Test Code: MIII (Objective type) 2001

Syllabus

Algebra - Permutations and combinations. Binomial Theorem. Theory of equations. Inequalities. Geometry of complex numbers and De Moivre's theorem. Elementary set theory. Functions and relations. Algebra of Matrices; Determinant, Rank & inverse of a matrix. Solutions of linear equations.

Coordinate geometry – Straight lines, Circles, Parabolas, Ellipses and Hyperbolas. Conic sections and their classification. Elements of three dimensional coordinate geometry– Straight lines, planes and spheres.

Calculus– Sequences and series. Taylor and Maclaurin series. Power series, Limit and continuity of functions of one or more variables. Differentiation and integration of functions of one variable with applications. Definite integrals. Areas using integrals. Maxima and minima. Differentiation of functions of several variables. Multiple integrals and their applications. Ordinary linear differential equations.

Sample Questions

<u>Note:</u> *For each question there are four suggested answers of which only one is correct.*

1. The number of permutations of $\{1, 2, 3, 4, 5\}$ that keep at least one integer fixed is

   (A) 81 (B) 76 (C) 120 (D) 60.

2. A club with x members is organized into four committees such that

   (a) each member is in exactly two committees,

   (b) any two committees have exactly one member in common.
   Then x has

   (A) exactly two values both between 4 and 8

   (B) exactly one value and this lies between 4 and 8

   (C) exactly two values both between 8 and 16

   (D) exactly one value and this lies between 8 and 16.

3. A subset S of the set of numbers $\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is said to be *good* if it has exactly 4 elements and their gcd $= 1$. Then the number of good subsets is

   (A) 126 (B) 125 (C) 123 (D) 121.

4. In how many ways can three persons, each throwing a single die once, make a score of 11?

(A) 22 (B) 27 (C) 24 (D) 38

5. $x^2 + x + 1$ is a factor of $(x+1)^n - x^n - 1$, whenever

(A) n is odd

(B) n is odd and a multiple of 3.

(C) n is an even multiple of 3.

(D) n is odd and not a multiple of 3.

6. The equation $x^3 - 5x + 3 = 0$

(A) has exactly one real root

(B) has exactly one positive root and two negative roots

(C) has exactly two positive roots and one negative root

(D) none of the above is true.

7. If $\alpha_1, \alpha_2, \ldots, \alpha_n$ be the roots of $x^n + 1 = 0$, then $(1-\alpha_1)(1-\alpha_2)\ldots(1-\alpha_n)$ is equal to

(A) 1 (B) 0 (C) n (D) 2

8. The equation

$$\frac{1}{3} + \frac{1}{2}s^2 + \frac{1}{6}s^3 = s$$

has

(A) exactly three solutions in $[0, 1]$

(B) exactly one solution in $[0, 1]$

(C) exactly two solutions in $[0, 1]$

(D) no solution in $[0, 1]$.

9. The equation $P(x) = \alpha$ where $P(x) = x^4 + 4x^3 - 2x^2 - 12x$ has four distinct real roots if and only if

(A) $P(-3) < \alpha$ (B) $P(-1) > \alpha$

(C) $P(-1) < \alpha$ (D) $P(-3) < \alpha < P(-1)$

10. If $a_1, a_2, \ldots, a_n$ are positive real numbers, then

$$\frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1}$$

is always

(A) $\geq n$ (B) $\leq n$ (C) $\geq n^{1/n}$ (D) $\leq n^{1/n}$

11. Let $X = \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + \cdots + \frac{1}{3001}$.
Then,

(A) $X < 1$ (B) $X > \frac{3}{2}$
(C) $1 < X < \frac{3}{2}$ (D) none of the above.

12. The inequality $\frac{2-gx-x^2}{1-x+x^2} \leq 3$ be true for all values of x if and only if

(A) $1 \leq g \leq 7$ (B) $-1 \leq g \leq 1$
(C) $-6 \leq g \leq 7$ (D) $-1 \leq g \leq 7$

13. Let $Z = \frac{1+z}{1-z}$. As z varies on the circle $|z - 1 - i| = \frac{1}{2}$, Z will lie on

(A) A circle with centre $(-1, \frac{8}{3})$.
(B) A circle with centre $(1, \frac{8}{3})$.
(C) A circle with centre $(-1, -\frac{8}{3})$.
(D) A circle with centre $(1, -\frac{8}{3})$.

14. For complex numbers $z_1 = x_1 + iy_1$, and $z_2 = x_2 + iy_2$, write $z_1 \preceq z_2$ if $x_1 \leq x_2$ and $y_1 \leq y_2$. Then for all complex numbers z with $1 \preceq z, (1 \neq z)$.

(A) $\left|\frac{1+z}{1-z}\right| \preceq 1$ (B) $\frac{1+z}{1-z} \preceq 0$
(C) $\frac{1-z}{1+z} \preceq 0$ (D) none of these.

15. Let a and c be two complex numbers. Then there is at least one complex number z such that

$$|z - a| + |z + a| = 2|c|$$

if and only if

(A) $|c| < |a|$ (B) $|c| \leq |a|$ (C) $|a| < |c|$ (D) $|a| \leq |c|$

16. Let $X \neq \phi$ be a set and let $\mathcal{P}(X)$ denote the collection of all subsets of X. Define $f : X \times \mathcal{P}(X) \to \Re$ by

$$f(x, A) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

Then

(A) $f(x, A \cup B) = f(x, A) + f(x, B)$

(B) $f(x, A \cup B) = f(x, A) + f(x, B) - 1$

(C) $f(x, A \cup B) = f(x, A) + f(x, B) - f(x, A).f(x, B)$

(D) $f(x, A \cup B) = f(x, A) + |f(x, A) - f(x, B)|$

17. The set $\{x : |x + \frac{1}{x}| > 6\}$ is:

(A) $(0, 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}, \infty)$

(B) $(-\infty, -3 - 2\sqrt{2}) \cup (-3 + 2\sqrt{2}, \infty)$

(C) $(-\infty, 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}, \infty)$

(D) $(-\infty, -3 - 2\sqrt{2}) \cup (-3 + 2\sqrt{2}, 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}, \infty)$

18. For a pair (A, B) of subsets of the set $X = \{1, 2, \ldots, 100\}$, let $A \triangle B$ denote the set of all elements of X which belong to exactly one of A or B. The number of pairs (A, B) of subsets of X such that $A \triangle B = \{2, 4, 6, \ldots, 100\}$ is

(A) 2^{151} (B) 2^{102} (C) 2^{101} (D) 2^{100}

19. The set of all real numbers x such that

$$|\,|3 - x| - |x + 2|\,| = 5$$

is

(A) $[3, \infty)$ (B) $(-\infty, -2]$

(C) $(-\infty, -2] \cup [3, \infty)$ (D) $(-\infty, -3] \cup [2, \infty)$

20. If

$$f(x) = \frac{\sqrt{3}\sin x}{2 + \cos x},$$

then the range of $f(x)$ is:

(A) $-1 \leq f(x) \leq \frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{3}}{2} \leq f(x) \leq 1$

(C) $-1 \leq f(x) \leq 1$ (D) none of the above.

21. If $f(x) = x^2$ and $g(x) = x \sin x + \cos x$ then

(A) f and g agree at no points

(B) f and g agree at exactly one point

(C) f and g agree at exactly two points

(D) f and g agree at more than two points.

22. For non-negative integers m, n define a function as follows:

$$f(m,n) = \begin{cases} n+1 & \text{if } m = 0 \\ f(m-1,1) & \text{if } m \neq 0, n = 0 \\ f(m-1, f(m, n-1)) & \text{if } m \neq 0, n \neq 0 \end{cases}$$

Then the value of $f(1,1)$ is

(A) 4 (B) 3 (C) 2 (D) 1.

23. The system of linear equations

$$\begin{aligned} kx_1 + \lambda x_2 + \lambda x_3 + \lambda x_4 &= 0 \\ \lambda x_1 + kx_2 + \lambda x_3 + \lambda x_4 &= 0 \\ \lambda x_1 + \lambda x_2 + kx_3 + \lambda x_4 &= 0 \\ \lambda x_1 + \lambda x_2 + \lambda x_3 + kx_4 &= 0 \end{aligned}$$

has a unique solution if, and only if

(A) $k - \lambda \neq 0$ (B) $k + 3\lambda \neq 0$

(C) $(k - \lambda)(k + 3\lambda) \neq 0$ (D) none of the above.

24. The rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{bmatrix}$ is less then 4 if, and only if

(A) $a = b = c = d$

(B) at least two of a, b, c, d are equal

(C) at least three of a, b, c, d, are equal

(D) a, b, c, d are distinct real numbers.

25. The value of the determinant

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 & \cdots & n^2 \\ 2^2 & 3^2 & 4^2 & \cdots & (n+1)^2 \\ \cdots & \cdots & \cdots & \cdots & \\ n^2 & (n+1)^2 & (n+2)^2 & \cdots & (2n-1)^2 \end{vmatrix}$$

is

(A) zero if and only if $n > 4$. (B) zero if and only if $n = 4$.

(C) zero if and only if $n \geq 4$. (D) none of the above.

26. The system of equations

$$\begin{aligned} x+y+z &= 6 \\ x+2y+3z &= 10 \\ x+2y+az &= b \end{aligned}$$

has no solution if

(A) $a \neq 3$ and b has any value
(B) $a = 3$ and $b \neq 10$.
(C) $a = 3$ and $b = 10$.
(D) none of the above.

27. Let $2x - y + 4 = 0$ be the equation of a diameter of the circle circumscribing the rectangle $ABCD$, the co-ordinates of A and B being (1,2) and (3,6) respectively. The area of the rectangle $ABCD$ is

(A) 16 (B) $2\sqrt{10}$ (C) $2\sqrt{5}$ (D) 20

28. If the tangent at the point P with co-ordinates (h, k) on the curve $y^2 = 2x^3$ is perpendicular to the straight line $4x = 3y$, then

(A) $(h, k) = (0, 0)$
(B) $(h, k) = \left(\frac{1}{8}, -\frac{1}{16}\right)$
(C) $(h, k) = (0, 0)$ or $(h, k) = \left(\frac{1}{8}, -\frac{1}{16}\right)$
(D) no such point (h, k) exists.

29. Let A be the matrix $\begin{pmatrix} x & 0 & 3 \\ -3 & y & y \\ 0 & 0 & 1 \end{pmatrix}$. If the determinant of A^n is equal to the determinant of A for all $n \geq 2$, then the locus of the points (x, y) with $xy \neq 0$ is

(A) a parabola (B) an ellipse
(C) a hyperbola (D) none of the above.

30. The circle $C_1 : x^2 + y^2 = 16$ intersects the circle C_2 of radius 5 in such a manner that the common chord is a diameter of C_1 and has a slope equal to $\frac{3}{4}$. Then a possible position of the centre of C_2 is:

(A) $\left(-\frac{12}{5}, \frac{9}{5}\right)$ (B) $\left(\frac{9}{5}, -\frac{12}{5}\right)$
(C) $\left(\frac{12}{5}, -\frac{9}{5}\right)$ (D) $\left(-\frac{9}{5}, \frac{12}{5}\right)$

31. A parabola intersects a rectange of area A at two diagonally opposite vertices and one side of the rectangle falls on the axis of the parabola. Then the parabola divides the rectangle into two pieces of areas

(A) $\frac{1}{2}A$ and $\frac{1}{2}A$
(B) $\frac{1}{3}A$ and $\frac{2}{3}A$
(C) $\frac{2}{5}A$ and $\frac{3}{5}A$
(D) none of the above.

32. The locus of the foot of the perpendicular from the origin to the planes which contain the point $(1, -1, 1)$ is:

(A) $x^2 + y^2 + z^2 + x - y + z = 0$
(B) $x^2 + y^2 + z^2 + x - y + z = 3$
(C) $x^2 + y^2 + z^2 - x + y - z = 0$
(D) $x^2 + y^2 + z^2 - x + y - z = 3$.

33. The equation of the plane that passes through $(1, 4, -3)$ and contains the line of interesection of the planes $3x-2y+4z-7 = 0$ and $x+5y-2z+9 = 0$ is:

(A) $11x + 4y + 8z - 3 = 0$
(B) $13x + 4y + 8z - 5 = 0$
(C) $13x + 5y + 9z - 6 = 0$
(D) $11x + 5y + 9z - 4 = 0$.

34. A variable plane passes through a fixed point (a, b, c) and cuts the coordinate axes at P, Q, R (where none of P, Q, R is the origin). The co-ordinates (x, y, z) of the centre of the sphere passing through P, Q, R and the origin satisfy the equation

(A) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$
(B) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$
(C) $ax + by + cz = 1$
(D) $ax + by + cz = a^2 + b^2 + c^2$

35. The sphere which passes through the point $(-3, 4, 0)$ and the circle $x^2 + y^2 + z^2 + 4x - 2y + 4z - 16 = 0,\ 2x + 2y + 2z + 9 = 0$ has radius

(A) 5 (B) 3 (C) 2 (D) 1

36. $\lim_{n\to\infty} \dfrac{1+\sqrt{2}+\sqrt[3]{3}+\cdots,+\sqrt[n]{n}}{n}$

(A) equals 0 (B) equals 1

(C) equals ∞ (D) does not exist.

37. The sum of the infinite series

$$1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\cdots$$

is

(A) e^2 (B) 3 (C) $\sqrt{5}$ (D) $\sqrt{8}$

38. If $0 < x < 1$, then the sum of the infinite series

$$\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\cdots$$

is

(A) $\log\frac{1+x}{1-x}$
(B) $\frac{x}{1-x}+\log(1+x)$
(C) $\frac{1}{1-x}+\log(1-x)$
(D) $\frac{x}{1-x}+\log(1-x)$

39. Let $\{a_n\}$ be a sequence of real numbers. Then $\lim_{n\to\infty} a_n$ exists if, and only if

(A) $\lim_{n\to\infty} a_{2n}$ and $\lim_{n\to\infty} a_{2n+2}$ exists
(B) $\lim_{n\to\infty} a_{2n}$ and $\lim_{n\to\infty} a_{2n+1}$ exist
(C) $\lim_{n\to\infty} a_{2n}$, $\lim_{n\to\infty} a_{2n+1}$, and $\lim_{n\to\infty} a_{3n}$ exist
(D) none of the above.

40. The series $\sum_{n=1}^{\infty} \dfrac{x^n}{n}$, where x is a real variable,

(A) converges if $|x| < 1$ and diverges if $|x| \geq 1$
(B) converges if $|x| \leq 1$ and diverges if $|x| > 1$
(C) converges if $|x| \leq 1, x \neq 1$ and diverges otherwise
(D) none of the above.

41. In the Taylor expansion of the function $f(x) = e^{x/2}$ about $x = 3$, the coefficient of $(x-3)^5$ is

(A) $e^{3/2} \cdot \frac{1}{5!}$

(B) $e^{3/2} \cdot \frac{1}{2^5 5!}$

(C) $e^{-3/2} \cdot \frac{1}{2^5 5!}$

(D) none of the above.

42. For $x > 0$ let

$$f(x) = \lim_{n \to \infty} n(x^{\frac{1}{n}} - 1).$$

Then

(A) $f(x) + f(\frac{1}{x}) = 1$,

(B) $f(xy) = f(x) + f(y)$,

(C) $f(xy) = xf(y) + f(x)$

(D) none of the above.

43. If

$$a = \lim_{n \to \infty} (1 + \frac{1}{2} + \cdots + \frac{1}{n}) \qquad b = \lim_{n \to \infty} \frac{1}{n}(1 + \frac{1}{2} + \cdots + \frac{1}{n})$$

then

(A) Both $a = \infty$ and $b = \infty$

(B) $a = \infty$ and $b = 0$

(C) $a = \infty$ and $b = 1$

(D) none of the above.

44. If

$$a = \lim_{n \to \infty} \frac{1}{\sqrt{n}}(1 + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}}),$$

then, a equals

(A) $\frac{1}{2}$ (B) 2 (C) 1 (D) does not exist.

45. Consider two series

$$(i) \sum_{n=1}^{\infty} \sin \frac{\pi}{n} \qquad (ii) \sum_{n=1}^{\infty} (-1)^n \cos \frac{\pi}{n}$$

Then

(A) Both (i) and (ii) converge.

(B) (i) converges but (ii) diverges.

(C) (i) diverges but (ii) converges.

(D) Both (i) and (ii) diverge.

46. If $0 < c < d$, then the sequence

$$a_n = (c^n + d^n)^{1/n}$$

(A) is bounded and monotone decreasing.

(B) is bounded and monotone increasing.

(C) is monotone increasing but unbounded for $1 < c < d$.

(D) is monotone decreasing but unbounded for $1 < c < d$.

47. The limit

$$\lim_{x \to 0+} \log \left(\frac{1+x}{1-x} \right)^{\frac{1}{x}}$$

(A) exists and is equal to 0 (B) exists and is equal to 1

(C) exists and is equal to 2 (D) does not exist.

48. Let

$$f(x,y) = \begin{cases} e^{-\frac{1}{x^2+y^2}}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Then $f(x,y)$ is

(A) not continuous at $(0,0)$

(B) continuous at $(0,0)$ but does not have first order partial derivatives

(C) continuous at $(0,0)$ and has first order partial derivatives, but not differentiable at $(0,0)$

(D) differentiable at $(0,0)$

49. Let $f(x)$ be the function

$$f(x) = \begin{cases} \frac{x^p}{(\sin x)^q}, & \text{if } x > 0 \\ 0, & \text{if } x = 0. \end{cases}$$

Then $f(x)$ is continuous at $x = 0$ if

(A) $p > q$ (B) $p > 0$ (C) $q > 0$ (D) $p < q$.

50.

$$\lim_{x \to 0} \sin \frac{e^x - x - 1 - x^2/2}{x^2}$$

(A) does not exist. (B) exists and equals 1.

(C) exists and equals 0. (D) exists and equals $\frac{1}{2}$.

51. If

$$F(x) = \begin{cases} \frac{1}{1+e^{1/(x-2)}+e^{-1/(x-3)^2}} & \text{if } x \neq 2 \text{ or if } x \neq 3 \\ 1 & \text{if } x = 2 \\ \frac{1}{1+e} & \text{if } x = 3. \end{cases}$$

Then:

(A) $F(x)$ is continuous at $x = 2$ but not at $x = 3$.

(B) $F(x)$ is not continuous at $x = 2$ but continuous at $x = 3$.

(C) $F(x)$ is neither continuous at $x = 2$ nor at $x = 3$.

(D) $F(x)$ is continuous at $x = 2$ and also at $x = 3$.

52. Let $p > 1$ and for $x > 0$ define $f(x) = (x^p - 1) - p(x - 1)$. Then:

(A) $f(x)$ is an increasing function of x on $(0, \infty)$

(B) $f(x)$ is a decreasing function of x on $(0, \infty)$

(C) $f(x) \geq 0$ for all $x > 0$

(D) $f(x)$ takes both positive and negative values for $x \in (0, \infty)$

53. Let $y^2 = F(x)$, where $F(x)$ is a polynomial of degree 3, then

$$2\frac{d}{dx}\left(y^3 \frac{d^2y}{dx^2}\right)$$

equals

(A) $F''F'''$ (B) $\frac{3}{2}FF' + (F'')^2F'''$

(C) $\frac{3}{2}F(F')^2$ (D) constant times F

54. The map

$$f(x) = a_0 \cos|x| + a_1 \sin|x| + a_2|x|^3$$

is differentiable at $x = 0$ if, and only if

(A) $a_1 = 0$ and $a_2 = 0$ (B) $a_0 = 0$ and $a_1 = 0$

(C) $a_1 = 0$ (D) a_0, a_1, a_2 can take any real value.

55. $f(x)$ is a differentiable function on the real line such that $\lim_{x \to \infty} f'(x) = 1$ and $\lim_{x \to \infty} f'(x) = \alpha$. Then

(A) α must be 0 (B) α need not be 0, but $|\alpha| < 1$
(C) $\alpha > 1$ (D) $\alpha < -1$.

56. Let f and g be two differentiable functions such that $f'(x) \leq g'(x)$ for all $x < 1$ and $f'(x) \geq g'(x)$ for all $x > 1$. Then

(A) if $f(1) \geq g(1)$, then $f(x) \geq g(x)$ for all x
(B) if $f(1) \leq g(1)$, then $f(x) \leq g(x)$ for all x
(C) $f(1) \leq g(1)$
(D) $g(1) \leq f(1)$.

57. $\int_0^\pi \min(\sin x, \cos x) dx$ equals

(A) $1 - 2\sqrt{2}$ (B) 1 (C) 0 (D) $1 - \sqrt{2}$

58. The length of the curve $x = t^3,\ y = 3t^2$ from $t = 0$ to $t = 4$ is

(A) $5\sqrt{5} + 1$ (B) $8(5\sqrt{5} + 1)$
(C) $5\sqrt{5} - 1$ (D) $8(5\sqrt{5} - 1)$

59. The value of

$$\int \int_D \frac{\sqrt{xy}}{x^2 + y^2} dx\ dy, \text{ where } D = \{(x, y) : x > 0,\ y > 0, xy < \frac{1}{2}\}$$

is:

(A) $\frac{\pi}{2\sqrt{2}}$ (B) $\frac{\pi}{\sqrt{2}}$ (C) $\frac{\pi}{4}$ (D) $\sqrt{\frac{\pi}{8}}$

60. If

$$M = \int_0^{\frac{\pi}{2}} \frac{\cos x}{x + 2} dx, \text{ and } N = \int_0^4 \frac{\sin x \cos x}{(x + 1)^2} dx,$$

then the value of $M - N$ is

(A) π (B) $\pi/4$ (C) $\frac{2}{\pi - 4}$ (D) $\frac{2}{\pi + 4}$

61. The value of the integral $\int_0^{10} (x - [x])^2 dx$, where $[x]$ is the largest integer $\leq x$, is

(A) 5 (B) 4 (C) 3 (D) 2.

62. The value of the integral

$$\int_{-2}^{2} \min\{|x-1|, |x+2|\} dx$$

is

(A) $\frac{11}{4}$ (B) $\frac{9}{4}$ (C) $\frac{11}{2}$ (D) $\frac{9}{2}$

63. The minimum value of the function

$$f(x,y) = 4x^2 + 9y^2 - 12x - 12y + 14$$

is

(A) 1 (B) 3 (C)14 (D) none of the above.

64. A metal plate of radius 1 is placed on the XY plane with its centre at the origin and its temperature distribution is given by the function

$$T(x,y) = e^x \cos\ y + e^y\ \cos\ x, \quad x^2 + y^2 \leq 1$$

Then the direction in which the temperature increases most rapidly at the centre is towards the point

(A) $(1,0)$ (B) $(0,1)$ (C) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (D) $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

65. The minimum value of

$$(\sqrt{3}\cos\theta + \sin\theta)(\sin\theta + \cos\theta)$$

in the interval $(0, \pi/2)$ is attained

(A) at exactly one point (B) at exactly two points
(C) at exactly three points (D) nowhere.

66. The maximum value of e^x/x^2 on the interval $[1,3]$ is

(A) e (B) $\frac{e^2}{4}$ (C) $\frac{e^3}{9}$ (D) e^2

67. A man is in a muddy field, 300 feet from the nearest point A of a straight road bordering the field. He wants to walk to a point B on the road 600 feet away from A. He can walk 3 feet per second in the muddy field and 5 feet per second on the road. Then the least time in which he can walk to B is

(A) 201.42 sec. (B) 200 sec. (C) 210 sec. (D) 220 sec.

68. A right circular cone is cut from a solid sphere of radius a, the vertex and the circumference of the base being on the surface of the sphere. The height of the cone when its volume is maximum is

(A) $\frac{4a}{3}$ (B) $\frac{3a}{2}$ (C) a (D) $\frac{6a}{5}$

69. The volume of the solid, generated by revolving about the horizontal line $y = 2$ the region bounded by $y^2 \leq 2x, x \leq 8$, and $y \geq 2$, is

(A) $2\sqrt{2\pi}$ (B) $\frac{28\pi}{3}$ (C) 84π (D) none of the above.

70. An inclined plane passing through a diameter of the base of a solid circular cylinder placed vertically, cuts the curved surface of the cylinder at a maximum height of H cms. Then the volume of the cutout portion is:

(A) $\frac{HD^2}{6}cc$ (B) $\frac{\pi HD^2}{8}cc$
(C) $\frac{7\pi HD^2}{6}cc$ (D) none of the above.

71. The coordinates of a moving point P satisfy the equations

$$\frac{dx}{dt} = \tan x \quad \frac{dy}{dt} = -\sin^2 x \quad t \geq 0$$

If the curve passes through the point $\left(\frac{\pi}{2}, 0\right)$ when $t = 0$, then the equation of the curve in rectangular co-ordinates is:

(A) $y = \frac{1}{2}\cos^2 x$ (B) $y = \sin 2x$
(C) $y = \cos 2x + 1$ (D) $y = \sin^2 x - 1$

72. Let y be a function of x satisfying

$$\frac{dy}{dx} = 2x^3 y^{1/2} - 4xy$$

If $y(0) = 0$ then $y(1)$ equals

(A) $\frac{1}{4e^2}$ (B) $\frac{1}{e}$ (C) $e^{1/2}$ (D) $e^{3/2}$.

73. Let $y(x)$ be a non-trivial solution of the second order linear differentialm equation

$$\frac{d^2y}{dx^2} + 2c\frac{dy}{dx} + ky = 0,$$

where $c < 0,\ k > 0$ and $c^2 > k$. Then

(A) $|y(x)| \to \infty$ as $x \to\ +\infty$
(B) $|y(x)| \to 0$ as $x \to\ +\infty$
(C) $\lim_{x\to\pm\infty}|y(x)|$ exists and is finite
(D) none of the above.