Test Code: CS(Short answer type) 2001

M.Tech. in Computer Science

The candidates for M.Tech. in Computer Science will have to take two tests – Test MIII (objective type) in the forenoon session and Test CS (short answer type) in the afternoon session. The CS test booklet will have two groups as follows.

GROUP A

A test for all candidates in mathematics at the B.Sc. (pass) level, carrying 30 marks.

GROUP B

A test, divided into several sections, carrying equal marks of 70 in mathematics, statistics, and physics at the B.Sc. (Hons.) level and in computer science, engineering and technology at the B.Tech. level. A candidate has to answer questions in *one* of these sections *only* according to his/her choice.

The syllabus and sample questions of the CS test are given below.

Syllabus

GROUP A

Elements of set theory. Permutations and combinations. Functions and relations. Theory of equations. Inequalities.
Limit, continuity, sequences and series, differentiation and integration with applications, maxima-minima, elements of ordinary differential equations, geometry of complex numbers and De Moivre's theorem.
Elementary number theory, divisibility, congruences, primality.
Determinants, matrices, solutions of linear equations, vector spaces, linear independence, dimension, rank and inverse.

GROUP B

Mathematics
(B.Sc. Hons. level)

In addition to the syllabus of Mathematics in Group A, the syllabus includes:

Calculus and real analysis – Real numbers, basic properties; convergence of sequences and series; limits, continuity, uniform continuity of functions; differentiability of functions of one or more variables and applications. Indefinite integral, fundamental theorem of Calculus, Riemann integration, improper integrals, double and multiple integrals and applications. Sequences and series of functions, uniform convergence.

Linear algebra - Vector spaces and linear transformations; matrices and systems of linear equations, characteristic roots and characteristic vectors, Cayley-Hamilton theorem, canonical forms, quadratic forms.

Graph Theory - Connectedness, trees, vertex colouring, planar graphs, eulerian graphs, hamiltonian graphs, digraphs and tournaments.

Abstract algebra – Groups, subgroups, cosets, Lagrange's theorem; normal subgroups and quotient groups; permutation groups; rings, subrings, ideals, integral domains, fields, characteristics of a field, polynomial rings, unique factorization domains, field extensions, finite fields.

Differential equations – Solutions of ordinary and partial differential equations and applications.

Linear programming including duality theory.

Statistics

(B.Sc. Hons. level)

Notions of sample space and probability, combinatorial probability, conditional probability, Bayes theorem and independence, random variable and expectation, moments, standard univariate discrete and continuous distributions, sampling distribution of statistics based on normal samples, central limit theorem, approximation of binomial to normal. Poisson law, Multinomial, bivariate normal and multivariate normal distributions.

Descriptive statistical measures, graduation of frequency curves, product-moment, partial and multiple correlation; regression (simple and multiple); elementary theory and methods of estimation (unbiasedness, minimum variance, sufficiency, maximum likelihood method, method of moments, least squares methods). Tests of hypotheses (basic concepts and simple applications of Neyman-Pearson Lemma). Confidence intervals. Tests of regression. Elements of non-parametric inference. Contingency Chi-square, ANOVA, basic designs (CRD/RBD/LSD) and their analyses. Elements of factorial designs. Conventional sampling techniques, ratio and regression methods of estimation.

Physics

(B.Sc. Hons. level)

Kinetic theory of gases. Equation of states for ideal and real gases. Specific heats of gases. First and Second laws of thermodynamics and thrmodynamical relations. Heat engines. Low temperature physics, Joule-Thomson effect.

Structure of atoms – Bohr-Sommerfeld's model, quantum number, e/m of electrons, mass spectrograph. Wave-particle dualism, De Broglie's equation, X-ray, Bragg's law, Compton effect. Schrodinger's equation, potential well and harmonic oscillator problems. Types of semiconductors, transport phenomena of electrons and holes in semiconductors, *p-n* Junction, radioactivity, special theory of relativity.
Simple harmonic motion. Mements of inertia. Conservation laws.

Coulomb's law, Gauss's Theorem, dielectrics, Biot-Savart's Law, Ampere's law. Electromagnetic induction, self and mutual inductance. Maxwell's equations. Fundamental laws of electric circuits, RC, RL, RLC circuits. Boolean algebra. Transistor and diode circuits. Amplifiers. Oscillators.

Computer Science
(B.Tech. level)

Data structures - stack, queue, linked list, binary tree, heap, AVL tree, B tree, design of algorithms, programming fundamentals, internal sorting, searching, switching theory and logic design, computer organization and architecture, operating systems, principles of compiler construction, formal languages and automata, databases, computer networks.

Engineering and Technology
(B.Tech. level)

Moments of inertia, motion of a particle in two dimentions, elasticity, surface tension, viscosity, gravity, acceleration due to gravity.
Problems of geometrical optics.
First and second laws of thermodynamics, thermodynamic relations and their uses, heat engines.
Electrostatics, magnetostatics, electromagnetic induction.
Magnetic properties of matter - Dia, para and ferromagnetism.
Laws of electrical circuits - RC, RL and RLC circuits, measurement of currents, voltages and resistance.
D.C. generators, D.C. motors, induction motors, alternators, transformers.
p-n junction, transistor amplifiers, oscillator, multivibrators, operational amplifiers.
Digital circuits - Logic gates, multiplexers, demultiplexers, counters, A/D and D/A converters.
Boolean algebra, minimization of switching functions, combinational and sequential circuits.

Note: The questions will be of the following standard as per the above syllabi. All questions in the sample set are not of equal difficulty. They may not carry equal marks in the test.

Sample Questions

GROUP A

Mathematics

A1. If $1, a_1, a_2, \ldots, a_{n-1}$ are the n roots of unity, find the value of

$$(1 - a_1)(1 - a_2)\ldots(1 - a_{n-1}).$$

A2. Let $A = (a_{ij})$ be an n x n matrix, where

$$a_{ij} = \begin{cases} b \text{ if } i = j, \\ c \text{ if } i \neq j, \end{cases}$$

and b, c are real numbers such that $b \neq c$. When is the matrix A invertible?

A3. (a) Let

$$S = \{(a_1, a_2, a_3, a_4): a_i \in \Re, i = 1,2,3,4 \text{ and } a_1 + a_2 + a_3 + a_4 = 0\} \text{ and}$$

$$\Gamma = \{(a_1, a_2, a_3, a_4): a_i \in \Re, i = 1,2,3,4 \text{ and } a_1 - a_2 + a_3 - a_4 = 0\}.$$

Find a basis for $S \cap \Gamma$.

(b) Provide the inverse of the following matrix:

$$\begin{pmatrix} c_0 & c_1 & c_2 & c_3 \\ c_2 & c_3 & c_0 & c_1 \\ c_3 & -c_2 & c_1 & -c_0 \\ c_1 & -c_0 & c_3 & -c_2 \end{pmatrix} \text{ where}$$

$$c_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}, \quad c_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}, \quad c_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}, \quad \text{and} \quad c_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}.$$

(Hint: What is $c_0^2 + c_1^2 + c_2^2 + c_3^2$?)

A4. Let $A_1, A_2, \ldots, A_n$ be n sets, $n > 4$. Let $|A_i|$ represent the number of elements in the set A_i. Suppose x is an element which belongs to each of A_1, A_2, A_3, A_4 but not to any other A_i for $i > 4$. How many times is x counted in the following expression?

$$\sum_{i=1}^{n} |A_i| - \sum_{i_1<i_2} |A_{i_1} \cap A_{i_2}| +$$
$$\sum_{i_1<i_2<i_3} |A_{i_1} \cap A_{i_2} \cap A_{i_3}| -$$
$$\sum_{i_1<i_2<i_3<i_4} |A_{i_1} \cap A_{i_2} \cap A_{i_3} \cap A_{i_4}|$$
$$+\ldots\pm |A_1 \cap A_2 \cap \ldots \cap A_n|$$

A5. Find

$$\lim_{n\to\alpha} (n!e - [n!e])$$

where $[x]$ denotes the largest integer $\le x$, and $e = 1 + 1 + 1/2! + 1/3! + \ldots$

A6. A sequence $\{x_n\}$ is defined by $x_1 = \sqrt{2},\ x_{n+1} = \sqrt{2+x_n},\ n = 1, 2, \ldots$
Show that the sequence converges and find its limit.

A7. Show that $\int_0^{\Pi/2} \sqrt{\cot x}\, dx = \pi/\sqrt{2}.$

A8. Find the total number of English words (all of which may not have proper English meaning) of length 10, where all ten letters in a word are not distinct.

A9. Let $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \ldots + \frac{a_n}{n+1} = 0$, where a_i's are some real constants. Prove that the equation $a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n = 0$ has at least one solution in the interval (0, 1).

A10. Let $\phi(n)$ be the number of positive integers less than n and having no common factor with n. For example, for $n = 8$, the numbers 1, 3, 5, 7 have no common factors with 8, and hence $\phi(8) = 4$. Show that

(i) $\phi(p) = p - 1$,
(ii) $\phi(pq) = \phi(p)\phi(q)$, where p and q are prime numbers.

A11. A set S contains integers 1 and 2. S also contains all integers of the form $3x + y$ where x and y are distinct elements of S, and every element of S other than 1 and 2 can be obtained as above. What is S? Justify your answer.

A12. Let f be a real-valued function such that $f(x+y) = f(x) + f(y)\ \forall x, y \in \mathbf{R}$. Define a function ϕ by $\phi(x) = c + f(x)$, $x \in \mathbf{R}$, where c is a real constant. Show that for every positive integer n,

$$\phi^n(x) = (c + f(c) + f^2(c) + ... + f^{n-1}(c)) + f^n(x);$$

where, for a real-valued function g, $g^n(x)$ is defined by

$$g^0(x) = 0,\ g^1(x) = g(x),\ g^{k+1}(x) = g(g^k(x)).$$

GROUP B

Mathematics

M1. Let $0 < x_1 < 1$. If $x_{n+1} = \dfrac{x_n + 3}{3x_n + 1}$, n = 1,2,3, …

(i) Show that $x_{n+2} = \dfrac{5x_n + 3}{3x_n + 5}$, n = 1,2,3, …

(ii) Hence or otherwise, show that $\lim\limits_{n \to \alpha} x_n$ exists.

(iii) Find $\lim\limits_{n \to \alpha} x_n$.

M2 Given that the power series $\sum\limits_{n=0}^{\infty} a_n x^n$ has radius of convergence 2, find radius of convergence of $\sum\limits_{n=0}^{\infty} a_n x^{n^2}$.

M3 (a) Prove the inequality $e^x > 1 + (1+x)\log(1+x)$, for $x > 0$.

(b) Show that the series $\Sigma \dfrac{x}{n(1+nx)^2}$ is uniformly convergent on [0,1].

M4. Consider the function of two variables

$$F(x,y) = 21x - 12x^2 - 2y^2 + x^3 + xy^2.$$

(a) Find the points of local minima of F.

(b) Show that F does not have a global minimum.

M5. Find the volume of the solid given by $0 \le y \le 2x,\ x^2 + y^2 \le 4$ and $0 \le z \le x$.

M6. (a) Find the matrix of the quadratic form $(2x_1 - x_2 + 3x_3)^2$ without squaring this. Find the rank and the signature of the quadratic form.

(b) Let S be the subspace of $\boldsymbol{R}^4$ defined by

$S = \{(a_1, a_2, a_3, a_4) : 5a_1 - 2a_3 - 3a_4 = 0\}$.

Find a basis for S.

M7. Let A be a 3x3 matrix with characteristic equation $\lambda^3 - 5\lambda^2 = 0$.

(i) Show that the rank of A is either 1 or 2.

(ii) Provide examples of two matrices A_1 and A_2 such that the rank of A_1 is 1, rank of A_2 is 2 and A_i has characteristic equation $\lambda^3 - 5\lambda^2 = 0$ for $i = 1,2$.

M8. Let X and Y be two finite sets and $J = \{T_i: 1 \le i \le m$ and each T_i is a bijection from X to $Y\}$. Suppose that for any $T_i, T_j, T_k, \in J$, $T_i\ T_j^{-1}\ T_k \in J$. Show that $G = \{T_i^{-1}\ T_j : T_i, T_j \in J\}$ forms a group of order m under composition.

M9. Let G be the group of all 2x2 non-singular matrices with matrix multiplication as the binary operation. Provide an example of a normal subgroup H of G such that $H \ne G$ and H is not a singleton.

M10. Let $\boldsymbol{R}$ be the field of reals. Let $\boldsymbol{R}[x]$ be the ring of polynomials over $\boldsymbol{R}$, with the usual operations.

(a) Let $I \subseteq \boldsymbol{R}[x]$ be the set of polynomials of the form $a_0 + a_1x + ... + a_nx^n$ with $a_0 = a_1 = 0$. Show that I is an ideal.

(b) Let $\boldsymbol{P}$ be the set of polynomials over $\boldsymbol{R}$ of degree ≤ 1. Define $\oplus$ and Θ on $\boldsymbol{P}$ by $(a_0 + a_1x) \oplus (b_0 + b_1 x) = (a_0 + b_0) + (a_1 + b_1)x$ and $(a_0 + a_1x) \Theta (b_0 + b_1x) = a_0b_0 + (a_1b_0 + a_0b_1)x$. Show that (P, $\oplus$, Θ) is a commutative ring. Is it an integral domain? Justify your answer.

M11. (a) If G is a group of order 24 and H is a subgroup of G of order 12, prove that H is a normal subgroup of G.

(b) Show that a field of order 81 cannot have a subfield of order 27.

M12. Solve the following differential equations: $(a)\ \ y'' - y = \dfrac{2}{1+e^x}$ and

$(b)\ \ x^2y'' - xy' + y = x$ with $x = 1$, $y = 0$ and $x = e$, $y = 1$.

M13. Consider the Linear Programming (LP) problem :
maximise $C^t x$ subject to $Ax = b, x \ge 0$, where A is of order m x n with rank m.

(a) Define feasible basis and basic feasible solution.

(b) State whether each of the following statements is true or false giving brief justification.

(i) If the above LP has a basic feasible solution, it has an optimal basic feasible solution

(ii) If the above LP has a feasible solution, it has an optimal solution.

(c) Suppose the first m columns of A form a feasible basis B and x is the corresponding basic feasible solution. Show that $x \ne ½\, s + ½\, t$ for any feasible solutions s and t different from x.

M14. a) Prove that there exists no 3-connected graph with 7 edges.
b) Are all hamiltonian graphs eulerian? What about the converse?

M15. χ(D), the chromatic number of a digraph D=(V,A), is defined as the minimum number of parts in a partition $V = V_1 \cup V_2 \cup \ldots \cup V_k$, such that, V_i is an independent set for $1 \le i \le k$. Justify whether every digraph D contains a path of length at lease $\chi(D) - 1$.

Statistics

S1. A safety device is designed to have a high conditional probability of operating when there is a failure (dangerous condition) and high conditional probability of not operating when a failure does not occur. For a particular brand of safety device both these probabilities are 0.98. Given that a dangerous condition occurs with probability 0.01 find the conditional probability that there was a failure when the safety device worked.

S2. Suppose that in a gathering of 1000 people there are 10 persons with Ph.D.; 20 persons are chosen at random with replacement. Show that the probability of finding at least two persons with Ph.D. in the sample, is approximately equal to 0.04 (you may use the approximation $e^{-x} \approx 1 - x$ for small x.)

S3. If a die is rolled m times and you had to bet on a particular number of sixes occurring, which number would you choose? Is there always one best bet, or could there be more than one?

S4. $X_1, X_2, \ldots, X_n$ are independently and identically distributed with common density

$$\frac{1}{\theta}\ e^{-x/\theta},\ x > 0,\ \theta > 0.$$

(a) Show that $T_0 = \sum \frac{x_i}{n}$ is the MVUE for θ.

(b) Let $T_c = c\sum X_i$. Show that $E_\theta(T_c - \theta)^2 = \theta^2 E_1(T_c - 1)^2$, and hence obtain a value of c which minimizes $E_\theta(T_c - \theta)^2$. Comment on the implications of (a) and (b).

S5. New laser altimeters can measure elevation to within a few inches, without bias. As a part of an experiment, 25 readings were made on the elevation of a mountain peak. These averaged out to be 73,631 inches with a standard deviation (SD) of 10 inches. Examine each of the following statements and ascertain whether the statement is true or false, giving reasons for your answer.

(a) 73631 ± 4 inches is a 95% confidence interval for the elevation of the mountain peak.

(b) About 95% of the readings are in the range 73631 ± 4 inches.

(c) There is about 95% chance that the next reading will be in the range of 73631 ± 4 inches.

S6. Consider the following block design with treatments *a, b, c* and *d*:

Block 1	*a b*
Block 2	*a c d*
Block 3	*b c d*

(a) Are all the treatment contrasts estimable? Justify.
(b) What is the BLUE for the treatment contrast $c - d$?

S7. Let X be a discrete random variable having the probability mass function

$$p(x) = \Lambda^x(1- \Lambda)^{1-x}, x = 0, 1,$$

where Λ takes values $\geq \frac{1}{2}$ only. Find the most powerful test, based on 2 observations, for testing H : $\Lambda = \frac{1}{2}$ against $H_1 : \Lambda = \frac{2}{3}$, with level of significance 0.05.

S8. *Let* $X_1, X_2, \ldots, X_n$ be n independent $N(\theta,1)$ random variables where $-1 \leq \theta \leq 1$. Find the maximum likelihood estimate of θ and show that it has smaller mean square error than the sample mean $\bar{X}$.

S9. Let $t_1, t_2, \ldots t_k$ be k independent and unbiased estimators of the same parameter θ with variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2$. Define $\bar{t}$ as $\sum_{i=1}^{k} t_i / k$. Find E($\bar{t}$) and the variance of $\bar{t}$. Show that $\sum_{i=1}^{k} (t_i - \bar{t})^2 / \{k(k-1)\}$ is an unbiased estimator of var ($\bar{t}$).

S10. Consider a simple random sample of n units, drawn without replacement from a population of N units. Suppose the value of Y_1 is unusually low whereas that of Y_n is very high. Consider the following estimator of $\bar{Y}$, the population mean.

$$\hat{\bar{Y}} = \begin{cases} \bar{y} + c, \text{ if the sample contains unit } 1 \text{ but not unit } N; \\ \bar{y} - c, \text{ if the sample contains unit } N \text{ but not unit } 1; \\ \bar{y}, \text{ for all other samples;} \end{cases}$$

where $\bar{y}$ is sample mean and c is a constant. Show that $\hat{\bar{Y}}$ is unbiased. Given that

$$V(\hat{\bar{Y}}) = (1 - f)\left[\frac{S^2}{n} - \frac{2c}{N-1}(Y_N - Y_1 - nc)\right] \text{ where } f = \frac{n}{N} \text{ and } S^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2,$$

comment on the choice of c.

S11. In order to compare the effects of four treatments *A,B,C,D,* a block design with 2 blocks each having 3 plots was used. Treatments *A,B,C* were given randomly to the plots of one block and treatments *A,B,D* were given randomly to the plots of the other block. Write down a set of 3 orthogonal contrasts with the 4 treatment effects and show that all of them are estimable from the above design.

S12. Let y_1, y_2 and y_3 be independent and identically distributed random variables with distribution $N(\mu,1)$. Find a_1,a_2 and b_1,b_2,b_3 such that $U_1=a_1y_1+a_2y_2$ and $U_2=b_1y_1+b_2y_2+b_3y_3$ are independent $N(0,1)$, hence express

$y_1^2+y_2^2+y_3^2-\dfrac{(y_1^2+y_2^2+y_3^2)^2}{3}$ in terms of U_1 and U_2 and show that

$y_1^2+y_2^2+y_3^2-\dfrac{(y_1^2+y_2^2+y_3^2)^2}{3}$ follows the x^2 distribution with two d.f.

S13. In a factory, the distribution of workers according to age-group and sex is given below.

Sex ↓	Age-group 20-40 yrs.	Age-group 40-60 yrs.	Row total
Male	60	40	100
Female	40	10	50
Column Total	100	50	150

Give a scheme of drawing a random sample of size 5 so that both the sexes and both the age-groups are represented. Compute the first-order inclusion probabilities for your scheme.

Physics

P1. A beam of X-rays of frequency v falls upon a metal and gives rise to photoelectrons. These electrons in a magnetic field of intensity H describe a circle of radius γ. Show that

$$h(v-v_0)=m_0c^2\left[\left(\frac{1+e^2\gamma^2H^2}{m_0^2c^4}\right)^{\frac{1}{2}}-1\right]$$

where v_0 is the frequency at the absorption limit and m_0 is the rest mass of the electron, e being expressed in e.s.u.

P2. To find C_p/C_v of a gas, one sometimes uses the following method. A certain amount of gas with an initial temperature T_0, pressure P_0, and volume V_0 is heated by a current flowing through a platinum wire for a time t. The experiment is done twice: First at a constant volume V_0 with the pressure changing from P_0 to P_1, and then at a constant pressure P_0 with the volume changing from V_0 to V_1. The time t is the same in both experiments. Considering the gas to be ideal, show that

$$\frac{C_p}{C_v} = \frac{P_1/P_0 - 1}{V_1/V_0 - 1}.$$

P3. Near the point A lying on the boundary between glass and vacuum the electric field strength in vacuum is equal to $|\vec{E}|$, the angle between the vector $\vec{E}_0$, and the normal $\vec{n}$ on the boundary line being α_0. Find the field strength $|\vec{E}|$ in the glass near the point A and also the angle α between $\vec{E}$ and $\vec{n}$ Also find the surface density of the bound charges at A, and the flux of the vector $\vec{E}$ through a sphere of radius R with center located at the surface of the dielectric.

P4. (a) Show that it is impossible for a photon to give up all its energy and momentum to a free electron.

(b) Find the proper length of a rod in the laboratory frame of reference if its velocity is $v = c/2$, its length is $l = 1m$, and the angle between the rod and its direction of motion is 45 deg.

P5. A simple form of underfloor heating for a room 6m long and 5.5m wide consists of wire netting through which current may be passed. All the holes in the net are hexagons of side 5cm. One pair of opposite sides of each hexagon lies parallel to the shorter side of the room and consists of double strands of wire twisted together; all other edges consist of single strands. This net is connected to two conducting bars along the longer side of the room. If the supply voltage is 220V r.m.s., what diameter of wire should be used to dissipate approximately 2 kW as heat within the room, if the resistivity of material of wire = $10^{-6}\,\Omega m$. [Neglect edge effects.]

P6. (a) Determine the energy levels and the normalized wave functions of a particle in a potential well,

$$\begin{aligned} V(x) &= \infty \qquad \text{for } x < 0 \text{ and for } x > a, \\ &= 0 \qquad \text{for } 0 < x < a. \end{aligned}$$

(b) Show that for the particle in the above well, the following relations hold:

$$\bar{x} = \frac{a}{2}, \quad \overline{(x-\bar{x})^2} = \frac{a^2}{2}\left(1 - \frac{6}{n^2\pi^2}\right).$$

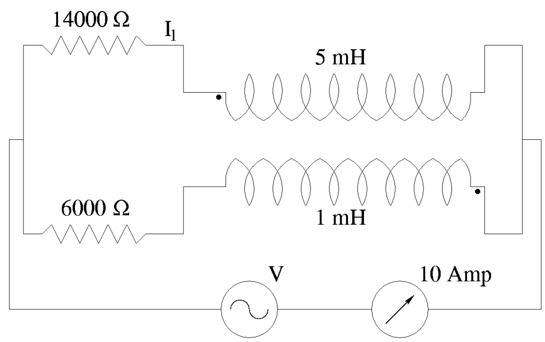
$\bar{x} \equiv$ expectation value of x

$n \equiv$ integer

(c) Show that for large n, the second equality agrees with the classical result. You can use the relation

$$\int x^2 \cos(bx)dx = \frac{x}{b^2}(bx\sin(bx) + 2\cos(bx)) - \frac{2}{b^3}\sin(bx)$$

P7. Consider the following circuit in which an a.c. source of V volts at a frequency of $10^6/\pi$ cycles/sec is applied across the combination of resistances and inductances. The total rms current flowing through the circuit as measured by an a.c. ammeter is 10 amp. Find the rms current I_1 flowing through the upper branch of impedances. The self inductance of the two coils are as shown in the figure. The mutual inductance between the coils is 2 mH and is such that the magnetization of the two coils are in opposition.

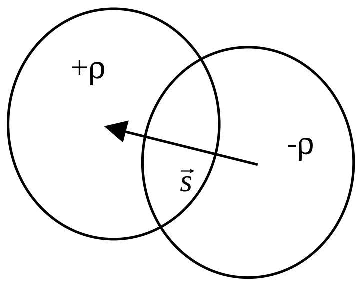


P8. A relay has a resistance $20\,\Omega$ and an inductance 0.5H. It is energised by a d.c. voltage pulse which rises from 0 to 10 volts instantaneously, remains constant for 0.25 s and then instantly falls to zero value. The relay closes when the current in it attains a value 200mA and opens when it drops to 100mA. Find the time for which the relay remains closed.

P9. A silicon based p-n junction has an equal concentration of donor and acceptor atoms. Its depletion zone of width *d* is symmetrical about its junction plane.

(a) What is the maximum electrical field E_{max} in depletion zone, if *k* is the dielectric constant of silicon?
(b) What is the potential difference, V, existing across the depletion zone?
(c) If the potential difference across the depletion zone is 0.4 volt and the concentration of donor/acceptor atoms 3 x 10^{22} m^{-3}, find the width *d* of the depletion zone and maximum electric field in the zone.

P10. (a) Two spheres, each of radius R and carrying uniform volume charge densities $+\rho$ and $-\rho$ respectively, are placed so that they partially overlap (see the following figure). Denote the vector from negative charge center to positive charge center by $\vec{s}$. Show that the field in the region of overlap is constant and find its value.



(b) Find the electrostatic energy of a uniformly charged spherical shell of total charge q and radius R.

P11. An elementary particle called Σ^-, at rest in laboratory frame, decays spontaneously into two other particles according to $\Sigma^- \to \pi^- + n$. The masses of Σ^-, π^- and n are M_1, m_1, and m_2 respectively.

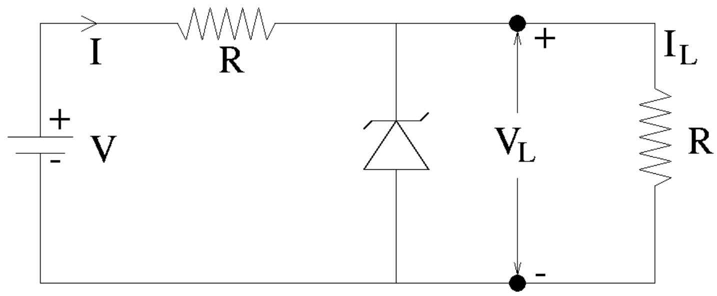
(a) How much kinetic energy is generated in the decay process?

(b) What are the ratios of kinetic energies and momenta of π^- and n?

P12. The Zener diode regulates at 40V over a range of diode currents from 5 to 40mA. The supply voltage $V = 200$V.

(a) Calculate R to allow voltage regulation from a load current $I_L = 0$ upto I_{max}, the maximum possible value of I_L. Find the value of I_{max}.

(b) With the value of R found above, and the load current $I_L = 25mA$, what are the limits between which V may vary without loss of regulation in the circuit ?



Computer Science

C1. Consider the following program:

procedure $P(X, Y, Z)$;
begin $Y \leftarrow Y + 1$;
$Z \leftarrow Z + X$;
end P;

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begin    A ← 2; B ← 3;
         P(A + B, A, A);
         print A

end;
```

What will be printed by the above program assuming the following methods of passing parameters:

(i) call by name
(ii) call by reference
(iii) call by value
(iv) copy restore.

C2. Algorithm Rearrange *(A, i, j)*

Begin

1. If $A(i) > A(j)$

   then exchange $A(i) \Leftrightarrow A(j)$
2. If i+1 $\geq j$

   then return
3. k $\leftarrow \lfloor$(j-i+1)/3$\rfloor$
4. Rearrange $(A, i, j-k)$
5. Rearrange $(A, i+k, j)$
6. Rearrange $(A, i, j-k)$

End

Let *A* = (5,3,2,7). What would be the output of Rearrange (A, 1, 4)? To justify your answer, show the configuration of the array *A* after each step.

C3. A relation *R* contains the following attributes, *R* (A, B, C, D, E). The following functional dependencies are defined on the attributes,

$A \longrightarrow B$, $B \longrightarrow C$, $C \longrightarrow D$, $D \longrightarrow E$ and $E \longrightarrow A$.

a) What are the candidate keys of *R*?
b) Justify that *R* is in BCNF.
c) If the last dependency $E \rightarrow A$ is not present, show that *R* cannot be decomposed under BCNF preserving the other dependencies. Use only the given dependencies without computing the transitive closure.

C4. Describe very briefly a data structure to present a set $S \subseteq \{1, 2, \ldots, n\}$ such that each of the following operations takes constant time [independent of |*S*|].

(i) insert an element
(ii) delete an element
(iii) return any element other than the minimum

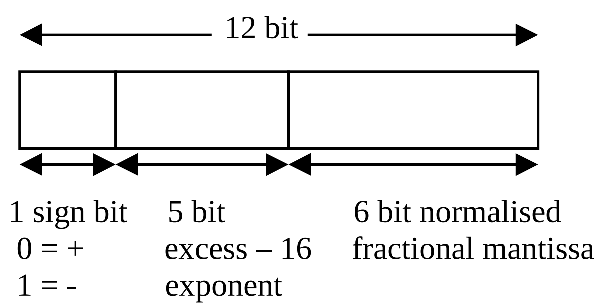
[Hint: You may possibly use an array and a doubly-linked list.]

C5. (a) Consider a pipelined processor with *m* stages. The processing time at every stage is the same. What is the speed-up achieved by the pipelining ?

(b) Define 'hit-ratio' in the context of a hierarchical memory organization.

(c) In a certain computer system, suppose 750 ns (nanosec) is the access time for main memory and 50 ns is the time for searching the set of associative registers. Find the percentage decrease in the effective access time if the hit ratio is increased from 80% to 90%.

C6. (a) Consider the floating point binary numbers represented in the 12-bit format as shown below.



(i) What does "normalized" mean in the context of this format ?

(ii) Represent the numbers +2.1 and -0.07 in this format.

(b) Following are the solutions for the two process (p_i and p_j) critical section problem. Find the errors (if any) in these solutions and rectify them. The notations have usual meanings and i = 0, 1; j = 1-i.

Solution 1

```
Pi:  repeat
       while flag [j] do skip;
       flag [i] = true;
       critical section;
            flag [i] = false;
       exit section;
       until false;
```

Solution 2

```
Pi:   repeat
        flag[i] = true;
        while flag [j] do skip ;
        critical section:
             flag [i] = false ;
        exit section;
```

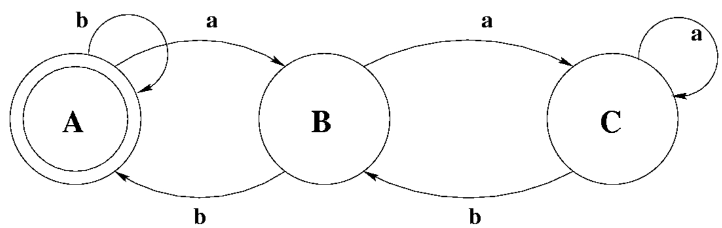
until false;

C7. (a) Data link protocols almost always put the CRC in the trailer, rather than header. Why?

(b) Sketch the Manchester encoding for the bit stream 0001110101.

(c) If delays are recorded in 8-bit numbers in a 50-router network, and delay vectors are exchanged twice a second, how much bandwidth per (full-duplex) line is consumed by the distributed routing algorithm? Assume that each router has 3 lines to other routers.

C8. (a) Let *S* be the set of strings that are accepted by the following automaton.

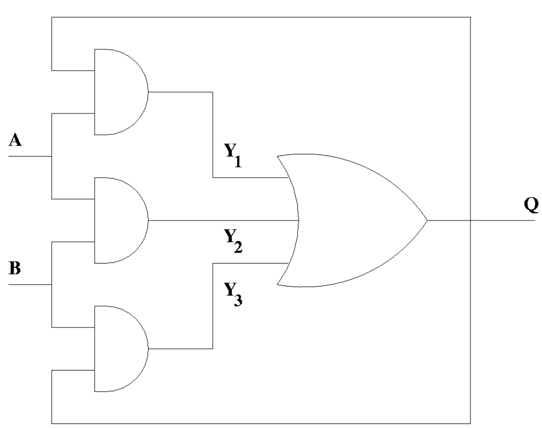


Give a regular expression that describes *S.*

(b) Consider the following grammar *G: {(S, A), (a, b), S, P}*

$P: \begin{Bmatrix} S \rightarrow Sa \,|\, Aa \,|\, b \\ A \rightarrow Aa \,|\, b \end{Bmatrix}$. Show by an example that *G* is ambiguous. Write an equivalent unambiguous grammar.

C9. (a) Using the circuit diagram given below, find the output *Q* for various values of *A* and *B*, assuming that the inputs do not change until the output is stable.



(b) A certain four-input gate *G* realizes the switching function *G(a, b, c, d) = abc + bcd*. Assuming that the input variables are available in both complemented and uncomplemented forms:

(i) Show a realization of the function $f(u, v, w, x) = \Sigma(0, 1, 6, 9, 10, 11, 14, 15)$ with only three G gates and one OR gate.

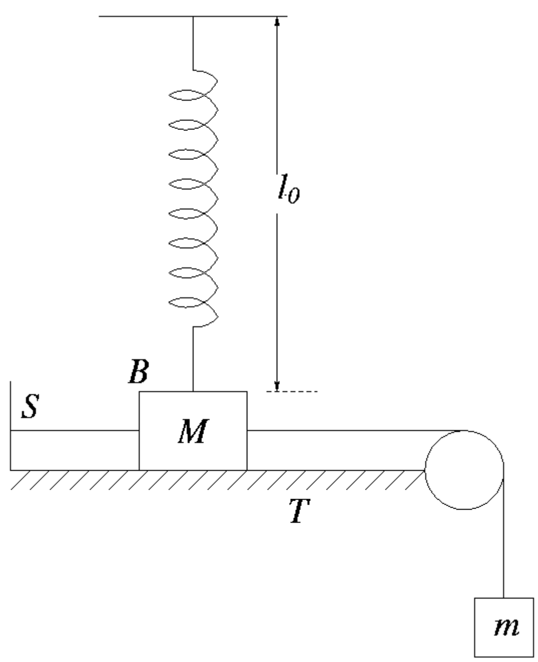
(ii) Can all switching functions be realized with $\{G, OR\}$ logic set ?

C10. Let L_1 and L_2 be two arrays each with $n = 2^k$ elements sorted separately in ascending order. If the two arrays are placed side by side as a single array of $2n$ elements, it may not be found sorted. All the $2n$ elements are distinct. Considering the elements of both the arrays, write an algorithm with $k + 1$ comparisons to find the n-th smallest element among the entire set of $2n$ elements.

Engineering and Technology

E1. A rocket weighing 50,000 kg has been designed so as to eject gas at a constant velocity of 250 meters/sec. Find the minimum rate at which the rocket should lose its mass (through ejection of gas) so that the rocket can just take off.

E2.



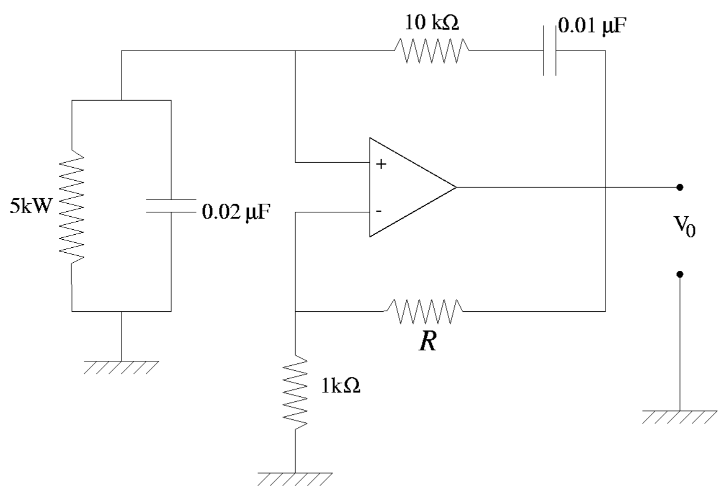
A bar B of mass M is resting on a smooth horizontal table T and is attached to a point S on the table by an inextensible thread as shown in the above figure. It is attached to another block of mass m by a weightless, frictionless pulley. The bar B is also attached to a massless spring of undeformed length l_0 and spring constant k = 5Mg/ l_0. The thread SB is snapped and then the bar starts moving. Find the velocity of the bar B at the time it just lifts off the plane.

E3. A light elastic string of length L and uniform area of cross-section A is suspended vertically from a point and carries a heavy mass *M* at its lower free end which stretches it through a distance *l*. If the string is pulled down through a small distance and then released, show that the vertical oscillation of the system will be a simple harmonic motion and find its time-period. Assume that the Young's modulus of the material of the string is *Y*.

E4. A cylinder of mass *M*, radius *r* and height *h* suspended by a spring whose upper end is fixed, is submerged in water. In equilibrium position the cylinder sinks half of its height. Before coming to the equilibrium position the cylinder was submerged to 2/3 of its height at a time point and then with no initial velocity started moving up vertically. Find the height of the cylinder submerged at any time *t* before reaching the equilibrium position. (*s* = density of water, *k* = coefficient of stiffness of the spring).

E5. A flywheel of mass 100 kg and radius of gyration 20 cm is mounted on a light horizontal axle of radius 2 cm, and is free to rotate on bearings whose friction may be neglected. A light string wounded on the axle carries at its free end a mass of 5 kg. The system is released from rest with the 5 kg mass hanging freely. If the string slips off the axle after the weight has descended 2 m, prove that a couple of moment of $10/\pi^2$ kg. wt.cm. must be applied in order to bring the flywheel to rest in 5 revolutions.

E6. In the circuit shown below, the Op-Amp is an ideal one.
(a) Show that the conditions for free oscillation can be met in the circuit.
(b) Find the ideal value of *R* to meet the conditions for oscillation.
(c) Find the frequency of oscillation. (Assume $\pi = 3.14$).



E7. Two bulbs of 500cc capacity are connected by a tube of length 20 cm and internal radius 0.15 cm. The whole system is filled with oxygen, the initial pressures in the bulbs before connection being 10 cm and 15 cm of Hg, respectively. Calculate the time taken for the pressures to become 12 cm and 13 cm of Hg, respectively. Assume that the coefficient of viscosity η of oxygen is 0.000199 cgs unit.

E8. A converging lens of focal length f_1 = 40 cm is immovably fixed at a distance of 40 cm from a screen. An object is placed 120 cm away from the lens on the other side as that of the screen. You are given another lens of focal length f_2 = 40 cm. Where should this second lens be placed in order to focus the image of the object onto the screen? (there are two possible positions and both of them should be reported.)

E9. (a) A piston of mass *m* divides a cylinder containing a gas into two equal parts. Each part contains gas at pressure P_0, and has length *l*, cross-sectional area *A* and volume V_0. The piston is slightly displaced to one side and released. Assuming that the temperature remains constant, find the frequency of the oscillation of the piston (assume that the initial displacement of the piston is very small).

(b) A piston can move inside a sealed horizontal cylinder. Initially, the piston separates the space inside the cylinder into two equal parts, each containing an ideal gas at P_0, V_0 and *T*. What work has to be performed in order to isothermally increase the volume of one part of gas to *N* times the other by slowly moving the piston?

E10. (a) A process can reach the equilibrium state (2) from an initial state (1) following three different paths *A* (Adiabatic), *B* (Constant volume followed by constant pressure) and *C* (Constant pressure followed by constant volume). Derive the energy flowing in the system along paths B and C if internal energy at state (1) is 100 kg-wt-m, W_A = 20 kg-wt-m, W_B = 17 kg-wt-m and W_C = 27 kg-wt-m (W_i, $i = A, B, C$ is the work done along the path *i*).

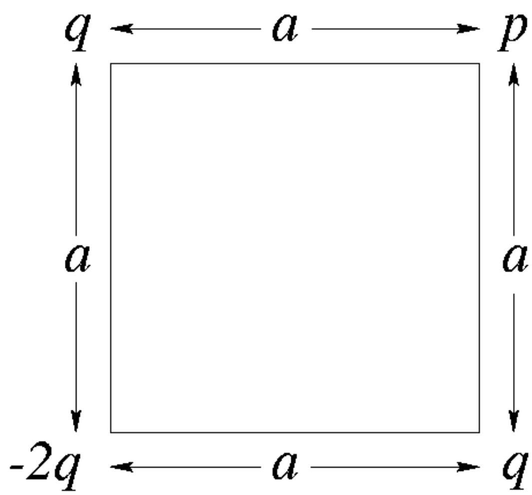
(b) A manufacturer claims the following specifications for his heat engine:
Power developed: 48 kW;
Fuel burnt/hr: 3 kg;
Heating value of the fuel: 60,000 kJ/kg;
Temperature limits: 477^0C and 27^0C.
Comment on the feasibility of the claim.

E11. A spherical charge distribution has a volume density ρ, which is a function of *r*, the radial distance from the center of the sphere, as given below.

$$\rho = \begin{cases} A/r, \ A \text{ is constant } \ for\ 0 \le r \le R \\ 0, \quad \text{for}\ r > R \end{cases}$$

Determine the electric field as a function of *r*, for $r \geq R$. Also deduce the expression for the electrostatic potential energy $U(r)$, given that $U(\infty) = 0$ in the region $r \geq R$.

E12. Consider the distribution of charges as shown in the figure below. Determine the potential and field at the point *p*.



E13. A proton of velocity 10^7 m/s is projected at right angles to a uniform magnetic induction field of 0.1 w/m^2. How much is the path of the particle deflected from a straight line after it has traversed a distance of 1 cm? How long does it take for the proton to traverse a 90^0 arc?

E14. Calculate the diamagnetic susceptibility of neon at standard temperature and pressure (0^0C and 1 atmospheric pressure) on the assumption that only the eight outer electrons in each atom contribute and their mean radius is 4.0 X 10^{-9} cm.

E15. A circular disc of radius 10cm is rotated about its own axis in a uniform magnetic field of 100 weber/m^2, the magnetic field being perpendicular to the plane of the disc. Will there be any voltage developed across the disc? If so, then find the magnitude of this voltage when the speed of rotation of the disc is 1200 rpm.

E16. A 3-phase, 50-Hz, 500-volt, 6-pole induction motor gives an output of 50 HP at 900 rpm. The frictional and windage losses total 4 HP and the stator losses amount to 5 HP. Determine the slip, rotor copper loss, and efficiency for this load.

E17. Two series traction motors run at 600 rpm and 750 rpm respectively when taking a current of 60 A from a 600 V supply. Total resistance of each motor is 0.3Ω. If the two motors are mechanically coupled and put in series across 600 V, at what speed will they run when still taking a current of 60 A?

E18. An alternator on open-circuit generates 360 V at 60 Hz when the field current is 3.6 A. Neglecting saturation, determine the open-circuit e.m.f. when the frequency is 40 Hz and the field-current is 24 A.

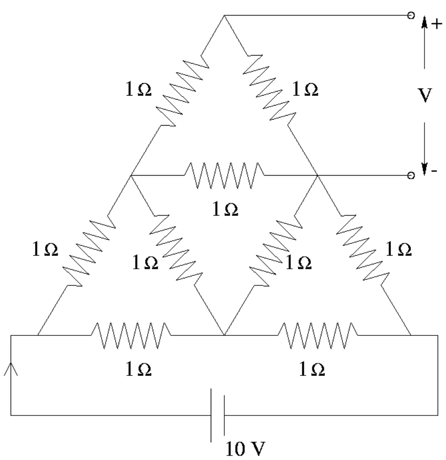
E19. A 150 KVA, 4400/440 volt single phase transformer has primary and secondary resistance and leakage reactance values as follows:

$R_p = 2.4\ \Omega$, $R_s = 0.026\Omega$, $X_p = 5.8\Omega$, and $X_s = 0.062\Omega$.

This transformer is connected with a 290 KVA transformer in parallel to deliver a total load of 330 KVA at a lagging power factor of 0.8. If the first transformer alone delivers 132 KVA, calculate the equivalent resistance, leakage reactance and percent regulation of the second transformer at this load. Assume that both the transformers have the same ratio of the respective equivalent resistance to equivalent reactance.

E20. The hybrid parameters of a *p-n-p* junction transistor used as an amplifier in the common-emitter configuration are: h_{ie} = 800Ω, h_{fe} = 46, h_{oe} = 8 x 10^{-5} mho, h_{re} = 55.4 x 10^{-4}. If the load resistance is 5 $k\Omega$ and the effective source resistance is 500 Ω, calculate the voltage and current gains and the output resistance.

E21. In the circuit below, determine the current *I* and the voltage *V*.

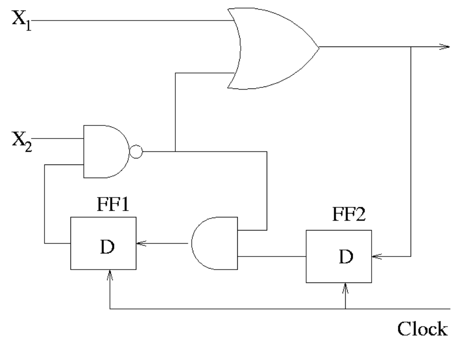


E22. (a) Show that a 2^n -to-1 multiplexer can be used to realize any arbitrary (n+1) variable Boolean function.

(b) Design a modulo-6 counter using JK flip-flops which counts in the way specified below.

0	0	0
0	0	1
0	1	1
0	0	1
1	1	0
1	0	0

E23. In the figure, consider that *FF*1 and *FF*2 cannot be set to a desired value by reset/preset line. The initial states of the flip-flops are unknown. Determine a sequence of inputs (x_1, x_2) such that the output is zero at the end of the sequence.



X_1
X_2
FF1
D
FF2
D
Clock