalent resistance between the points A and D of the circuit shown in the diagram.



Each resistance = $r \Omega$

- E22. (a) Design a special purpose counter to count from 6 to 15 using a decade counter. Inverter gates may be used if required.
 (b) For a 5 variable Boolean function the following minterms are true: (0, 2, 3, 8, 10, 11, 16, 17, 18, 24, 25 and 26). Find a minimized Boolean expression using Karnaugh map.
- E23. In the figure, consider that $FF1$ and $FF2$ cannot be set to a desired value by reset/preset line. The initial states of the flip-flops are unknown. Determine a sequence of inputs (x_1, x_2) such that the output is zero at the end of the sequence.

