Test Code MS (Short answer type) 2010

Syllabus for Mathematics

Permutations and combinations. Binomial and multinomial theorem. Theory of equations. Inequalities.

Vectors and vector spaces. Matrices. Determinants. Solution of linear equations.

Trigonometry. Co-ordinate geometry.

Geometry of complex numbers and De Moivres theorem.

Elements of set theory.

Convergence of sequences and series. Functions. Limits and continuity of functions of one or more variables. Power series.

Differentiation. Leibnitz formula. Maxima and minima. Taylors theorem. Differentiation of functions of several variables. Applications of differential calculus.

Indefinite integral. Fundamental theorem of calculus, Riemann integration and properties. Improper integrals. Double and multiple integrals and applications.

Syllabus for Statistics

Probability and Sampling Distributions : Notions of sample space and probability. Combinatorial probability. Conditional probability and independence. Random variables and expectations. Moments and moment generating functions. Standard univariate discrete and continuous distributions. Joint probability distributions. Multinomial distribution. Bivariate and multivariate normal distributions. Sampling distributions of statistics. Weak law of large numbers. Central limit theorem.

Descriptive Statistics : Descriptive statistical measures. Contingency tables and measures of association. Product moment and other types of correlation. Partial and multiple correlation. Simple and multiple linear regression.

Inference : Elementary theory of estimation (unbiasedness, minimum variance, sufficiency). Methods of estimation (maximum likelihood method, method of moments). Tests of hypotheses (basic concepts and simple applications of Neyman Pearson Lemma). Confidence intervals. Inference related to regression. ANOVA. Elements of nonparametric inference.

Design of Experiments and Sample Surveys : Basic designs (CRD/ RBD/ LSD) and their analyses. Elements of factorial designs. Conventional sampling techniques (SRSWR/SRSWOR) including stratification. Ratio and regression methods of estimation.

Sample Questions

1. Let A be a 4×4 matrix with non-negative entries such that the sum of the entries in each row of A equals 1. Find the sum of all entries in the matrix A^5.

2. Find the total number of isosceles triangles such that the length of each side is a positive integer less than or equal to 40.
(Here equilateral triangles are also counted as isosceles triangles.)

3. Let $f : (-1, 1) \to R$ be a continuous function with the property that $f(x) = f(x^4)$ for all x and $f(1/2) = a$. Show that the only function f satisfying this property is the constant function $f(x) = a$.

4. Let $X_1, X_2, ..., X_n$ be a random sample from a Bernoulli distribution with parameter p. Let $\bar{X}_n$ be the sample average given by $\bar{X}_n = \frac{1}{n}(X_1 + \quad + X_n)$. Find the expected value of $(\bar{X}_n - p)^3$.

5. A rectangle is drawn where the lengths of the sides are chosen randomly from $[0, 10]$ and independently of one another. Find the probability that the length of its diagonal is smaller than or equal to 10.

6. A person throws a fair dice repeatedly, each throw being independent of the other throws. At each trial he observes one of the numbers $1, 2, ..., 6$. What is the expected number of throws to observe

   (a) two distinct numbers for the first time?

   (b) three distinct numbers for the first time?

7. Let $Y_i = \alpha + \beta(\frac{i}{n}) + \epsilon_i$, $i = 1, ..., n$, where $\epsilon_1, ..., \epsilon_n$ are i.i.d. $N(0, \sigma^2)$. Here α, β, σ^2 are unknown parameters; $-\infty < \alpha, \beta < \infty$, $0 < \sigma^2 < \infty$, $n \geq 3$.

   (a) Find the least squares estimators of α and β.

   (b) Are the least squares estimates of α, β based on the observed values of Y_i's, UMVUE? Justify your answer.

8. A population contains 10 units, labelled $U_1, U_2, ..., U_{10}$. The value, of a character Y under study, for U_i is Y_i $(1 \leq i \leq 10)$. In order to estimate the population mean, $\bar{Y}$, a sample of size 4 is drawn in the following manner:

   (i) a simple random sample of size 2 is drawn without replacement from the units $U_2, U_3, ..., U_9$;

   (ii) the sample drawn in step (i) is augmented by the units U_1 and U_{10}. Based on the above sample in (ii), suggest an unbiased estimator of $\bar{Y}$ and obtain its variance.

9. Let $X_1, X_2, ..., X_n$ be i.i.d. $N(\mu, 1)$ and independently $Y_1, Y_2, ..., Y_m$ be i.i.d. $N(\mu, 4)$ random variables.

   (a) Test $H_0 : \mu = 0$ against the alternative $H_1 : \mu > 0$ based on the combined sample.

   (b) Find the power function of the test.

10. (a) Let $X \sim N(\mu, 1)$ and $Y = e^X$. Find the probability density function of Y.

    (b) Find $\mu_Y = E(Y)$; here $E(\cdot)$ stands for expectation.

    (c) Let $Y_1, Y_2, ..., Y_n$ be a random sample from the distribution of Y. Find the maximum likelihood estimator of μ_Y based on this sample.

11. Let X be a random variable uniformly distributed over $(0, 2\theta)$, $\theta > 0$ and $Y = \max(X, 2\theta - X)$.

    (a) Find $\mu = E(Y)$.

    (b) Let $X_1, ..., X_n$ be a random sample from the above distribution with unknown θ. Find two distinct unbiased estimators of μ, as defined in (a), based on the entire sample.

* For more sample questions, visit http://www.isical.ac.in/~deanweb/MSTATSQ.html