Test Code : RP : (Short Answer type) 2005

Junior Research Fellowship in Theoretical Physics and Applied Mathematics

The candidates for Junior Research Fellowships in Applied Mathematics and Theoretical Physics will have to take two tests−Test MIII (objective type) in the forenoon session and Test RP (short answer type) in the after noon session.

The RP test booklet will consist of two parts. The candidates are required to answer from Part I and only one of the remaining parts II, III and IV.

The syllabus and sample questions for the test are as follows.

PART-I

Mathematical and logical reasoning

Syllabus

B.Sc. Pass Mathematics syllabus of Indian Universities.

Sample Questions

1. Let f be a real valued function defined on the interval $[-2, 2]$ as:

$$f(x) \;=\; \begin{cases} (x+1)2^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & \text{for} \quad x \neq 0 \\ 0 & \text{for} \quad x = 0 \end{cases}$$

   i) Find the range of the function.

   ii) Is f continuous at every point in $(-2, 2)$? Justify your answer.

2. Let $g : R \to R$ be a continuous function such that $g(x) = g\left(\dfrac{x-1}{2}\right)$ for all x. Show that g must be a constant function.

3. Find the minimum value of

$$2^{\cos x} + 2^{\sin x} \qquad (0 \leq x \leq 2\pi)\,.$$

4. Evaluate the following limit : $\lim\limits_{x\to 0}\left[\dfrac{1}{\log(1+x)} - \dfrac{1}{x}\right]$.

5. If $z_0,\, z_1,\, \cdots\, z_{n-1}$ are the roots of the equation $z^n - 1 = 0$, and $z_0 = 1$, then find the value of

$$(1-z_1)\,(1-z_2)\cdots(1-z_{n-1})$$

6. X is a uniformly distributed random variable with probability density function

$$f(x) = \begin{cases} \dfrac{5}{a} & \text{for} \quad -\frac{a}{10} \leq x \leq \frac{a}{10} \\ 0 & \text{for} \quad \text{otherwise} \end{cases}$$

where a is a nonnegative constant.
If $P(|x| < 2) = 2P(|x| > 2)$, then find a.

7. Let X be a Poisson random variable with parameter λ. Show that

$$P\,(X \text{ is even}) = \frac{1}{2}\left(1 + e^{-2\lambda}\right).$$

8. If $f(x)$ is continuous on $(0, \infty)$ and for $x \neq 0$

$$f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 2$$

find $\int_2^3 f(x)dx$.

9. If $f(x) \geq 0$ for $a \leq x \leq b$, we know that $\int\limits_a^b f(x)dx \geq 0$. However, by actual calculation we find that $\int\limits_{-2}^{1} \frac{dx}{x^2} = -\frac{3}{2}$, which is clearly absurd. What went wrong ?

10. Let $f(x)$ be a given differentiable function. Then find the general solution of the following differential equation in y

$$f(x)\frac{dy}{dx} = yf'(x) - y^2.$$

11. If the equation

$$x^4 - 14x^2 - 24x - c = 0$$

has four real and unequal roots, then show that $8 < c < 11$

12. A point moves in the (x, y)-plane such that at time $t(> 0)$ it has the co-ordinates $(1/t, (1 + t)/\sqrt{2})$. Find when it comes closest to the origin.

13. Solve graphically the LPP

$$\begin{array}{ll} \text{Maximize} & z = 3x_1 + 2x_2 \\ \text{subject to} & 2x_1 - x_2 \geq -1, \\ & x_1 \leq 2, \\ & x_1 + x_2 \leq 3, \\ & x_1, x_2 \geq 0. \end{array}$$

14. If $f(x, f(y)) = x^p y^q$ all x, y, then show that $p^2 = q$ and find $f(x)$.

15. If $\prod_{j=1}^{n}(a_j + ib_j) = 1 + i$, where $a_1, \cdots, a_n$ and $b_1, \cdots, b_n$ are real numbers and $i = \sqrt{-1}$, then find the value of $\tan\left(\sum_{j=1}^{n} \tan^{-1}\frac{b_j}{a_j}\right)$.

16. Find the maximum possible value of xy^2z^3 subject to the conditions x, y, $z \geq 0$ and $x + y + z = 3$.

17. If the lines $3x - 4y + 4 = 0$, $6x - 8y - 7 = 0$ are tangents to the same circle, find the radius of the circle.

18. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, find the trace of A^{100}.

PART-II

Applied Mathematics

Syllabus

1. *Abstract algebra* : Groups, rings, fields.

2. *Real analysis* : Functions of single and several variables, metric space, normed linear space, Riemann Integral, Fourier series.

3. *Differential equations* : ODE − Existence of solution, fundamental system of integrals, elementary notions, special functions. PDE upto second order, equations of parabolic, hyperbolic and elliptic type.

4. *Dynamics of particles and rigid bodies* : Motion of a particle in a plane and on a smooth curve under different laws of resistance, kinematics of a rigid body, motion of a solid body on an inclined smooth or rough plane.

5. *Functions of complex variables* : Analytic function, Cauchy's theorem Taylor and Laurent series, singularities, branch-point, contour integration, analytic continuation.

6. *Numerical analysis* : Solution of a system of linear equations, polynomial interpolation, numerical integration formula (Newton-Cotes type).

7. *Linear programming* : Formulation of a LPP, graphical method of solution, simplex method. Transportation and assignment problems.

8. *Fluid Mechanics* : Kinematics of fluid, equation of continuity, irrotational motion, velocity potential, dynamics of ideal fluid, Eulerian and Lagrangian equations of motion, stream function, sources, sinks and doublets, vortex, surface waves, group velocity, Viscous flow − Navier Stokes equation, boundary layer theory, simple problems.

9. *Probability and statistics* : Probability axioms, conditional probability, probability distribution, mathematical expectations, characteristic functions, covariance, correlation coefficient. Law of large numbers, central limit theorem. Random samples, sample characteristics, estimation, statistical hypothesis, Neyman pearson theorem, likelihood ratio testing.

Sample Questions

1. (a) Let G be a group such that

$$(ab)^m = a^m b^m$$

for three consecutive integers m, $m+1$ and $m+2$ for all $a, b \in G$. Show that G is abelian.

(b) Let R be a ring with a unit element. Form another ring R' by defining

$$a \oplus b = a + b + 1, \quad a.b = ab + a + b$$

Determine the zero element and unit element of R'. (7 + 7)

2. (a) Let $X_1 = [1,2]$ and $X_2 = [0,1]$. Let d_1 denote the Euclidean metric in X_1 and let $d_2(x,y) = 2|x-y|$ in X_2. Show that (X_1, d_1) and (X_2, d_2) are equivalent metric spaces.

   b) Two different metrics on the space $X = x \in R : 0 < x \leq 1$ are defined by $d_1(x,y) = |x-y|$ and $d_2(x,y) = |\frac{1}{x} - \frac{1}{y}|$. Are the spaces (X, d_1) and (X, d_2) equivalent? Give reasons for your answer.

3. A uniform flat disc of mass M and radius r rotates about a horizontal axis through its centre with angular speed ω_0. A chip of mass m breaks off the edge of the disc at an instant such that the chip rises vertically above the point at which it broke off. How high does the chip rise above the point before it starts to fall off ? What is the final angular momentum of the disc ?

4. Show that if the solution of the ODE

$$2xy'' + (3-2x)y' + 2y = 0$$

is expressed in the form $y = \sum_{n=0}^{\infty} a_n\, x^{n+\sigma}$, σ can take two possible values. Find the relation between a_n and a_{n+1}, and show that one solution reduces to a polynomial.

5. Reduce the partial differential equation

$$(a-1)^2 \frac{\partial^2 u}{\partial x^2} - y^{2a} \frac{\partial^2 u}{\partial y^2} = a y^{a-1} \frac{\partial u}{\partial y}$$

to canonical form and find its general solution.

6. a) Let S be a region of the complex z plane and f: $S \rightarrow R$ is a harmonic function. Show that $\frac{\partial f}{\partial x} - i \frac{\partial f}{\partial y}$ is an analytic function in S.

   b) For what value of a is the function $f(z) = \int_1^z (\frac{1}{\zeta} + \frac{a}{\zeta}) \cos \zeta d\zeta$ $(z = x+iy)$ is single valued in the complex z-plane.

7. Prove with the aid of residue theorem of complex variable, that,
$$\int_0^{\infty} \frac{dz}{z^4+1} = \frac{\pi}{2\sqrt{2}}.$$

8. The function $u(t)$ satisfies the differential equation

$$t\frac{d^2u}{dt^2} + (2t+1)\frac{du}{dt} + u = 0$$

and $u(0) = 1$. Show that the Laplace transform of $u(t)$ is $(s^2 + 2s)^{-1/2}$. Explain why the condition $u(0) = 1$ is sufficient to determine a particular solution of this second-order differential equation.

9. Construct an integration rule of the form

$$\int_{-1}^{1} f(x)dx \simeq c_0 f\left(-\frac{1}{2}\right) + c_1 f(0) + c_2 f(\frac{1}{2})$$

which is exact for all polynomials of degree ≤ 2. Find the error term.

10. Solve the following transportation problem for minimum cost starting with the degenerate solution $x_{12} = 30$, $x_{21} = 40$, $x_{32} = 20$, $x_{43} = 60$.

	D_1	D_2	D_3	
0_1	4	5	2	30
0_2	4	1	3	40
0_3	3	6	2	20
0_4	2	3	7	60
	40	50	60	

11. The area of cross section of a large tank is $0.5m^2$. It has an opening near the bottom having area of cross section $1cm^2$. A load of $20kg$ is applied on the water at the top. Find the velocity of water coming out of the opening at the time when the height of water level is $50cm$ above the bottom. Take $g = 10m/\text{sec}^2$.

12. A viscous fluid flows along a circular pipe with diameter D and length L. Assuming one dimensional flow, show that the pressure drop is given by

$$\Delta p = \frac{32\mu L\bar{u}}{D^2}$$

where $\bar{u}$ is the mean velocity of flow and μ is the viscocity of the fluid.

13. The random variables X_1, X_2 ... have $E(X_i) = 0$ for $i = 1, 2, \ldots$ and $E(X_iX_j) = \beta^{j-i}$ for $1 \leq i \leq j$ where $0 < \beta < 1$. If $Y_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, show that

$$\text{Var } (Y_n) = \frac{1+\beta}{n(1-\beta)} - \frac{2\beta(1-\beta^n)}{n^2(1-\beta)^2}$$

PART-III

Theoretical Physics

Syllabus

1. **Classical Mechanics**

   Mechanics of a particle and system of particles−conservation laws− scattering in a central field − Lagrange's equation and their applications. Hamilton's equation, canonical transformation − special theory of relativity, small oscillation, vibration & accoustics.

2. **Electromagnetic theory**

   Classical electrodynamics, Maxwell's equations − gauge transformation − Poynting's theorem − wave equation and plane waves − radiating system and scattering.

3. **Statistical Physics & Condensed Matter Physics**

   Thermodynamic equilibrium, partition functions, density matrix, phase transition, spin systems, Models. Statistical fluctuations, Band theory of electrons, Semiconductor Physics.

4. **Quantum Mechanics and Quantum Field Theory**

   Inadequacy of classical physics − Schrodinger wave equation − general formalism of wave mechanics − exactly soluble eigenvalue problems. Approximation methods − scattering theory − time dependent perturbation theory. Symmetries and Conservation Laws : Relativistic equations : Klein-Gordon/Dirac equations, Lagrangian field theory, Examples of quantum field theory − ϕ^4, Quantum electrodynamics.

5. **Elementary Particles**

   Elementary particles − weak and strong interactions − selection rules − CPT theorem − Symmetry Principles in Particle Physics.

Sample questions for Theoretical Physics

1. A body of mass m was suspended by a non-streched spring (massless) and then set free without push. The stiffness of the string is k. Find the law of motion $y(t)$ where y=displacement of the body from the equilibrium position and the maximum and minimum tensions of the spring during the motion.

2. A wire bent as a parabola $y = ax^2$ is located in a uniform magnetic field of induction B, the vector $\vec{B}$ being perpendicular to the plane (x, y). At time $t = 0$, a connector starts sliding translation wise from the vertex of the parabola with a constant acceleration f. Find the e.m.f. of electromagnetic induction in the loop thus formed as a function of y.

3. (a) One mole of an ideal gas goes through a process in which the entropy S of the gas changes with temperature T as $S = aT + C_v\ lnT$, $a(> 0)$ being a constant, and C_v is the molar heat capacity of the gas at constant volume. Find the volume dependence of the gas temperature in this process, given $T = T_0$ when $V = V_0$.

   (b) An ideal gas undergoes a quasi-static thermodynamic process consisting of two isobaric and two isothermal steps. Show that the net work done in a complete cycle is $W = P_1(V_2 - V_1)\ ln\ \frac{P_2}{P_1}$.

   Take the cycle as :

$$(P_1, V_1) \overset{\text{isothermal}}{\longrightarrow} (P_2, V_1') \overset{\text{isobaric}}{\longrightarrow} (P_2, V_2') \overset{\text{isothermal}}{\longrightarrow} (P_1, V_2) \overset{\text{isobaric}}{\longrightarrow} (P_1, V_1)$$

4. Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30^o with each other. When suspended in a liquid of density ($800kg/m^3$), the angle remains the same. What is the dielectric constant of the liquid? (The density of the material of the sphere is $1600kg/m^3$).

5. (a) A light beam is propagating through a block of glass with refractive index 1.5. If the block is moving with constant velocity in the same direction as the beam, what is the velocity of light in the block as measured by an observer in the laboratory?

   (b) A particle of mass $m_1 = 1gm$ travelling at a speed of $.9c$, c being the speed of light, collides head on with a stationary particle of mass $m_2 = 10gms$ and is embedded in it. What is the rest mass and velocity of the resulting composite particle?

6. A particle of mass m and charge e enters a homogeneous and stationary electric field E with velocity v perpendicular to the direction of the field. Calculate the particle's path.

7. (a) If the effective density of states in valence band is eight times that in convection band in a pure semiconductor at 27^oC, find the shift of Fermi level from the middle of the energy gap assuming low concentration of electrons and holes in the semicondctor. (Boltzmann's constant $K_B = 1.33 \times 10^{-23} J/\text{ mole}^o\ K$, In 2 = .693)

(b) Consider a line of $2N$ ions of alternating charges $\pm q$ with a repulsive potential A/R^n between nearest neighbours in addition to the usual Coulomb potential. Neglecting surface effects find the equilibrium separation R_0 for such a system. Let the crystal be compressed so that R_0 becomes $R_0(1-\delta)$. Calculate work done in compressing a unit length of the crystal to order δ^2.

8. a) A hypothetical semi-conductor has a conduction band (cb) that can be described by $E_{cb} = E_1 - E_2\cos(ka)$ and valence band (vb) which is represented by $E_{vb} = E_3 - E_4\sin^2\left(\frac{ka}{2}\right)$ where $E_3 < (E_1 - E_2)$ and $-\pi/a < k < \pi/a$. Find out the expressions for

   i) the band-widths of the conduction band and the valence band,

   ii) the band-gap of the material,

   iii) the effective mass of the electrons at the bottom of the conduction band.

   (b) A beam of electrons with kinetic energy 1 keV is diffracted as it passes through a polycrystalline metal foil. The metal has a cubic crystal structure with a spacing of $1A^0$. Calculate the wave length of electron and the Bragg angle for the first order diffraction.Take m, h, c the mass of the electron, Planck's constant and speed of light respectively as follows: $mc^2 = .5$Mev, $c = 3\times 10^8$m/s, $h = 6.6\times 10^{-34}$Js. Also take $1eV = 1.6\times 10^{-19}$J.

9. (a) State with reasons which of the following reactions are allowed or forbidden, according to the various conservation rules in particle physics:

   i) $\mu^+ \to e^+ + \tau$

   ii) $n \to p + e^- + \bar{\nu}_e$

   iii) $\Omega^- \to \Xi^0 + K^-$

   iv) $P + P \to P + \Sigma^+ + K^-$

   (b) Consider a Hamiltonian $H = \dfrac{p^2}{2m} + \dfrac{kx^2}{2} + qE_0x\cos(\omega t)$. Determine $\dfrac{d}{dt} < x >$, $\dfrac{d}{dt} < p >$, $\dfrac{d}{dt} < H >$. (Symbols have their usual meanings).

10. a) An electron is moving freely inside a one dimensional potential box with walls at $x = 0$ and $x = a$. If the electron is initially in the ground state of the box and if we suddenly double the size of the box by moving the boundary on the right hand side to $x = 2a$, find the probability of finding the electron in the ground state of the new box.

   (b) A Hamiltonian is of the form

$$H = A^+A^- + \epsilon x^6$$

where $A = \left(\frac{d}{dx} + x\right)$ and $A^\dagger = \left(-\frac{d}{dx} + x\right)$. Taking ϵ small, calculate the ground state energy of the above Hamiltonian to $O(\epsilon)$.

11. Consider the theory of Dirac fermions interacting with photons (QED).

    i) Find expression for the charge current density and show that it is conserved.

    ii) Using Wick's theorem, derive amplitudes for the following processes:

       (a) electron - positron annihilation process of $O(e^2)$ where e is the coupling constant.

       (b) Same process of $O(e^4)$.

       (c) Draw the Feynman diagrams of the above processes.

    iii) Consider the energy projection operators $\Lambda^{\pm}(p) = \pm\frac{\not{p} + mc}{2mc}$, where $\not{p} = \gamma_\mu p^\mu$. Prove the following identities :

       (a) $(\Lambda^+)^2 = \Lambda^+$

       (b) $\Lambda^+\Lambda^- = 0$ (3 + 7 + 4)

12. The points of suspension of two identical simple pendulums each of mass m and length ℓ are located on one horizontal straight line. Points of these pendulums at a distance $h(0 < h < 1)$ from the point of suspension are connected by a spring of rigidity γ. The spring is released when the pendulums occupy the vertical positions. For motion in the vertical plane, write down the Lagrangian of the system in terms of angles ϕ_1, ϕ_2 formed by the pendulums with the vertical at any time t. Show that the frequencies of small oscillations are given by $\sqrt{g/\ell}$ and $\sqrt{g/\ell + \frac{2\gamma h^2}{m\ell^2}}$ respectively.

13. A thin fixed ring of radius 1 metre has a positive charge of 10^{-5} coulomb uniformly distributed over it. A particle of mass .9 gm and having a negative charge of 10^{-6} coulomb is placed on the axis of the ring at a distance of 1 cm from the centre of the ring. Show that the motion of the particle is approximately simple harmonic. Find the time period of oscillation.

14. (a) Show that the magnetic vector potential for two long and straight parallel wires carrying the same current I in opposite direction is given by

$$\vec{A} = \frac{\mu_0 I}{2\pi} ln\left(\frac{r_2}{r_1}\right)\vec{k}$$

where $\vec{k}$ is a unit vector parallel to the wires and r_1, r_2 are the distances of the field points from the wires. (Symbols have their usual meanings).

(b) Given the electromagnetic waves

$$\vec{E} = \vec{i}E_0 \cos \omega t\,(\sqrt{\epsilon_\mu} z - t)$$
$$+\vec{j}E_0 \sin \omega t\,(\sqrt{\epsilon_\mu} z - t)$$

where E_0 is a constant and $\vec{i}$, $\vec{j}$ are unit vectors along x and y axes respectively, find the corresponding magnetic field $\vec{B}$ and the Poynting vector $\vec{S}$.

PART-IV

Statistics
Syllabus

Probability and Sampling Distributions : Notion of sample space, combinatorial probability, conditional probability and independence, random variable and expectation, moments, standard discrete and continuous distributions, sampling distribution of statistics based on normal samples.

Descriptive Statistics (including Numerical Analysis) : Descriptive measures, graduation of frequency curves, correlation and regression (bivariate and multivariate), polynomial interpolation, numerical integration.

Inference : Elementary theory and methods of estimation (unbiasedness, minimum variance, sufficiency, mle, method of moments). Testing of hypotheses (basic concepts and simple applications of Neyman-Pearson lemma).

Designs (including elementary ANOVA) and Sample Surveys : Basic designs, (CRD/RBD/LSD) and their analysis, conventional sampling techniques (srswr/wor) including stratification.

Sample Questions

1. The standard deviation of two sets containing n_1 and n_2 members are σ_1 and σ_2 respectively, being measured from their respective means m_1 and m_2. If the two sets are grouped together as one set of $(n_1 + n_2)$ members, show that the standard deviation σ of this set measured from its mean is given by

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)^2}(m_1 - m_2)^2.$$

2. (a) A pair of dice is thrown. If it is known that one dice shows a 4, calcutate the probability that the total of both the dice is greater than 7.

(b) Let X have a probability distribution with a density at x as

$$\begin{aligned} f(x) &= k_0 \sqrt{x}, \qquad 0 < x < 1 \\ &= 0, \quad \text{elsewhere} \end{aligned}$$

Calcutate the probability $P(0.3 < X < 0.6)$, numerically.

3. A ball is drawn from an urn containing 9 balls numbered 0, 1, 2, $\cdots$, 8, of which the first 4 are white, the next 3 red and the last 2 black. If the colours white, red and black are reckoned as colour numbered 0, 1 and 2 respectively, find the joint distribution of the random variables $-$ the number on the ball drawn and its colour number.

4. Find the maximum likelihood estimator of θ for a random sample from a distribution having the p.d.f.

$$f_\theta(x) = \begin{cases} (\theta+1)x^\theta & \text{if } \ 0 \le x \le 1; \\ \\ 0 & \text{otherwise.} \end{cases}$$

Find also the estimator of θ by the method of moments.

5. In a bolt factory, machines A, B, C manufacture respectively 25, 35 and 40 per cent of the total. Of their output 5, 4 and 2 per cent are defective bolts. A bolt is drawn at random from the produce and is found non defective. What are the probabilities that it was manufactured by machines A, B or C ?

6. (a) Show that for any $t > 0$,

$$P\left(|X - E(X)| \ge t\right) \le \frac{\text{van }(X)}{t^2}.$$

(b) Using (a), show that if X_1, X_2, $\cdots$ are independent and identically distributed random variables with mean μ and finite variance σ^2, then $\bar{X}_n \to \mu$ in probability.

7. Let X, Y have the joint p.d.f.

$$f(x,y) = \begin{cases} \dfrac{6-x-y}{8} & \text{if } \ 0 \le x \le 2, \ 2 \le y \le 4; \\ \\ 0 & \text{otherwise.} \end{cases}$$

Find $P(X+Y<3)$ and $P(X<1|Y=3)$.

8. Consider a multiple choice test of 20 questions, each with 5 choices. A candidate scores 1 for correct answer and 0 otherwise. What would be the most likely score for a candidate who guesses each time ? What would be the mean score for such students ? If a particular candidate is able to answer 70% of the questions correctly and guesses the rest, find the mean score of such candidates.

9. To estimate the total number of workers employed in an industry comprising 90 factories in all, a sample of 10 factories are selected in the following manner : The smallest two factories are included in the sample and in addition a simple random sample (without replacement) of size 8 is selected from the remaining 88 factories. The estimator proposed is $90\bar{y}$ where $\bar{y}$ is the sample mean of number of workers of all the 10 factories in the sample.

   Show that the estimator is not unbiased. Suggest an unbiased estimator and prove that your estimator is unbiased.

10. Let X_1 and X_2 constitute a random sample of size 2 from the population with a density of the form $f(x|\theta) = \theta x^{\theta-1}$ for $0 < x < 1$.

    If the critical region "$x_1 x_2 \geq \frac{3}{4}$" is used to test the null hypothesis $H_0 : \theta = 1$ against the alternative $H_1 : \theta = 2$, what is the power of this test at $\theta = 2$?

11. Let the probability p_n that a family has exactly n children be αp^n where $n \geq 1$ and $p_0 = 1 - \alpha p(1 + p + p^2 \cdots)$. Suppose that all sex distributions of n children have the same probability. For a positive integer k, find the probability that a family has exactly k boys.

12. In estimating the mean of a finite survey population on drawing a sample of a given size from it explain how one may involve an 'analysis of variance' arrgument to justify the efficicacy of stratified sampling. Develop a principle in formation of strata. If you are given a stratified simple random sample taken without replacement independently stratum-wise, show how you may derive an unbiased estimator for the population variance.

13. From a population of N units of varying sizes $X(i = 1, 2, \cdots, N)$, a sample of size n is selected with probability of selection of a unit proportional to the size and with replacement. A technician calculates the sample mean y of the observed y value under study. Is y unbiased for the population mean of the y values ? If so, verify the unbiased property. If not, calculate the bias.

14. Based on a simple random sample of size 200, a 95% confidence interval for the population mean turned out to be (10, 20). Find a 90% confidence interval for the population mean based on this information.

15. Let X_1, X_2 and X_3 be independent and identically distributed random variables following the gamma distribution with the parameter p. Find the distribution of

$$\frac{X_1}{X_1 + X_2 + X_3}.$$