

# A kinetic approach to the formation of wealth distribution curves

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## Summary

Kinetic market models have been proposed recently to account for the redistribution of wealth in simple market economies. These models allow to develop a qualitative theory, which is based on methods borrowed from the kinetic theory of rarefied gases. The aim of this lecture is to present a unifying approach to the study of the evolution of wealth in the large-time regime. The considered models are divided into two classes: the first class is such that the society's mean wealth is conserved, while for models of the second class, the mean wealth grows or decreases exponentially in time. In both cases, it is possible to classify the most important feature of the steady (or self-similar, respectively) wealth distributions, namely the fatness of the Pareto tail. We shall also discuss the tails' dynamical stability in terms of the model parameters. Our results are derived by means of a qualitative analysis of the associated homogeneous Boltzmann equations. The key tools are suitable metrics for probability measures, and a concise description of the evolution of moments. A recent extension to economies, in which different groups of agents interact, is presented in detail. We conclude with numerical experiments that confirm the theoretical predictions. The ideas of using methods valid for the Boltzmann equation for Maxwell molecules come back to a paper jointly written with L. Pareschi [6]. From this pioneering paper on, various contribution have been done, jointly written with B. Düring and D. Matthes [1, 2, 3, 4, 5].

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## References

- [1] B. Düring, D. Matthes and G. Toscani, Exponential and algebraic relaxation in kinetic models for wealth distribution. In: “*WASCOM 2007*” - *Proceedings of the 14th Conference on Waves and Stability in Continuous Media*, N. Manganaro et al. (eds.), pp. 228–238, World Sci. Publ., Hackensack, NJ, 2008.
- [2] B. Düring, D. Matthes and G. Toscani, Kinetic Equations modelling Wealth Redistribution: A comparison of Approaches. *Phys. Rev. E* **78**, (2008), 056103.
- [3] B. Düring and G. Toscani, International and domestic trading and wealth distribution. *Commun. Math. Sci.* **6**,(4) (2008), 1043–1058.
- [4] D. Matthes and G. Toscani, On steady distributions of kinetic models of conservative economies. *J. Stat. Phys.* **130** (2008), 1087–1117.
- [5] D. Matthes and G. Toscani, Analysis of a model for wealth redistribution. *Kinetic Rel. Mod.* **1** (2008), 1–22.
- [6] L. Pareschi and G. Toscani, Self-similarity and power-like tails in nonconservative kinetic models. *J. Stat. Phys.* **124**(2-4) (2006), 747–779.