Test Code: QR (Short answer type) 2010

M.Tech. in Quality, Reliability and Operations Research

The candidates applying for M.Tech. in Quality, Reliability and Operations Research will have to take two tests : **Test MIII** (objective type) in the forenoon session and **Test QR** (short answer type) in the afternoon session.

For Test **MIII**, see a different Booklet. For Test **QR**, refer to this Booklet **ONLY.**

If you are from **Statistics / Mathematics Stream,** you will be required to **ANSWER PART I.**

If you are from **Engineering Stream**, you will be required to **ANSWER PART II.**

In **PART I**, a **TOTAL of TEN [10]** questions, are divided into **TWO Groups**: **S1**: **Statistics and S2: Probability** – **each group carrying FIVE [5]** questions. You will be required to answer a **TOTAL of SIX [6]** questions, taking **AT LEAST TWO [2]** from **each group**.

In **PART II**, there will be **SIX Groups: E1-E6**. **E1** will contain **THREE [3]** questions from **Engineering Mathematics** and each other group will contain **TWO [2]** questions from **Engineering and Technology**. You will be required to answer a total of **SIX [6]** questions taking **AT LEAST TWO [2]** from group **E1**.

Syllabus

PART I: STATISTICS / MATHEMATICS STREAM

Statistics (S1)

Descriptive statistics for univariate, bivariate and multivariate data.
Standard univariate probability distributions [Binomial, Poisson, Normal] and their fittings, properties of distributions. Sampling distributions.
Theory of estimation and tests of statistical hypotheses.
Multiple linear regression and linear statistical models, ANOVA.
Principles of experimental designs and basic designs [CRD, RBD & LSD].
Elements of non-parametric inference.
Elements of sequential tests.
Sample surveys – simple random sampling with and without replacement, stratified and cluster sampling.

Probability (S2)

Classical definition of probability and standard results on operations with events, conditional probability and independence.
Distributions of discrete type [Bernoulli, Binomial, Multinomial, Hypergeometric, Poisson, Geometric and Negative Binomial] and continuous type [Uniform, Exponential, Normal, Gamma, Beta] random variables and their moments.
Bivariate distributions (with special emphasis on bivariate normal), marginal and conditional distributions, correlation and regression.
Multivariate distributions, marginal and conditional distributions, regression, independence, partial and multiple correlations.
Order statistics [including distributions of extreme values and of sample range for uniform and exponential distributions].
Distributions of functions of random variables.
Multivariate normal distribution [density, marginal and conditional distributions, regression].

Syllabus

Weak law of large numbers, central limit theorem.
Basics of Markov chains and Poisson processes.

PART II : ENGINEERING STREAM

Mathematics (E1)

Elementary theory of equations, inequalities.
Elementary set theory, functions and relations, matrices, determinants, solutions of linear equations.
Trigonometry [multiple and sub-multiple angles, inverse circular functions, identities, solutions of equations, properties of triangles].
Coordinate geometry (two dimensions) [straight line, circle, parabola, ellipse and hyperbola], plane geometry, Mensuration.
Sequences, series and their convergence and divergence, power series, limit and continuity of functions of one or more variables, differentiation and its applications, maxima and minima, integration, definite integrals areas using integrals, ordinary and partial differential equations (upto second order), complex numbers and De Moivre’s theorem.

Engineering Mechanics (E2)

Forces in plane and space, analysis of trusses, beams, columns, friction, principles of strength of materials, work-energy principle, moment of inertia, plane motion of rigid bodies, belt drivers, gearing.

Syllabus

Electrical and Electronics Engineering (E3)

D.C. circuits, AC circuits (1-ϕ), energy and power relationships, Transformer, DC and AC machines, concepts of control theory and applications.
Network analysis, 2 port network, transmission lines, elementary electronics (including amplifiers, oscillators, op-amp circuits), analog and digital electronic circuits.

Theromodynamics (E4)

Laws of thermodynamics, internal energy, work and heat changes, reversible changes, adiabatic changes, heat of formation, combustion, reaction, solution and dilution, entropy and free energy and maximum work function, reversible cycle and its efficiency, principles of internal combustion engines. Principles of refrigeration.

Engineering Properties of Metals (E5)

Structures of metals, tensile and torsional properties, hardness, impact properties, fatigue, creep, different mechanism of deformation.

Engineering Drawing (E6)

Concept of projection, point projection, line projection, plan, elevation, sectional view (1^{st} angle/3^{rd} angle) of simple mechanical objects, isometric view, dimensioning, sketch of machine parts.
(Use of set square, compass and diagonal scale should suffice).

SAMPLE QUESTIONS

PART I: STATISTICS / MATHEMATICS STREAM

GROUP S–1: Statistics

1. Let X_1 and X_2 be independent χ^2 variables, each with n degrees of freedom. Show that $\frac{\sqrt{n}\left(X_1 - X_2\right)}{2\sqrt{X_1 X_2}}$ has the t distribution with n degrees of freedom and is independent of $X_1 + X_2$.

2. Let $[\{x_i\ ;\ i = 1, 2, \ldots, p\};\ \{y_j\ ;\ j = 1, 2, \ldots, q\};\ \{z_k\ ;\ k = 1, 2, \ldots, r\}]$ represent random samples from $N(\alpha + \beta, \sigma^2)$, $N(\beta + \gamma, \sigma^2)$ and $N(\gamma + \alpha, \sigma^2)$ populations respectively. The populations are to be treated as independent.
   (a) Display the set of complete sufficient statistics for the parameters $(\alpha, \beta, \gamma, \sigma^2)$.
   (b) Find unbiased estimator for β based on the sample means only. Is it unique?
   (c) Show that the estimator in (b) is uncorrelated with all error functions.
   (d) Suggest an unbiased estimator for σ^2 with maximum d.f.
   (e) Suggest a test for H_0: $\beta = \beta_0$.

3. Consider the linear regression model : $y = \alpha + \beta x + e$ where e's are iid $N(0, \sigma^2)$.
   (a) Based on n pairs of observations on x and y, write down the least squares estimates for α and β.
   (b) Work out exact expression for Cov($\hat{\alpha}$, $\hat{\beta}$).
   (c) For a given y_0 as the "predicted" value, determine the corresponding predictand "x_0" and suggest an estimator "$\hat{x}_0$" for it.

4. A town has N taxis numbered 1 through N. A person standing on roadside notices the taxi numbers on n taxis that pass by. Let M_n be the largest number observed. Assuming independence of the taxi numbers and sampling with replacement, show that

$$\hat{N} = (n + 1)\, M_n / n$$

   is an approximately unbiased estimator of N for large N.

5.(a) Let $x_1, x_2, \ldots, x_n$ be a random sample from the rectangular population with density

$$f(x) = \begin{cases} \dfrac{1}{\theta}, & 0 < x < \theta \\ 0, & otherwise \end{cases}$$

Consider the critical region $x_{(n)} > 0.8$ for testing the hypothesis $H_0 : \theta = 1$, where $x_{(n)}$ is the largest of $x_1, x_2, \ldots, x_n$. What is the associated probability of type I error and what is the power function?

(b) Let $x_1, x_2, \ldots, x_n$ be a random sample from a population having p.d.f.

$$f(x,\theta) = \begin{cases} \dfrac{\theta^3}{\Gamma(3)} e^{-\theta x} x^2, & 0 < x < \infty \\ 0, & otherwise \end{cases}$$

Obtain the maximum likelihood estimate of θ and also obtain the Cramer Rao lower bound to the variance of an unbiased estimator of θ.

6.(a) Give an example of a Latin Square Design of order 4 involving 4 rows, 4 columns and 4 treatments. Give the general definition of "treatment connectedness" in the context of a Latin Square Design and show that the Latin Square Design that you have given is indeed treatment connected.

(b) In a CRD set-up involving 5 treatments, the following computations were made:

$n = 105$, Grand Mean = 23.5, SSB = 280.00, SSW = 3055.00

(i) Compute the value of the F-ratio and examine the validity of the null hypothesis.

(ii) It was subsequently pointed out that there was one additional treatment that was somehow missed out! For this treatment, we are given sample size = 20, Sum = 500 and Sum of Squares (corrected) = 560.00. Compute revised value of F-ratio and draw your conclusions.

7. If X_1, X_2, X_3 constitute a random sample from a Bernoulli population with mean p, show why $[X_1 + 2X2 + 3X_3] / 6$ is *not* a sufficient statistic for p.

8. If X and Y follow a trinomial distribution with parameters n, θ_1 and θ_2, show that

(a) $E(Y / X = x) = \dfrac{(n-x)\theta_2}{1-\theta_1}$,

(b) $V(Y / X = x) = \dfrac{(n-x)\theta_2(1-\theta_1-\theta_2)}{(1-\theta_1)^2}$

9. Life distributions of two independent components of a machine are known to be exponential with means μ and λ respectively. The machine fails if at least one of the components fails. Compute the chance that the machine will fail due to the second component. Out of n independent prototypes of the machine m of them fail due to the second component. Show that $m/(n-m)$ approximately estimates the odds ratio $\theta = \lambda/\mu$.

GROUP S–2: Probability

1. A boy goes to his school either by bus or on foot. If one day he goes to the school by bus, then the probability that he goes by bus the next day is 7/10. If one day he walks to the school, then the probability that he goes by bus the next day is 2/5.

   (a) Given that he walks to the school on a particular Tuesday, find the probability that he will go to the school by bus on Thursday of that week.

   (b) Given that the boy walks to the school on both Tuesday and Thursday of that week, find the probability that he will also walk to the school on Wednesday.

   [You may assume that the boy will not be absent from the school on Wednesday or Thursday of that week.]

2. Suppose a young man is waiting for a young lady who is late. To amuse himself while waiting, he decides to take a random walk under the following set of rules:

   He tosses an imperfect coin for which the probability of getting a head is 0.55. For every head turned up, he walks 10 yards to the north and for every tail turned up, he walks 10 yards to the south.

   That way he has walked 100 yards.

   (a) What is the probability that he will be back to his starting position?

   (b) What is the probability that he will be 20 yards away from his starting position?

3. (a) A coin is tossed an odd number of times. If the probability of getting more heads than tails in these tosses is equal to the probability of getting more tails than heads then show that the coin is unbiased.

   (b) For successful operation of a machine, we need at least three components (out of five) to be in working phase. Their respective chances of failure are 7%, 4%, 2%, 8% and 12%. To start with, all the components are in working phase and the operation is initiated. Later it is observed that the machine has stopped but the first component is found to be in working phase. What is the likelihood that the second component is also in working phase?

   (c) A lot contains 20 items in which there are 2 or 3 defective items with probabilities 0.4 and 0.6 respectively. Items are tested one by one from the lot unless all the defective items are tested. What is the probability that the testing procedure will continue up to the twelfth attempt ?

4.(a) Let S and T be distributed independently as exponential with means $1/\lambda$ and $1/\mu$ respectively. Let $U = \min\{S,T\}$ and $V = \max\{S,T\}$. Find $E(U)$ and $E(U+V)$.

   (b) Let X be a random variable with $U(0,1)$ distribution. Find the p.d.f. of the random variable $Y = X / (1 + X)$.

5.(a) Let U and V be independent and uniformly distributed random variables on [0,1] and let θ_1 and θ_2 (both greater than 0) be constants.

Define $X = -\frac{1}{\theta_1}\ln U$ and $Y = -\frac{1}{\theta_2}\ln V$. Let $S = \min\{X,Y\}$, $T=max\{X,Y\}$ and $R = T - S$.

(i) Find $P[S=X]$.

(ii) Show that S and R are independent.

(b) A sequence of random variables $\{X_n \mid n = 1, 2, \ldots\}$ is called a *martingale* if

(i) $E(|X_n|) < \infty$

(ii) $E(X_{n+1} \mid X_1, X_2, \ldots, X_n) = X_n$ for all $n = 1, 2, \ldots$

Let $\{Z_n \mid n = 1, 2, \ldots\}$ be a sequence of iid random variables with $P[Z_n = 1] = p$ and $P[Z_n = -1] = q = 1-p, 0 < p < 1$. Let $X_n = Z_1 + Z_2 + \ldots + Z_n$ for $n = 1, 2, \ldots$. Show that $\{X_n \mid n = 1, 2, \ldots\}$, so defined, is a martingale if and only if $p = q = ½$..

6.(a) Let X be a random variable with density

$$f_X(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

For the minimum $X_{(1)}$ of n iid random observations $X_1, X_2, \ldots, X_n$ from the above distribution, show that $n^{1/4} X_{(n)}$ converges in distribution to a random variable Y with density

$$f_Y(y) = \begin{cases} 4e^{-y^4} y^3, & y > 0 \\ 0, & otherwise \end{cases}$$

(b) A random sample of size n is taken from the exponential distribution having p.d.f.

$$f(x) = \begin{cases} e^{-x}, & 0 \le x < \infty \\ 0, & otherwise. \end{cases}$$

Find the p.d.f. of the sample range.

7.(a) In a recent study, a set of n randomly selected items is tested for presence of colour defect. Let A denote the event colour defect is present" and B denote the event "test reveals the presence of colour defect". Suppose $P(A) = \alpha$, $P(B|A) = 1-\beta$ and $P(\text{Not } B| \text{ Not } A) = 1-\delta$, where $0 < \alpha, \beta, \delta < 1$. Let X be the number of items in the set with colour defects and Y be the number of items in the set detected having colour defects.

(i) Find $E(X|Y)$.

(ii) If the colour defect is very rare and the test is a very sophisticated one such that $\alpha = \beta = \delta = 10^{-9}$, then find the probability that an item detected as having colour defect is actually free from it.

(b) Consider the following bivariate density function

$$f(x, y) = \begin{cases} c \cdot xy\,, & x > 0, y > 0, x + y < 1 \\ 0 & otherwise \end{cases}$$

i) Find c.

ii) Find the conditional expectation, $E(Y|X = x)$, for $0 < x < 1$.

8. Suppose in a big hotel there are N rooms with single occupancy and also suppose that there are N boarders. In a dinner party to celebrate the marriage anniversary of one of the boarders they start drinking alcohol to their hearts' content and as a consequence they become unable to identify their own rooms. What is the probability that at the end of the dinner party none of the boarders occupies the room originally assigned to them? What is the limiting value of this probability as $N \to \infty$?

9. (a) Consider a Markov Chain with state space $I = \{1,2,3,4,5,6\}$ and transition probability matrix P given by

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/4 & 3/4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1/8 & 7/8 & 0 & 0 \\ 1/4 & 0 & 1/2 & 0 & 1/8 & 1/8 \\ 0 & 0 & 1/4 & 3/4 & 0 & 0 \end{bmatrix}$$

Find the various classes of this chain and classify them as recurrent or transient.

(b) Pulses arrive at a Geiger counter according to a Poisson Process with parameter $\lambda > 0$. The counter is held open only a random length of time T (independent of the arrival time of the pulses), where T is exponentially distributed with parameter $\beta > 0$. Find the distribution of N = Total number of pulses registered by the counter.

PART II: ENGINEERING STREAM

GROUP E-1: Engineering Mathematics

1(a) Let $f(x)$ be a polynomial in x and let a, b be two real numbers where $a \neq b$. Show that if $f(x)$ is divided by $(x - a)(x - b)$ then the remainder is

$$\frac{(x-a)f(b) - (x-b)f(a)}{b-a}.$$

(b) Find $\frac{dy}{dx}$ if $x^{\cos y} + y^{\sin x} = 1$.

2.(a) Let A be the fixed point (0,4) and B be a moving point ($2t$, 0). Let M be the mid-point of AB and let the perpendicular bisector of AB meets the y-axis at R. Find the equation of the locus of the mid-point P of MR.

(b) Inside a square ABCD with sides of length 12 cm, segment AE is drawn where E is the point on DC such that DE = 5 cm. The perpendicular bisector of AE is drawn and it intersects AE, AD and BC at the points M, P and Q respectively. Find the ratio PM: MQ.

3.(a) Evaluate the value of $3.9^{1/2}.27^{1/4}.81^{1/8}\ldots\ldots\infty$.

(b) Let f be a twice differentiable function such that

$$f''(x) = -f(x);\ f'(x) = g(x) \text{ and } h(x) = f^2(x) + g^2(x).$$

Given that $h(5)=11$, find $h(10)$.

4.(a) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\cdots(upto\,[n/2]\,terms)\right]=\frac{1}{2}.$$

(c) Test the convergence of the series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\cdots\infty$. Assume $x > 0$ and examine all possibilities.

5.(a) Find the limit of the following function as $x \to 0$.

$$\frac{|x|}{\sqrt{\left(x^4+4x^2+7\right)}}\sin\left(\frac{1}{3\sqrt{x}}\right).$$

(b) If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$ then show that $a \cdot b < 0$.

6.(a) If ω is a complex cube root of unity then show that

$$a^3+b^3+c^3-3abc=\left(a+b+c\right)\left(a+b\omega+c\omega^2\right)\left(a+b\omega^2+c\omega\right).$$

(b) Show that $\left[\frac{\sum\limits_{r>s} x^r}{r!}\right]\div\left[\frac{\sum\limits_{r>s} y^r}{r!}\right]>\frac{x^s}{y^s}$, whenever $x > y > 0$.

7.(a) Cable of a suspension bridge hangs in the form of a parabola and is attached to the supporting pillars 200 m apart. The lowest point of the cable is 40 m below the point of suspension. Find the angle between the cable and the supporting pillars. State all the assumptions involved.

(b) Let A, B and C be the angles of a triangle with angle C as the smallest of them.
Show that

(i) $\sin\left(\frac{C}{2}\right) \le \frac{1}{2}$

(ii) Hence, or otherwise, show that $\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) < \frac{1}{4}$.

8(a) Evaluate the following two integrals directly and compare them.

$$\iint\limits_{ax^2+by^2\le 1} dxdy \quad \text{and} \quad \iint\limits_{\sqrt{a}|x|\le 1, \sqrt{b}|y|\le 1} dxdy.$$

(b) Determine *x*, *y* and *z* so that the 3×3 matrix with the following row vectors is orthogonal : $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, $(1/\sqrt{2}, -1/\sqrt{2}, 0)$, (x, y, z).

GROUP E-2: Engineering Mechanics

1.(a) The simple planar truss in the given Fig.1 consists of two straight two-force members AB and BC that are pinned together at B. The truss is loaded by a downward force of P=12 KN acting on the pin at B. Determine the internal axial forces F_1 and F_2 in members AB and BC respectively. (Neglect the weight of the truss members).

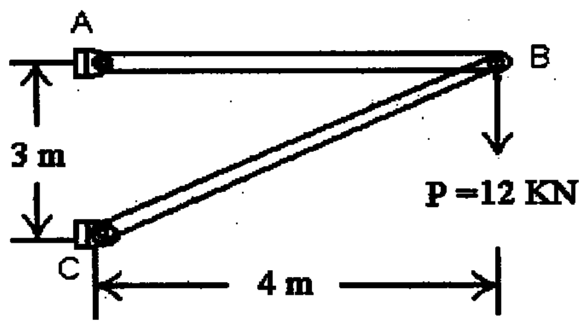


Fig. 1

(b) Derive the expression for moment of inertia I_{YY} of the shaded hollow rectangular section (Fig. 2).

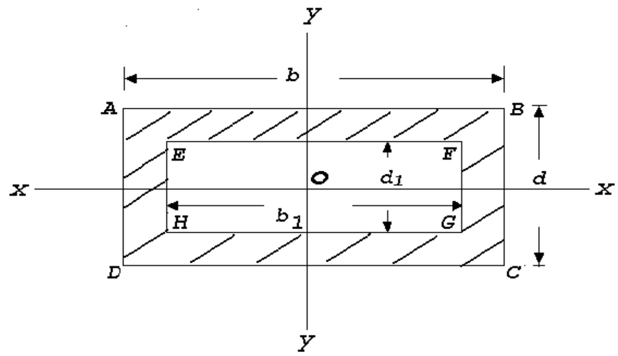


Fig. 2

2.(a) A turbine rotor weighs 20 tonnes and has a radius of gyration of 1.75 meter when running at 200 rpm. It is suddenly relieved of part of its load and its speed rises to 205 rpm in 1 sec. Find the unbalanced uniform turning moment.

(b) An Aluminium thin-walled tube (radius/thickness = 20) is closed at each end and pressurized by 6 MPa to cause plastic deformation. Neglect the elastic strain and find the plastic strain in the circumferential (hoop) direction of the tube. The plastic stress-strain curve is given by $\overline{\sigma} = 170\left(strain\ rate\right)^{0.25}$.

3.(a) A uniform ladder 5 m long and 14 kg mass is placed against a vertical wall at an angle 50^0 to the horizontal ground. The co-efficient of friction between ladder and wall is 0.2 and between ladder and ground in 0.5. Calculate how far up the ladder a man of 63 kg. can climb before the ladder shifts.

(b) Determine the diameter of a steel shaft rotating at an angular velocity of 300 rpm transmitting 500 HP. The allowable stress = 800 kg/cm^2. The allowable angle of twist = 0.5^0 per m, G = 8×10^5 kg/cm^2 . What would be the savings if a hollow shaft is used to transmit the same power under the same condition, the ratio of diameters being 0.9?

4.(a) For the beam and loading shown in Fig.3, determine the equation defining the shear and bending moment at any point and at point D.

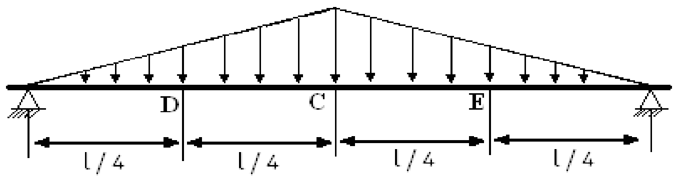


Fig. 3

(b) As illustrated in the given Fig.4 a metal punch (similar in principle to a paper punch) is used to punch holes in thin steel sheet that will be used to make a metal cabinet. To punch a 60 mm diameter disk or "slug" out of the sheet metal that is 10 mm thick requires a punch force of P=500 kg. Determine the average shear stress in the sheet metal resulting from the punching operation.

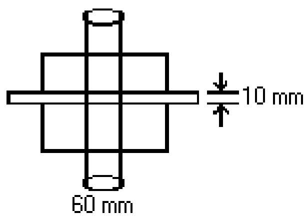


Fig. 4

5. (a) A tie rod in the suspension of a car is to be constructed from a grade of steel, which has 0.1% proof stress equal to 250 MN/m^2. The tie rod is to be constructed as a solid round bar of length 350 mm long. If the tie rod is subjected to a maximum axial force of 10 KN,

i) Determine the minimum diameter of the tie rod
ii) The extension of the tie rod under load (E= 2094 GN/m^2)
iii) The minimum diameter of the tie rod if a factor of safety of 2.5 is applied to the proof stress

(b) Find the width of the belt necessary to transmit 11.25 KW power to a pulley of diameter 300mm when the pulley makes 1600 rpm. Assume the co-efficient of friction between the belt and the pulley is 0.22 and angle of contact is 210^o. Maximum tension in the belt will not exceed 10N/mm width.

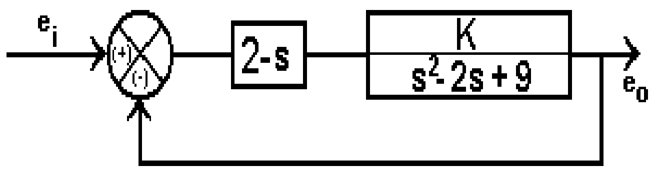
GROUP E-3: Electrical and Electronics Engineering

1.(a) A centrifugal pump, which is gear-driven by a DC motor, delivers 810 kg of water per minute to a tank of height 11 meter above the level of the pump. Draw the block diagram of the overall arrangement. Determine input power across the gearing and current taken by the motor operated at 220 volt provided the efficiency of the pump, gearing and motor respectively be 70%, 70% and 90% only. (Take $g = 9.8\ ms^{-2}$).

(b) The rms value of a sinusoidal alternating voltage at a frequency of 50 Hz is 155volt. If at $t = 0$ it crosses the zero axis in a positive direction, determine the time taken to attain the first instantaneous value of 155 volt. How much time it takes to fall from the maximum peak value to its half? Explain with suitable waveform.

2.(a) On full-load unity power factor test, a meter having specification of 235 V and 5A makes 60 revolutions in 6 minutes, but its normal speed is 520 revolution/KWh. Does the meter have any inaccuracy? If so, find the percentage error.

(b) Write down the transfer function of the given system (as shown in the following figure) and find the values of K for which the system will be stable but under damped.



3 (a) By intelligent selection of loop currents write down the mesh equations of the given circuit (as shown in Fig. 5) and determine the current flowing through that branch of the circuit containing capacitor. (All resistances/ reactance's are in ohms).

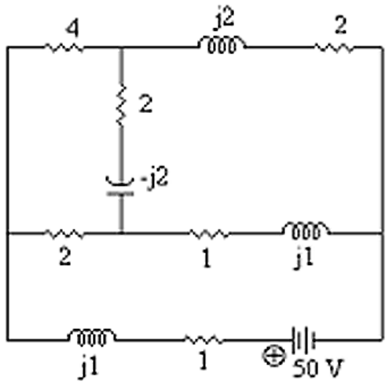


Fig. 5

(b)

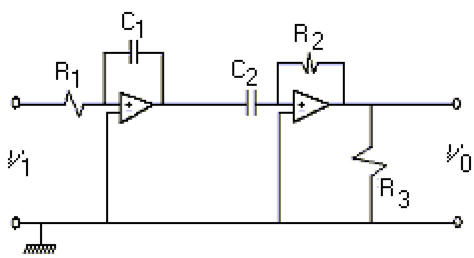


Fig. 6

Refer Fig. 6. Find the expression for V_0. What would be the nature of V_0 when $R_1 = R_2$ and $C_1 = C_2$? (Consider the Op-amps to be identical).

4. (a) A series ac circuit that resonates at 48 Hz consists of a coil (having R = 30 Ω and L = 500 mH) and a capacitor. If the supply voltage is 100 volt determine the value of the capacitor.

(b) Calculate the value of a capacitor which when connected across the circuit (as of Q. 4 (a) above), enhances the resonant frequency to 60 Hz. Compare the value of the source current in both the cases.

5 (a) A 200/400 - V, 10KVA, 50Hz single phase transformer has, at full load, a Cu loss of 120W. If it has an efficiency of 98% at full load unity power factor, determine the iron losses. What would be the efficiency of the transformer at half load 0.8 power factor lagging?

(b) In the 2-port network given below, the parameters at two parts are related by the equations,

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

i) Find expressions for A, B, C and D
ii) Show that AD - BC = 1
iii) What are the physical interpretations of the above coefficients?

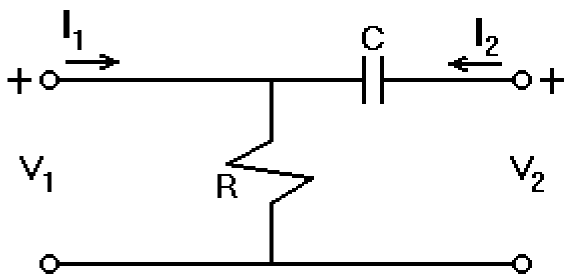


Fig 7

GROUP E-4: Thermodynamics

1 (a) In a thermodynamic system of a perfect gas, let $U = f(V,T)$ where U, V and T refer to internal energy, volume of a gram-molecule of the substance and temperature (in absolute scale) respectively. An amount of heat δQ is added so that the volume expands by δV against a pressure P. Prove that:

$$C_P - C_V = \left[P + \left(\frac{\delta U}{\delta V}\right)_T\right]\left(\frac{\delta V}{\delta T}\right)_P$$

where C_p and C_V stand for specific heat at constant pressure and specific heat at constant volume respectively.

(b) 0.15 cu.m. of air at a pressure of 1.06 kg/cm^2 is compressed to a volume of 0.008 cu.m. at 361 kg/cm^2. Calculate (i) the quantity of heat rejected, (ii) change in internal energy if the process of compression is a) Adiabatic b) Polytropic with $n = 1.3$.

2 (a) A compression ignition engine has a stroke of 28 cm and a cylinder diameter of 18 cm. The clearance volume is 475 cm^3. The fuel injection takes place at constant pressure for 4.5% of the stroke. Find the air standard efficiency of the engine assuming that it works on diesel cycle. If the fuel injection takes place at 10% of the stroke, find the loss in air standard efficiency.

(b) A diesel engine has a compression ratio 14 to 1 and the fuel supply is cut off at 0.08 of the stroke. If the relative efficiency is 0.52, estimate the weight of fuel of a calorific value 10400 k.cal per kg that would be required per horsepower.

3.(a) Calculate the change in entropy of saturated steam at a given pressure such that the boiling point = 152.6 ^{o}C and the latent heat at this temperature = 503.6 cal/gm. [Use $\log_e 1.56 = 0.445$.]

(b) Draw the pv and $T-\Phi$ diagrams for a diesel cycle in which 1 kg of air at 1 kg / cm^2 and 90 ^{0}C is compressed through a ratio of 14 to 1. Heat is then added until the volume is 1.7 times the volume at the end of compression, after which the air expands adiabatically to its original volume. Take C_v = 0.169 and $\gamma = 1.41$.

4.(a) The approximated equation for adiabatic flow of super heated steam through a nozzle is given by $pv^n = \text{constant.}$ Show that

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{n/(n+1)}$$

where $p_1 =$ pressure of steam at entry ; $p_2 =$ pressure of steam at throat and p_2 / p_1 is the critical pressure ratio.

(b) The dry saturated steam is expanded in a nozzle from pressure of 10 bar to pressure of 4 bar. If the expansion is super saturated, find the degree of under cooling.

GROUP E-5: Engineering Properties of Metals

1. (a) Distinguish between modulus of rigidity and modulus of rupture. Give an expression for the modulus of rigidity in terms of the specimen geometry, torque, and angle of twist. Is the expression valid beyond the yield strength (torsion)?

   (b) A steel bar is subjected to a fluctuating axial load that varies from a maximum of 340 kN to a minimum of 120 kN compression. The mechanical properties of the steel are σ_u = 1090 MPa, σ_0 = 1010 MPa and σ_e = 510 MPa. Determine the bar diameter to give infinite fatigue life based on a safety factor of 2.5

2 (a) A cylindrical bar is subjected to a torsional moment M_T at one end. The twisting moment is resisted by shear stress μ set up in the cross section of the bar. The shear stress is zero at the centre of the bar and increases linearly with the radius. Find the maximum shear stress at the surface of the bar.

Given $J = \dfrac{\pi D^4}{32}$ (assuming that the torsional deformation is restricted within the zone of elasticity)

where, J : Polar moment of inertia
D : Diameter of cylinder.

(b) Consider a flat plane containing a crack of elliptical cross-section. The length of the crack is 2c and stress is perpendicular to the major axis of the ellipse. Show that

$$\sigma = \sqrt{\frac{2\gamma E}{\pi c}}$$

σ : stress
γ : surface energy
E : Young's modulus of elasticity

3. (a) Consider a tension specimen, which is subjected to a total strain ε at an elevated temperature where creep can occur. The total strain remains constant and the elastic strain decreases. Show that

$$\frac{1}{\sigma^{n-1}} = \frac{1}{\sigma_o^{\,n-1}} + BE(n-1)t$$

where,

$\varepsilon = \varepsilon_e + \varepsilon_p$ — ε_e : elastic strain

$\varepsilon_e = \sigma / E$ — ε_p : plastic strain

$\frac{d\varepsilon_p}{dt} = B\sigma^n$ — t : time

$\sigma = \sigma_o$ at $t = 0$.

(b) Distinguish between slip and twinning with diagrams.

4. (a) Suppose a crystalline material has *fcc* structure with atomic radius of 1.278 Å Determine the density of the crystalline material. Assume number of atoms per unit cell and molecular weight are *n* and *M* gm respectively.

(b) Suppose there is an electron in an electric field of intensity 3200 volts/m. Estimate the force experienced by the electron. If it moves through a potential difference of 100 volts, find the kinetic energy acquired by the electron.

GROUP E-6: Engineering Drawing

1.(a) A hollow cube of 5cm side is lying on H.P. and one of its vertical face is touching V.P. A slim rod, to be taken as its solid diagonal, is placed within it. Draw top and front / side views of solid diagonal and, from the drawn figure determine its true length.

(b) Two balls are vertically erected to 18cm and 30 cm respectively above the flat ground. These balls are away from a 3 cm thick wall (on the ground) by 12 cm and 21 cm respectively but on either side of the wall. The distance between the balls, measured along the ground and parallel to the wall is 27 cm. Determine their approximate distance.

2. (a) Sketch the profile of a square thread, knuckle thread and a white-worth thread showing all relevant dimensions in terms of the pitch.

   (b) Sketch:

      i) single riveted lap joint,
      ii) double riveted lap joint chain-riveting,
      iii) double riveted lap joint zigzag-riveting, and
      iv) single cover single riveted butt joint.

3.(a) Draw the isometric view of an octahedron erected vertically up on one of its vertices. (Distinct free hand sketch only.)

   (b) You are given two square prisms of same height of 10cm. Prism A has side 7cm and prism B has side of 5cm respectively. Longer face of B is lying on H.P. with its base perpendicular to V.P. Base of A is lying on H.P. but equally inclined to V.P. You are instructed to remove by cutting a portion of bottom base of A so that within the cavity maximum of B may be placed accordingly. Note that vertical face of B may be parallel to V.P. but just touch the central axis of A. Draw the sectional view of the combination and determine the volume of material to be removed from A.

4. A parallelepiped of dimension 100×60×80 is truncated by a plane which passes through 85, 45 and 65 unit distance on the associated edges from the nearest top point of the object. Draw the isometric view of the truncated solid object. In third angle projection method, draw its plan. (All dimensions are in mm).

<u>Note</u>: A copy of one of the previous year's QR Test Question paper is appended in the following pages to give the candidate a rough idea.

BOOKLET No. TEST CODE: QR

Afternoon

Time: 2 hours

Group	Questions: Total	Questions: To be answered	Maximum marks
Part I (for Statistics/Mathematics Stream)			
S1 (Statistics)	5	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM EACH GROUP.	120
S2 (Probability)	5		
Part II (for Engineering Stream)			
E1 (Mathematics)	3	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM E1	120
E2 (Engineering Mechanics)	2		
E3 (Electrical and Electronics Engineering)	2		
E4 (Thermodynamics)	2		
E5 (Engineering Properties of Metals)	2		
E6 (Engineering Drawing)	2		

On the answer-booklet write your Name, Registration Number, Test Code, Number of this booklet, etc. in the appropriate places.

There are two parts in this booklet as detailed above. Candidates having Statistics background are required to answer questions from Part I as per instructions given. Those having engineering background are required to answer questions from Part II as per instructions given.

USE OF CALCULATORS IS NOT ALLOWED. SLIDE RULE MAY BE USED

STOP! WAIT FOR THE SIGNAL TO START

PART I (FOR STATISTICS / MATHEMATICS STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS, TAKING AT LEAST TWO [2] FROM EACH GROUP.

GROUP S1
Statistics

1. (a) X, Y and Z are independent random variables with the same variance. If

$X_1 = \frac{1}{\sqrt{2}}(X - Z)$, $X_2 = \frac{1}{\sqrt{3}}(X + Y + Z)$ and $X_3 = \frac{1}{\sqrt{6}}(X + 2Y + Z)$, show that X_1, X_2 and X_3 have equal variances. Calculate $r_{12 \cdot 3}$ and $R_{1 \cdot 23}$.

[12]

(b) Let X have the p.m.f.

$$f(x,\theta) = \begin{cases} \theta^x (1-\theta)^{1-x}, & x = 0, 1 \\ 0, & elsewhere \end{cases}$$

We test the simple null hypothesis $H_o : \theta = \frac{1}{4}$ against the alternative composite hypothesis $H_1 : \theta < \frac{1}{4}$ by taking a random sample of size 10 and rejecting $H_o : \theta = \frac{1}{4}$ if and only if the observed values $x_1, x_2, ..., x_{10}$ of the sample observations are such that $\sum_{i=1}^{10} x_i \leq 1$. Find the power function $K(\theta), 0 < \theta \leq \frac{1}{4}$ of this test.

[8]

2. An unbiased coin is to be used to select a Probability Proportional to Size With Replacement (PPSWR) sample in 2 draws from the following population of 3 units, where X is the size measure.

i	1	2	3
X_i	2	4	1

Consider the following procedure

(a) Toss the coin thrice independently.

(b) If the outcome is $\{HHH\}$ or $\{HHT\}$, then select the first unit.

(c) If the outcome is $\{HTH\}$, $\{HTT\}$, $\{THH\}$ or $\{THT\}$, then select the second unit.

(d) If the outcome is $\{TTH\}$, then select the third unit.

(e) If the outcome is $\{TTT\}$, do not select any of the units, go back to (a) above.

(f) Continue till a unit is selected.

Show that the above procedure is a PPS method. If W = number of tosses required to select a unit, find $E(W)$.

[20]

3. Show that $A^2 - G^2 = s^2$, for any set of positive values, $x_1, x_2, \ldots, x_n$, where A, G and s^2 are the arithmetic mean, geometric mean and sample variance respectively, provided the deviations $y_i = (x_i - A)$, i = 1, 2, …, n, are small compared to A so that $(y/A)^3$ and higher powers of (y/A) can be ignored.

[20]

4. A food products company – Wonderful Bakery Products, sells its produce to bakery shops. The sales are made by four sales persons Pratap, Quereshi, Rajinder and Sandip. The products are sold to four bakery shops namely Amit Bakeries (A), Bakeman Products (B), Chocolates and Breads (C) and, Delight Confectioners (D). The owner of Wonderful Bakery Products wanted to assess the effectiveness of the sales persons and carried out an experiment. All the sales persons were sent to the four shops three times each on different days and the quantity sold was noted. The actual sales figures for the different sales persons at different shops on different days are given below:

Pratap			Quereshi			Rajinder			Sandip		
Day	Bakery	Qty	Day	Bakery	Qty	Day	Bakery	Qty	Day	Bakery	Qty
2	A	11	2	B	18	6	D	74	2	C	35
11	C	48	8	D	40	7	C	42	2	D	47
15	B	68	9	A	29	10	A	6	3	A	8
18	D	36	15	C	57	13	D	104	4	C	35
21	A	24	16	A	24	16	D	56	8	B	64
25	C	57	25	B	58	18	B	28	8	C	20
27	C	33	26	D	136	21	C	93	11	A	9
30	D	77	28	A	34	27	B	22	19	B	15
34	D	64	29	C	27	31	A	14	19	D	27
35	B	42	37	D	60	31	B	18	29	B	27
37	A	11	38	B	28	34	C	72	31	D	28
40	B	18	39	C	70	35	A	21	38	A	10
Total Sales		489	Total Sales		581	Total Sales		550	Total Sales		325

While analyzing this data keep in mind that the sales made to a particular shop will depend on the skill of the sales person as well as the level of inventory of the shop. We also assume here that these bakeries buy their products from these four sales persons only and from no other sources.

How will you analyze this data to compare the performance of the sales persons? Which salesman do you think has performed best? State your assumptions clearly.

[20]

5. a) Let $x_1, x_2, \ldots, x_n$ be independent random sample of size $n > 3$ from $N(\mu, \sigma^2)$. Find, $E\left(\frac{\bar{x}^2}{s^2}\right)$ where $\bar{x}$ and s^2 are sample mean and variance respectively. Hence find an UMVUE of $\frac{\mu^2}{\sigma^2}$.

[10]

b) Let x_1, x_2 be a random sample of size 2 from the distribution with p.d.f.

$$f(x) = \frac{1}{\lambda} e^{-x/\lambda}, \quad x \geq 0, \lambda > 0$$

Consider the statistics $T_1 = x_1 + x_2$ and $T_2 = x_2$. Show that $E(T_2 | T_1)$ is an unbiased estimator for λ. Is $E(T_2 | T_1)$ the unique unbiased estimator for λ? Justify your answer.

[8+2=10]

GROUP S2
Probability

6 Ali, Bikram and Charlie are three antagonists engaged in a three-way duel. There are two rounds. In the first round each player is given one shot; first Ali, then Bikram and finally Charlie shoots. After the first round the survivors are given a second round again beginning with Ali, then Bikram and then Charlie.

From the point of view of any person fighting the duel, being the sole survivor is the most preferred outcome. Being one of the two survivors is the next best possibility. A case when everyone survives is the third best and obviously not surviving is the worst possible case.

It is known that Ali is a poor shot with a probability of 0.30 of hitting the target. Bikram is a much better shot and hits the target with probability 0.80. Charlie is a crack-shot. He never misses.

Let the optimal strategy for any person be a way of playing the game, i.e. choosing to shoot a particular target or deliberately miss so that the chance of his survival is maximized. What is the optimal strategy of Ali? Assuming that the players follow the optimal strategy, who has the greatest chance of survival and what is the probability?

[20]

7. a) Three urns contain, respectively, 4 white, 2 black balls; 2 white, 4 black balls; and 3 white, 3 black balls. One of the urns is chosen on the results of two throws of a coin: the first urn if head appears on each throw; the second urn if tail appears on each throw and the third urn in case head appears on one throw and tail on the other. Finally, a ball is drawn at random from the chosen urn.

(i) What is the probability for the ball being white?
(ii) If a ball taken at random from the chosen urn is found to be white, what is the probability that the first urn was chosen?

[10]

(b) Two teams *A* and *B* play a series of independent games until one of them wins 4 games. The probability of each team winning in each game is $\frac{1}{2}$. Find the probability that the series will end in

i) at most 6 games, and
ii) 6 games given that team *A* won the first two games.

[10]

8. Let the joint p.d.f of *X* and *Y* be given by

$$f(x,y)=\begin{cases}\dfrac{2}{(1+x+y)^3}, & 0<x<\infty, 0<y<\infty\\ 0, & elsewhere\end{cases}$$

a) Compute the marginal p.d.f of *X* and the conditional p.d.f of *Y*, given $X = x$.
b) For a fixed $X = x$, compute $E\left[1+x+Y|x\right]$ and use the result to compute $E\left(Y|x\right)$.

[20]

9 a) A letter in known to have come either from TATANAGAR or from CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter came from CALCUTTA?

[10]

b) Suppose that you are in a game show and you are given choice of three doors. Behind one door, there is a pot of gold, but the other two do not have any reward. You have chosen a door but have not opened it yet. The host of the game show opens another door and shows that there is nothing behind it. He now offers you a chance to switch doors. Should you switch? Explain your answer. State your assumptions clearly.

[10]

10 a) *A* and *B* throw alternately a pair of dice. *A* wins if he scores 6 points before *B* gets 7 points, in which case *B* wins. If *A* starts the game, what is his probability of winning?

[10]

b) A particle counter is recording two streams of particles, high-energy ones and low-energy ones. Assume that the two streams of particles arrive at the counter according to independent Poisson processes with intensities and respectively.

i) Find the conditional probability mass function of the number of particles registered in $(0,t]$ given that a total of *n* particles were registered in this time interval.

ii) If a low-energy particle has energy e_L and a high-energy particle has energy e_H, find the expected energy of a particle just registered.

[6+4=10]

PART II (FOR ENGINEERING STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS TAKING AT LEAST TWO [2] FROM E1.

GROUP E1
Mathematics

1. (a) Show that

$$\sqrt{{}^nC_1} + \sqrt{{}^nC_2} + \ldots \sqrt{{}^nC_n} \le 2^{n-1} + \frac{n-1}{2}.$$

[10]

(b) Find the inverse of the following matrix

$$\begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_2 & c_3 & c_0 & c_1 \\ c_3 & -c_2 & c_1 & -c_0 \\ c_1 & -c_0 & c_3 & -c_2 \end{bmatrix}$$

where $c_0 = \dfrac{1+\sqrt{3}}{4\sqrt{2}}$, $c_1 = \dfrac{3+\sqrt{3}}{4\sqrt{2}}$, $c_2 = \dfrac{3-\sqrt{3}}{4\sqrt{2}}$ and $c_3 = \dfrac{1-\sqrt{3}}{4\sqrt{2}}$.

[10]

2. (a) Show that

$$\int_0^\infty\int_0^\infty e^{-\left(x^2+y^2\right)} dx\, dy = \frac{\pi}{4}$$

[10]

(b) Show that $\displaystyle\int_0^\infty \frac{dx}{(x+1)(x+2)}$ is convergent.

[10]

3. (a) Let $f(t)$ be a continuous function on $(-\infty, \infty)$ and let

$$F(x) = \int_0^x (x-t)\, f(t)\, dt .$$

Find $F'(x)$.

[10]

(b) If $\alpha_1, \alpha_2, \ldots, \alpha_n$ are the roots of the equation

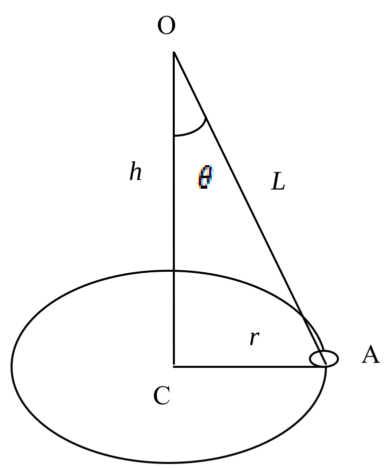
$$x^n + nax - b = 0 ,$$

then show that $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)\ldots(\alpha_1 - \alpha_n) = n(\alpha_1^{n-1} + a)$.

[10]

GROUP E2
Engineering Mechanics

4. (a) A particle of weight W attached to a fixed point O by a string of length L whirls in a horizontal circular path of radius r with uniform speed V so that the string generates a height h. Show that the relation between V, r, h and the tensile force T in the string is $T = W\sqrt{1 + \left(\frac{r}{h}\right)^2}$.

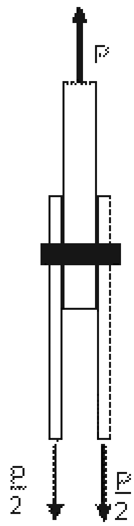


[10]

(b) A slender rod 2 m long and having a mass of 4 kg increases its speed about a vertical axis through one end from 20 r.p.m to 50 r.p.m in 10 revolutions. Find the required constant moment M.

[10]

5. A 20 mm diameter bolt (black) holds together a lap joint composed of a main member transferring a force P to two side members. The main member thickness is 12 mm and side member thickness is 9.5 mm. The load P = 4.5 Tons. Determine the average shear stress on the bolt and the bearing stresses on the members.



[20]

GROUP E3
Electrical & Electronics Engineering

6. (a) Explain why a dc motor should not be started direct-on-line.

(b) What is the relation between phase voltage and line voltage, phase current and line current in a star-connected and in a delta-connected system?

(c) Draw the complete phasor diagram of transformer on-load.

(d) Why does the rotor of a three-phase induction motor rotate?

[4 + 5 + 6 + 5 = 20]

7. (a) Consider a parallel L-C-R circuit across a supply voltage *V*. Write down the equation for overall current flowing through the combinations. Simulate currents by using operational amplifiers (op-amp) and requisite passive circuit elements. Design the overall system.

[12]

(b) Utilizing the above concept, design a buffer amplifier mentioning all sorts of necessary conditions to be considered. Let the initial input signal be connected across the inverting terminal of the op-amp.

[8]

GROUP E4
Thermodynamics

8. (a) Over a certain range of pressures and temperatures the equation of a certain substance is given by the relation $v = \frac{RT}{p} - \frac{C}{T^3}$ where C is a constant. Show that

(i) The change of enthalpy of this substance in an isothermal process is $(h_2 - h_1)_T = \frac{4C}{T^3}(p_1 - p_2)_T$

(ii) The change of entropy of this substance in an isothermal process is

$$(S_2 - S_1)_T = R\ln\frac{p_1}{p_2} + \frac{3C}{T^4}(p_1 - p_2)_T .$$

[6+4=10]

(b) In an air-saturated four-stroke Otto engine the minimum temperature T_1 is governed by the ambient atmosphere and the maximum temperature T_3 is dictated by the material of construction of the piston and cylinder. For fixed values of T_1 and T_3, show that the compression ratio r_0 for obtaining maximum net work per unit mass of air undergoing the cyclic change is $r_0 = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2(\gamma-1)}}$.

[10]

9. (a) If an ideal gas undergoes a reversible adiabatic process, trace the paths of the process on temperature versus volume diagram, pressure versus

volume diagram and temperature versus pressure diagram subsequent to establishing the corresponding thermodynamic relations.

[10]

(b) If an ideal gas is changed from P_1, v_1, T_1 to P_2, v_2, T_2, show that the relation to estimate the entropy change without devising a reversible path is $\Delta S = C_P \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$.

[10]

GROUP E5
Engineering Properties of Metals

10. (a) While dealing with the relation between the internal forces, deformation and external loads in strength of material, some very important assumptions are made about the body (engineering material) that is subjected to deformation. State these assumptions.

(b) In a real metallic body (unit solid cube), any application of load results in three-dimensional state of stress and hence 3 components of strain along the three directions (x, y, z). Write the expressions for the components of strain along the three directions in terms of stresses along the three directions. Write also the expression for shearing stresses acting on the cube and the resulting shearing strains.

(c) Are the modulus of elasticity, modules of elasticity in shear and the bulk modulus different names of the same parameter? Justify your answer.

(d) "The use of a thin-walled tubular test specimen is ideal for precisely measuring the torsional elastic limit or, yield strength in torsion". Explain why?

(e) A tensile specimen with an initial diameter of 15 mm and 45 mm gauge length reaches maximum load at 100 kN and fractures at 80 kN. The maximum diameter at fracture is 10 mm. Determine the engineering stress at the maximum load, true fracture stress and % elongation.

[3+5+5+2+5 = 20]

11. (a) When you are required to determine the hardness over a very small area (such as, different phases in the microstructure of an alloy), how would you measure the hardness? Describe the procedure in detail with the type of equipment, load etc. to be used.

(b) What is the *S-N* relationship in fatigue and how this curve is determined for a particular engineering material? Name the parameters that are needed for determining this curve. What are the differences between a *S-N* curve in low cycle region and *S-N* curve in high cycle region? How do you deal with the statistical nature of the fatigue life in *S-N* curve?

(c) Derive the condition of instability at torsion having considered that the torque is a function of strain, strain rate and temperature, and at constant strain rate, the normalized rate of hardening or softening of the torque is given by $V = \frac{1}{M_T}\left(\frac{dM_T}{d\theta}\right)_{\dot{\theta}}$ where, M_T = Torque, θ = strain, $\dot{\theta}$ = strain rate.

(d) A steel bar is subjected to a fluctuating axial load that varies from a maximum of 380 kN tension to a minimum of 120 kN compression. The mechanical properties of the steel are σ_u = 1090 MPa, σ_o =1010 MPa, σ_e = 510 MPa. Determine the bar diameter to give infinite fatigue life based on a safety factor of 1.7.

[4+6+5+5 = 20]

GROUP E6
Engineering Drawing

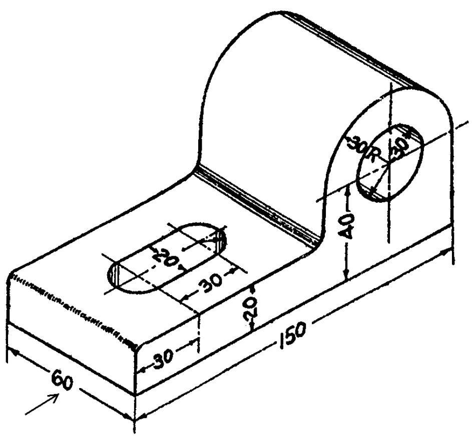
12. A cube of 10 cm is placed on H.P. and two of its adjoining side surfaces are equally inclined to V.P. There is a hollow cylindrical cavity centrally spaced along the axis of solid diagonal of the cube. The height and the radius of the cylindrical cavity are 10 cm and 1 cm respectively.

    (a) Draw the front view, side view and top view of the composite cube mentioning the angle of projection.

    (b) Draw the isometric view of the composite cube.

[12 + 8 = 20]

13. (a) Sketch the elevation, plan and left-side view of the given object mentioning the dimensions and angle of projection (First angle / Third angle projection).



All dimensions are in mm

    (b) Draw a Knuckle Joint used for fastening together two or more rods subjected to tensile or compressive stress.

[12 + 8 = 20]