POST-GRADUATE DIPLOMA IN STATISTICAL METHODS AND ANALYTICS

TEST CODE: DAA (Objective type) 2013

SYLLABUS

Algebra—Arithmetic, Geometric and Power series, sequences. Permutations and combinations. Binomial theorem. Theory of quadratic equations. Inequalities. Elementary set theory. Vectors and matrices. Determinant, rank and inverse of a matrix. Solutions of linear equations.

Coordinate geometry — Straight lines, circles, parabolas, ellipses and hyperbolas.

Calculus—Taylor and Maclaurin series. Limits and continuity of functions of one real variable. Differentiation and integration of functions of one real variable with applications. Definite integrals. Areas using integrals. Maxima and minima and their applications.

SAMPLE QUESTIONS

Note: *For each question there are four suggested answers of which only one is correct.*

1. If a, b are positive real variables whose sum is constant λ, then the minimum value of $\sqrt{(1+1/a)(1+1/b)}$ is

   (A) $\lambda - 1/\lambda$

   (B) $\lambda + 2/\lambda$

   (C) $\lambda + 1/\lambda$

   (D) none of the above.

2. Suppose in a competition 11 matches are to be played, each having one of 3 distinct outcomes as possibilities. The number of ways one can predict the outcomes of all 11 matches such that exactly 6 of the predictions turn out to be correct is

   (A) $\binom{11}{6} \times 2^5$ (B) $\binom{11}{6}$ (C) 3^6 (D) none of these.

3. Let A be a set of n elements. The number of ways we can choose an ordered pair (B, C), where B, C are disjoint subsets of A, equals

   (A) n^2 (B) n^3 (C) 2^n (D) 3^n.

4. Let $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, n being a positive integer. The value of

$$\left(1+\frac{C_0}{C_1}\right)\left(1+\frac{C_1}{C_2}\right)\cdots\left(1+\frac{C_{n-1}}{C_n}\right)$$

is

(A) $\left(\frac{n+1}{n+2}\right)^n$ (B) $\frac{n^n}{n!}$ (C) $\left(\frac{n}{n+1}\right)^n$ (D) $\frac{(n+1)^n}{n!}$.

5. The number of positive integers which are less than or equal to 1000 and are divisible by none of 17, 19 and 23 equals

(A) 854 (B) 153 (C) 160 (D) none of these.

6. Let $a_n = \left(1 - \frac{1}{\sqrt{2}}\right) \cdots \left(1 - \frac{1}{\sqrt{n+1}}\right), \quad n \geq 1$. Then $\lim_{n\to\infty} a_n$

(A) equals 1 (B) does not exist (C) equals $\dfrac{1}{\sqrt{\pi}}$ (D) equals 0.

7. $\displaystyle\lim_{x\to\infty} \left(\frac{3x-1}{3x+1}\right)^{4x}$ equals

(A) 1 (B) 0 (C) $e^{-8/3}$ (D) $e^{4/9}$.

8. $\displaystyle\lim_{n\to\infty} \frac{1}{n}\left(\frac{n}{n+1} + \frac{n}{n+2} + \cdots + \frac{n}{2n}\right)$ is equal to

(A) ∞ (B) 0 (C) $\log_e 2$ (D) 1.

9. The set $\{x : \left|x + \frac{1}{x}\right| > 6\}$ equals the set

(A) $(0, 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}, \infty)$
(B) $(-\infty, -3 - 2\sqrt{2}) \cup (-3 + 2\sqrt{2}, \infty)$
(C) $(-\infty, 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}, \infty)$
(D) $(-\infty, -3 - 2\sqrt{2}) \cup (-3 + 2\sqrt{2}, 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}, \infty)$.

10. Consider the sets defined by the real solutions of the inequalities

$$A = \{(x,y) : x^2 + y^4 \leq 1\} \qquad B = \{(x,y) : x^4 + y^6 \leq 1\}.$$

Then

(A) $B \subseteq A$
(B) $A \subseteq B$
(C) Each of the sets $A - B$, $B - A$ and $A \cap B$ is non-empty
(D) none of the above.

11. If $f(x)$ is a real valued function such that

$$2f(x) + 3f(-x) = 15 - 4x,$$

for every $x \in \mathbb{R}$, then $f(2)$ is

(A) -15 (B) 22 (C) 11 (D) 0.

12. If $f(x) = \dfrac{\sqrt{3}\sin x}{2 + \cos x}$, then the range of $f(x)$ is

(A) the interval $[-1, \sqrt{3}/2]$ (B) the interval $[-\sqrt{3}/2, 1]$

(C) the interval $[-1, 1]$ (D) none of these.

13. If M is a 3×3 matrix such that

$$[0 \quad 1 \quad 2]M = [1 \quad 0 \quad 0] \quad \text{and} \quad [3 \quad 4 \quad 5]M = [0 \quad 1 \quad 0]$$

then $[6 \quad 7 \quad 8]M$ is equal to

(A) $[2 \quad 1 \quad -2]$ (B) $[0 \quad 0 \quad 1]$ (C) $[-1 \quad 2 \quad 0]$ (D) $[9 \quad 10 \quad 8]$.

14. The values of η for which the following system of equations

$$\begin{array}{ccccccc} x & + & y & + & z & = & 1 \\ x & + & 2y & + & 4z & = & \eta \\ x & + & 4y & + & 10z & = & \eta^2 \end{array}$$

has a solution are

(A) $\eta = 1, -2$ (B) $\eta = -1, -2$ (C) $\eta = 3, -3$ (D) $\eta = 1, 2$.

15. Suppose the circle with equation $x^2 + y^2 + 2fx + 2gy + c = 0$ cuts the parabola $y^2 = 4ax$, $(a > 0)$ at four distinct points. If d denotes the sum of ordinates of these four points, then the set of possible values of d is

(A) $\{0\}$ (B) $(-4a, 4a)$ (C) $(-a, a)$ (D) $(-\infty, \infty)$.

16. If a sphere of radius r passes through the origin and cuts the three coordinate axes at points A, B, C respectively, then the centroid of the triangle ABC lies on a sphere of radius

(A) r (B) $\dfrac{r}{\sqrt{3}}$ (C) $\sqrt{\dfrac{2}{3}}\,r$ (D) $\dfrac{2r}{3}$.

17. The number of divisors of 6000, where 1 and 6000 are also considered as divisors of 6000 is

(A) 40 (B) 50 (C) 60 (D) 30.

18. Let x_1 and x_2 be the roots of the quadratic equation $x^2 - 3x + a = 0$, and x_3 and x_4 be the roots of the quadratic equation $x^2 - 12x + b = 0$. If x_1, x_2, x_3 and x_4 $(0 < x_1 < x_2 < x_3 < x_4)$ are in G.P., then ab equals

(A) 64 (B) 5184 (C) -64 (D) -5184.

19. The integral

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{50} x}{\sin^{50} x + \cos^{50} x}\, dx$$

equals

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) none of these.

20. Let the function $f(x)$ be defined as $f(x) = |x-1| + |x-2|$. Then which of the following statements is true?

(A) $f(x)$ is differentiable at $x = 1$

(B) $f(x)$ is differentiable at $x = 2$

(C) $f(x)$ is differentiable at $x = 1$ but not at $x = 2$

(D) none of the above.

21. $x^4 - 3x^2 + 2x^2y^2 - 3y^2 + y^4 + 2 = 0$ represents

(A) A pair of circles having the same radius

(B) A circle and an ellipse

(C) A pair of circles having different radii

(D) none of the above.

22. Let $\mathbb{N} = \{1, 2, 3, \ldots\}$ be the set of natural numbers. For each $n \in \mathbb{N}$, define $A_n = \{(n+1)k, k \in \mathbb{N}\}$. Then $A_1 \cap A_2$ equals

(A) A_3 (B) A_4 (C) A_5 (D) A_6.

23. The sum of the series

$$\frac{1}{1.2} + \frac{1}{2.3} + \ldots + \frac{1}{n(n+1)} + \ldots\ldots$$

is

(A) 1 (B) 1/2 (C) 0 (D) non-existent.

24. $\lim_{x \to 2} \dfrac{1}{1 + e^{\frac{1}{x-2}}}$
is

(A) 0 (B) 1/2 (C) 1 (D) non-existent.

25. ${}^nC_0 + 2\ {}^nC_1 + 3\ {}^nC_2 + \ldots + (n+1)\ {}^nC_n$
equals

(A) $2^n + n2^{n-1}$ (B) $2^n - n2^{n-1}$ (C) 2^n (D) none of these.

26. It is given that $e^a + e^b = 10$ where a and b are real. Then the maximum value of $(e^a + e^b + e^{a+b} + 1)$ is

(A) 36 (B) ∞ (C) 25 (D) 21.

27. If $A(t)$ is the area of the region bounded by the curve $y = e^{-|x|}$ and the portion of the x-axis between $-t$ and t, then $\lim_{t\to\infty} A(t)$ equals

(A) 0 (B) 1 (C) 2 (D) 4.

28. Suppose that the function $h(x)$ is defined as $h(x) = g(f(x))$ where $g(x)$ is monotone increasing, $f(x)$ is concave, and $g''(x)$ and $f''(x)$ exist for all x. Then $h(x)$ is

(A) always concave (B) always convex (C) not necessarily concave (D) None of these.

29. The conditions on a, b and c under which the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, $b \neq 0$ and $c \neq 0$, are of unequal magnitude but of opposite signs, are the following:

(A) a and c have the same sign while b has the opposite sign;
(B) b and c have the same sign while a has the opposite sign; or a and b have the same sign while c has the opposite sign;
(C) a and c have the same sign;
(D) a, b and c have the same sign.

30. The sum of the series $3 + 11 + \ldots + 8n - 5$
is

(A) $4n^2 - n$ (B) $8n^2 + 3n$ (C) $4n^2 + 4n - 5$ (D) $4n^2 + 2$.

31. Let $f(x) = \frac{2x}{x-1}$, $x \neq 1$. State which of the following statements is true.

(A) For all real y, there exists x such that $f(x) = y$;
(B) For all real $y \neq 1$, there exists x such that $f(x) = y$;
(C) For all real $y \neq 2$, there exists x such that $f(x) = y$;
(D) None of the above is true.

32. The determinant $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$ equals

(A) $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ (B) $2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ (C) $3\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$
(D) None of these.

33. Let $x_1 > x_2 > 0$. Then which of the following is true?
(A) $log(\frac{x_1+x_2}{2}) > \frac{logx_1+logx_2}{2}$;
(B) $log(\frac{x_1+x_2}{2}) < \frac{logx_1+logx_2}{2}$;
(C) There exist x_1 and x_2 such that $x_1 > x_2 > 0$ and $log(\frac{x_1+x_2}{2}) = \frac{logx_1+logx_2}{2}$;
(D) None of these.

34. Let $y^2 - 4ax + 4a = 0$ and $x^2 + y^2 - 2(1+a)x + 1 + 2a - 3a^2 = 0$ be two curves. State which one of the following statements is true.

(A) These two curves intersect at two points;
(B) These two curves are tangent to each other;
(C) These two curves intersect orthogonally at one point;
(D) These two curves do not intersect.

35. If $f(x) = sin\left(\frac{1}{x^2+1}\right)$, then

(A) $f(x)$ is continuous at $x = 0$, but not differentiable at $x = 0$;
(B) $f(x)$ is differentiable at $x = 0$, and $f'(0) \neq 0$;
(C) $f(x)$ is differentiable at $x = 0$, and $f'(0) = 0$;
(D) None of the above.

36. Consider the equation $P(x) = x^3 + px^2 + qx + r = 0$ where p, q and r are all real and positive. State which of the following statements is always correct.

(A) All roots of $P(x) = 0$ are real;
(B) The equation $P(x) = 0$ has at least one real root;
(C) The equation $P(x) = 0$ has no negative real root;

(D) The equation $P(x) = 0$ must have one positive and one negative real root.

37. The area enclosed by the curve $|x| + |y| = 1$ is

(A) 1 (B) 2 (C) $\sqrt{2}$ (D) 4.

38. For real α, the value of $\int_{\alpha}^{\alpha+1}[x]dx$, where $[x]$ denotes the largest integer less than or equal to x, is

(A) α (B) $[\alpha]$ (C) 1 (D) $\frac{[\alpha]+[\alpha+1]}{2}$.

39. Consider 30 multiple-choice questions, each with four options of which exactly one is correct. Then the number of ways one can get only the alternate questions correctly answered is

(A) 3^{15}, (B) 2^{31}, (C) $2 \times \binom{30}{15}$, (D) 2×3^{15}.

40. The following sum of $n+1$ terms

$$2 + 3 \times \binom{n}{1} + 5 \times \binom{n}{2} + 9 \times \binom{n}{3} + 17 \times \binom{n}{4} + \ldots \text{ up to } n+1 \text{ terms}$$

is equal to

(A) $3^{n+1} + 2^{n+1}$ (B) $3^n \times 2^n$ (C) $3^n + 2^n$ (D) 2×3^n.

41. The value of the integral $I = 4 \times \left(\int\limits_0^{\frac{\pi}{2}} \frac{e^{cot^{-1}(sinx)}}{e^{cot^{-1}(sinx)} + e^{cot^{-1}(cosx)}} dx \right)$ equals

(A) 1 (B) $\frac{\pi}{2}$ (C) π (D) 2π.

42. Let $f(x)$ be a continuous function from $[0,1]$ to $[0,1]$ satisfying the following properties.

(a) $f(0) = 0$,

(b) $f(1) = 1$, and

(c) $f(x_1) < f(x_2)$ for $x_1 < x_2$ with $0 < x_1, x_2 < 1$.

Then the number of such functions is

(A) 0 (B) 1 (C) 2 (D) ∞.

43. Let A and B be disjoint sets containing m and n elements respectively, and let $C = A \cup B$. Then the number of subsets S (of C) which contains p elements and also has the property that $S \cap A$ contains q elements, is

(A) $\binom{m}{q}$ (B) $\binom{n}{q}$ (C) $\binom{m}{q} \times \binom{n}{p-q}$ (D) $\binom{m}{p-q} \times \binom{n}{q}$.

44. Consider any integer $I = m^2 + n^2$, where m amd n are odd integers. Then

(A) I is never divisible by 2;
(B) I is never divisible by 4;
(C) I is never divisible by 6;
(D) None of the above.

45. Let $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ be a continuous function, $f(x) \to +\infty$ as $x \to \frac{\pi}{2}-$ and $f(x) \to -\infty$ as $x \to -\frac{\pi}{2}+$. Which one of the following functions satisfies the above properties of $f(x)$?

(A) $cosx$ (B) $tanx$ (C) $tan^{-1}x$ (D) $sinx$.

46. Suppose that A is a 3×3 real matrix such that for each $u = (u_1, u_2, u_3)' \in \mathbb{R}^3$, $u'Au = 0$ where u' stands for the transpose of u. Then which one of the following is true?

(A) $A' = -A$ (B) $A' = A$ (C) $AA' = I$ (D) None of these.

47. The function $f(x) = x^{\frac{1}{x}}$, $x \neq 0$ has

(A) a minimum at $x = e$;
(B) a maximum at $x = e$;
(C) neither a maximum nor a minimum at $x = e$;
(D) None of the above.

48. Let the following two equations represent two curves A and B.

$$A : 16x^2 + 9y^2 = 144 \quad \text{and} \quad B : x^2 + y^2 - 10x = -21$$

Further, let L and M be the tangents to these curves A and B, respectively, at the point $(3, 0)$. Then the angle between these two tangents, L and M, is

(A) 0^0 (B) 30^0 (C) 45^0 (D) 90^0.

49. The number of permutations of the letters a, b, c and d such that b does not follow a, c does not follow b, and c does not follow d, is

(A) 11 (B) 12 (C) 13 (D) 14.

50. Let $f(x) = sinx^2$, $x \in \mathbb{R}$. Then

(A) f has no local minima;
(B) f has no local maxima;
(C) f has local minima at $x = 0$ and $x = \pm\sqrt{(k + \frac{1}{2})\pi}$ for odd integers

k and local maxima at $x = \pm\sqrt{(k+\frac{1}{2})\pi}$ for even integers k;
(D) None of the above.