Proposed M.S. in Statistics and Applications

Structure and Syllabus

Semester I:
- [C1] Analysis - I
- [C2] Probability and Stochastic Processes - I
- [C3] Methods of Statistical Inference - I*
- [C4] Linear Algebra
- [C5] Elements of Sample Surveys and Design of Experiments*

Semester II:
- [C6] Probability and Stochastic Processes - II
- [C7] Linear Models and GLM*
- [C8] Statistical Inference - II*
- [C9] Multivariate Analysis*
- [C10] Regression Techniques*

Semester III:
- [C11] Analysis - II
- [C12] Statistical Computing*
- [C13] Time Series Analysis*
- [E1] Elective - I
- [E2] Elective - II

Semester IV:
- [C14] Probability and Stochastic Processes - III
- [C15] Project
- [E3] Elective - III
- [E4] Elective - IV
- [E5] Elective - V

- Suggested syllabi for all the courses follow.
- A partial list of elective courses and their syllabi is also enclosed. This list is not exhaustive and can be added upon.
- In the courses marked ' * ' there will be emphasis on practicals using suitable statistical packages.

Compulsory Courses

1. Analysis - I (Compulsory course - Semester I)
   Real numbers, countability. limit points of sets, lim sup and lim inf. open and closed sets, sequences and series, basic tests for convergence of a series, power series, product series.

   limit, continuity, properties of continuous functions.

   derivatives. basic properties. mean value theorem and applications, L'Hopital's rule, Taylor's theorem.

   Riemann integration, first and second fundamental theorems of calculus, change of variable, integration by parts, mean value theorems for integrals.

   Books recommended:

   (a) Introduction to Real Analysis by R.G. Bartle and D.R. Sherbert, John Wiley

   (b) Principles of Mathematical Analysis by W. Rudin, McGraw Hill

2. Probability and Stochastic Processes - I (Compulsory course - Semester I)
   Review of basic concepts of probability, random variables and common distributions.

   Random vectors, joint distributions, joint m.g.f., mixed moments, variance covariance matrix. Independence, sums of independent random variables, Poisson approximation to the normal distribution, multinomial distribution, Conditional expectation and variances, Order statistics, Sampling distributions, Characteristic functions

   Concepts of convergence of a sequence of r.v.s.

   Kolmogorov strong law of large numbers, monotone convergence theorem and dominated convergence theorem (all without proof).

   Continuity theorem for characteristic functions. Lindeberg's CLT and its particular cases.

   Books recommended:

   (a) Introduction to Mathematical Statistics by R. V. Hogg and T. T. Craig.

   (b) Introduction to Probability Theory and Mathematical Statistics by V. K. Rohatgi.

(c) Introduction to probability theory by P. G. Hoel, S. C. Port and C. J. Stone

(d) Probability and Measure by P. Billingsley.

(e) Introduction to Probability and its Applications vol.II by W. Feller.

(f) Probability Theory by M. Loeve.

(g) Probability Theory by B. V. Gnedenko.

3. **Methods of Statistical Inference - I** (Compulsory course - Semester I)

Sufficient statistics and minimal sufficient statistics, ancillary statistics, completeness, exponential families and their properties, Basu's theorem. Jensen's inequality.

Different kinds of loss functions, risk function, Rao-Blackwell Theorem with general loss functions, Information inequality: multiparameter case and extensions, UMVUE. Maximum likelihood estimation.

Elements of Bayesian Inference: Bayes Risk, Posterior, Bayes Credible Region, Minimax Estimates

Neyman Pearson Lemma, distributions with MLR property, UMP unbiased tests, LMP tests, Neyman structure, P-values and its examples, Interval estimation, Sequential Probability Ratio test, Stein's two-stage procedure

Books recommended:

(a) Statistical Inference by Casella and Berger

(b) Linear Statistical Inference by C. R. Rao

(c) Theory of Point Esimation by Lehmann and Casella

(d) Testing Statistical Hypotheses by Lehmann

(e) Mathematical Statistics by Bickel and Doksum

4. **Linear Algebra** (Compulsory course - Semester I)

Vector spaces with real and complex scalars; subspaces, linear dependence and independence, basis, dimension, sum and intersection of subspaces. Linear equations: homogeneous and nonhomogeneous systems, solution space, consistency and general solution, numerical examples.

Inner product and norm: geometric interpretation, Gram-Schmidt orthogonalization process, orthogonal projection, projection on a subspace.

Linear transformations and Matrices : rank, trace, elementary operations, canonical reductions, orthogonal matrices, symmetric matrices, inverse, sweep-out method. Operations with partitioned matrices.

Determinant: definition and properties, computation. Characteristic roots and vectors: numerical examples.

Quadratic forms: classification and transformations, canonical reduction of real symmetric matrices, spectral decomposition, Cayley-Hamilton theorem.

Generalized inverse: properties, applications. Projection operators on R^n as idempotent matrices: properties.

Books recommended:

(a) Linear Statistical Inference (LSI) by C. R. Rao

(b) Linear Algebra (LA) by A. R. Rao and P. Bhimsankaram

(c) Linear Algebra and Linear Models by R. B. Bapat

5. **Elements of Sample Surveys and Design of Experiments** (Compulsory course - Semester I)

Sample Surveys

Review of simple random sampling. stratification, ratio and regression estimators, two-stage sampling. Sampling designs, homogeneous linear unbiased estimators, Godambe's non-existence theorem. PPS sampling with and without replacement. Horvitz-Thompson estimator and its variance estimators. Some sampling schemes with inclusion probability proportional to size. Non-sampling errors and their handling.

Design of Experiments

Review of standard designs; Intra-block analysis of a general block design; concepts of connectedness and variance-balance. Balanced incomplete block designs and elementary construction of such designs. Factorial experiments and designs, concepts of orthogonal factorial structure and balance. Response surface designs.

Books recommended: W.G.Cochran, M.N.Murthy, P.Mukhopadhyay

Aloke Dey, Raghavarao, Dean and Voss, P.W.M.John, M.C.Chakravarty

6. **Probability and Stochastic Processes - II** (Compulsory course - Semester II)

Markov chains with stationary transition probabilities, properties of transition functions. classification of states, Stationary distribution of a Markov chain, existence and uniqueness, convergence to the stationary distribution.

Methods based on Markov chains for simulation of random vectors. MCMC algorithm.

Random Walks, Queueing Processes, Branching processes,

Gamblers ruin problem, Transient states.

Books recommended:

(a) An Introduction to Probability Theory and its Applications by W. Feller.

(b) A First Course in Stochastic Processes by S. Karlin and H. M. Taylor.

(c) Introduction to Stochastic Processes by P. G. Hoel, S. C. Port and C. J. Stone.

(d) Introduction to Probability Models by S. Ross.

7. **Linear Models and GLM** (Compulsory course - Semester II)

Linear statistical models, illustrations, Gauss-Markov model, normal equations and least squares estimators, estimable linear functions, g-inverse and solution of normal equations. Error space and estimation space. Variances and covariances of BLUEs. Estimation of error variance. Fundamental theorems of least squares and applications to tests of linear hypotheses.
Fisher-Cochran theorems. distribution of quadratic forms.
Generalized Linear Models. Logistic regression. Log-linear models.
Practical on matrix computations, least squares estimation and testing using statistical packages.

Books recommended:

C.R. Rao, Searle, T.P.Ryan, Montgomery.

8. Methods of Statistical Inference - II (Compulsory course - Semester II)

Delta method. Aymptotic properties of sample quantiles, Asymptotic relative efficiency of estimators, M.L.E.s and their large sample properties, Method of scoring.

Transformations and variance stabilising formulae, Asymptotic distributions of functions of sample moments. Likelihood ratio tests and their large sample properties, Pearson's chi-square, contingency tables.

Non-parametrics: Kolmogorov-Smirnov tests, sign test, signed rank test, run test, Kruskal-Wallis test, Mann-Whitney test, Hodges-Lehmann estimators, Censored data, Kaplan-Meier estimators.

Books recommended:

(a) Statistical Inference by Casella and Berger

(b) Mathematical Statistics - A Decision Theoretic Approach by Ferguson

(c) Theory of Point Esimation by Lehmann and Casella

(d) Statistical Inference Based On Ranks by Hettmansperger

(e) Linear Staistical Inference by C.R. Rao

(f) Elements of Large Sample Theory by Lehmann

(g) Approximation Theorems of Mathematical Statistics by Robert Serfling.

(h) Large Sample Methods in Statistics by P. K. Sen and J. M. Singer

(i) Theory of Rank tests by Hajek, Sidak and Sen.

9. **Multivariate Analysis** (Compulsory course - Semester II)
Same as in present M Stat.

10. **Regression Techniques** (Compulsory course - Semester II)
Same as in present M Stat.

11. **Analysis - II** (Compulsory course - Semester III)

Improper Riemann integrals, tests of convergence.

Sequence and series of functions and their behaviour vis-a-vis continuity, differentiability and integration.

Multivariate Calculus: Limit of a function, difference with the single variable case. differentiation,

Implicit function theorem and the inverse function theorem.

Differentiation with respect to a vector, Jacobians

Books recommended:

(a) Principles of Mathematical Analysis by W. Rudin, McGraw Hill

(b) Analysis by Terence Tao (Forthcoming in the TRIM Series).

12. **Statistical Computing** (Compulsory course - Semester III)

C++ programming, use of packages as available at the time, like S+, SyStat, SAS etc., simulation, resampling techniques like Jack-Knife and Boot-strap, EM algorithm, IRLS.

Books recommended:

(a) Statistical Computing by Kundu and Basu

(b) Simulation by S.M. Roos

13. **Time Series Analysis** (Compulsory course - Semester III)

Same as in present M Stat.

14. **Probability and Stochastic Processes - III** (Compulsory course - Semester IV)
Markov pure jump processes, Poisson process, Birth and Death processes. Finite state continuous time Markov chains.
Renewal processes, Poisson process as a renewal process, elementary renewal theorem. Statement (without proof) of other renewal theorems. Simple queueing systems. Introduction to Brownian Motion.

Books recommended:

(a) An Introduction to Stochastic Modelling by H. M. Taylor and S. Karlin.

(b) Stochastic Processes by J. Medhi.

(c) Introduction to Probability Models by S. Ross.

15. **Project** (Compulsory course - Semester IV)
It is envisaged that students, individually or in a group, are assigned to a faculty member. They will need to work on suitable projects arising out of real life problems and which involve application of statistical techniques. Help may be sought from colleagues in other units and industry in this regard.

Elective Courses

1. **Metric Topology and Complex Analysis** (Elective course)
As in M. Stat. First Year.

2. **Measure Theory** (Elective course)
   σ-fields and monotone class theorem, Measures, Caretheodory Extension theorem, Lebesgue measure, Integral of a measurable function with respect to a measure.

   Fatou's lemma, Monotone Convergence theorem, Dominated Convergence theorem.

   Hahn Jordan decomposition, Lebesgue decomposition, Radon Nikodym derivative, Product measure, Fubini's theorem.

   Convergence in measure, almost everywhere convergence, Kolmogorov Inequality. Kolmogorov three series criterion, strong law of large numbers.

   Conditional Probability and conditional expectations, their simple properties. Martingales (discrete parameter), uniform integrability.

   Books recommended:

   (a) Probability and Measure by P. Billingsley

   (b) Real Analysis and Probability by R. B. Ash.

3. **Advanced Linear Algebra / Matrix Analysis** (Elective course)
   (This is a prerequisite for Functional Analysis)
   Norms, limits and convergence in matrix spaces.
   Analysis of matrix decompositions : QR, LR, SVD and Spectral.
   Extremal Principles for eigenvalues.
   Sensitivity of eigenvalues and eigenvectors.
   Nonnegative matrices and their applications.

   Books recommended:

   (a) Nonnegative Matrices and Applications by R. B. Bapat and T. E. S. Raghavan.

   (b) Matrix Analysis by R. Bhatia.

   (c) Introduction to Matrix Computations by G. W. Stewart.

4. **Functional Analysis** (Elective course)
   Same as in present M Stat.

5. **Topics in Fourier Analysis** (Elective course)

   This topic is important in itself and at the same time a course on it reinforces the student's grasp of analysis, linear algebra etc.

(a) Fourier Series of continuous functions, their convergence and divergence, Cesaro and Abel convergence, mean convergence. Applications.

Topics to be chosen from:

(b) Fourier Transforms on L_1 and L_2 spaces. Basic theorems and their applications.

(c) Discrete Fourier Transform, Fast Fourier Transform, Wavelets on $\mathbb{Z}$ and $\mathbb{Z}^n$.

(d) Boundary value problems.

Books recommended:

(a) Fourier Series by R. Bhatia.

(b) An Introduction to Wavelets through Linear Algebra by M. W. Frazier.

(c) Fourier Transforms by R. R. Goldberg.

(d) Fourier Analysis by T. W. Korner.

6. **Applied Multivariate Analysis** (Elective course)
Same as in present ASDA.

7. **Advanced Sample Surveys** (Elective course)
Same as in present ASDA.

8. **Topics in Design of Experiments** (Elective course)
Recovery of interblock information in block designs. Optimality criteria and discussion of optimal block designs. Symmetric and asymmetric factorials. Fractional factorials and orthogonal arrays, including optimality of fractional factorials based on orthogonal arrays and construction of orthogonal arrays. Repeated measurement designs.

9. **Survival and Actuarial Models** (Elective course)
Concepts of Time, Order and Random Censoring.

Life distributions - Exponential Gamma, Weibull, Lognormal, Pareto, Linear Failure rate. Parametric inference Point estimation, Confidence Intervals, scores, tests based on LR , MLE

Life tables, Failure rate, mean residual life and their elementary properties. Ageing classes - IFR, IFRA, NBU, NBUE, HNBUE and their duals, Bathtub Failure rate.

Estimation of survival function - Actuarial Estimator, Kaplan - Meier Estimator, Estimation under the assumption of IFR/DFR.

Tests of exponentiality against non-parametric classes - Total time on test, Deshpande test. Two sample problem - Gehan Test, Log rank test. Mantel - Haenszel Test, Tarone - Ware tests. Rank tests with censored data.

Binomial and Poisson models for discrete data.

Markov Models: Two-state model, illness-death model, maximum likelihood estimator and its properties.

Semi-parametric regression for failure rate - Cox's proportional hazards model with one and several covariates.

Survival data with competing risks.

Books recommended:

(a) Analysis of Survival Data by D. R. Cox and D. Oakes.

(b) Survival Distributions: Reliability Applications in the Biomedical Sciences by A. J. Gross and V. A. Clark.

(c) Survival models and Data Analysis by R. E. Elandt-Johnson and N. L. Johnson.

(d) Survival Analysis by R. G. Miller.

(e) Classical competing risks by M. Crowder.

10. **Nonparametric Inference** (Elective course)
    Same as existing syllabus

11. **Actuarial Methods** (Elective course)
    Same as in present AS.

12. **Generalized inverses and applications** (Elective course)
    Basic properties of generalized inverse (g-inverse),
    least-squares and minimum-norm g-inverse and Moore-Penrose inverse.
    Partitioned matrices, constrained g-inverse, Group inverse and Drazin inverse
    Determinantal identities
    g-inverse over general algebraic structures
    Applications in statistics, Markov chains, iterative methods
    Computational aspects.

    Books recommended:

    (a) Compbell and Meyer.

    (b) Ben-Isreal and Greville

13. **Graph Theory** (Elective course)

Graph : isomorphism, adjacency and incidence matrices, degree sequence, Havel-Hakimi Theorem. Erdös-Gallai Theorem.

Paths, connectedness, trees; characterization. Minimal connector problem. Kruskal's algorithm. Cut vertices, cut edges.

Euler chain characterization theorem; Hamiltonian graph. Different degree sequences vs. Hamiltonicity. Travelling salesman problem. Applications.

Edge colouring, vertex colouring numbers. Vizing-Gupta theorem. Bipartitie graphs. Applications. Planar graphs, statement of Kuratowski's characterization. Examples. The five-colour theorem and the statement of the four-colour thoerem, graphs on surfaces, Directed graphs. Applications.

Flows, cuts, max-flow min-cut theorem; applications and examples

Ramsey theorey

Eigenvalues of graphs

Books recommended:

(a) West

(b) Bondy and Murty

14. **Microeconomics-I** (Elective course)

Same as existing syllabus.

15. **Macroeconomics I** (Elective course)

Same as existing syllabus of Macroeconomic Theory I.

Money: monetary and financial institutions

16. **Microeconomics-II** (Elective course)

General equilibrium theory: existence of competitive equilibrium, stability, core equivalence, Welfare economics: Fundamental theorems of welfare economics, market failure, public goods and externalities, Social choice theory: majority voting, Arrow's Impossibility Theorem, Economics of information: Moral hazard and adverse selection.

17. **Game Theory** (Elective course)