

# **Friedman-Ball Hypothesis Revisited in the Framework of Regime-Based Model for Inflation: Evidence from G7 and some Euro Zone Countries**

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## **Abstract**

This paper revisits the Friedman-Ball hypothesis which states that high inflation leads to high inflation uncertainty, and for which the empirical evidence is mixed. To this end, the paper also considers two important issues hitherto not properly and adequately addressed to in the existing models covering this relationship. These are: (i) the possible existence of a threshold level of inflation to be determined endogenously and the consequent identification of two regimes characterised by high and low inflation, and (ii) whether or not the coefficient capturing the link between inflation and inflation uncertainty is different in the two different inflation regimes. This paper has proposed a model where these issues have been incorporated in the specifications of the conditional mean and conditional variance. Using monthly time series data on consumer price indices of 13 countries - comprising Euro Zone and G7 – the proposed model has been found to be better than some of the existing ones. The findings also show that threshold levels of inflation exist in eight countries. Further, the Friedman-Ball hypothesis holds for five countries (Canada, Germany, Finland, Greece and Luxembourg) in the high inflation regime only, one country (Belgium) in the low inflation regime only, and one country (Spain) in both the regimes. Moreover, it is observed that inflation uncertainty increases in the low inflation regime for five countries (France, Japan, the U.K., the U.S.A. and Greece), thus yielding an empirical finding which goes counter to the Friedman-Ball hypothesis.

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*Keywords:* Inflation; Inflation uncertainty; Regime switching model

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## 1. Introduction

It is well established that uncertainty about future inflation put a greater burden on the decision-making of consumer and business by distorting their efficient allocation of resources and thus reducing their economic well-being. Economists have frequently argued that a rise in current inflation leads to a greater uncertainty about future inflation, the origin of which are due to Johnson (1967) and Okun (1971). In his Nobel address in 1977, Friedman argued that a rise in average inflation creates uncertainty about future monetary policy to counter it, leading to wide variations in actual and anticipated inflation, and thus resulting in economic inefficiency and lower output growth. As formalised by Ball (1992), when inflation is low, public is almost certain about the future policy because the policy makers- both conservative and tough- will try to keep it low, and hence uncertainty concerning future inflation will also be low. However, when inflation is high, policy makers respond differently to counter high inflationary pressure and consequently uncertainty about the future monetary policy and the future path of inflation becomes greater. Thus, the basic idea regarding the relationship between inflation and inflation uncertainty, as proposed by Friedman (1977), and thereafter its formalisation in a game theoretic framework by Ball (1992), is known as Friedman-Ball hypothesis.

As argued by Pourgerami and Maskus (1987) and Ungar and Zilberfarb (1993), it is also possible that a higher level of inflation will lead to a lower level of inflation uncertainty. The idea behind this argument is that in presence of higher inflation, an economic agent may invest more to generate accurate prediction about inflation and hence this may lead to lower uncertainty about inflation. Reversing the causation link of the Friedman-Ball view, Cukierman and Meltzer (1986) and also Cukierman (1992) showed that higher inflation uncertainty will raise the average inflation. Holland (1995), on the other hand, argued about a negative causal effect of inflation

uncertainty on inflation. Thus, from theoretical consideration, there may exist bidirectional causal relationships between inflation and inflation uncertainty while the sign of interdependence between these two could be ambiguous.

These observations and consequent arguments have given rise to many empirical studies examining the link between inflation and inflation uncertainty during the last two decades. Early empirical studies to verify these linkages have used variance or moving average of standard deviation as a measure of inflation uncertainty. Obviously, these measure inflation variability, and not inflation uncertainty.<sup>‡</sup> It is only after the introduction of autoregressive conditional heteroskedasticity (ARCH) process by Engle (1982) that researchers started measuring inflation uncertainty as the conditional variance of unanticipated shocks to inflation process. With an ARCH process, Engle (1982) found weak evidence to support Friedman's hypothesis. Subsequently, other researchers like Bollerslev (1986) and Cosimano and Jansen (1988) also found no statistical support for the Friedman-Ball hypothesis.

While dealing with inflation uncertainty, the issue of regime switching behaviour of inflation was first raised by Evans and Wachtel (1993). They also claimed that the resultant model will seriously underestimate both the degree of uncertainty and its impact if one neglects this regime switching feature of inflation. One obvious limitation with the GARCH model is that it does not incorporate the possibility of structural instability due to regime changes. While this argument has been put forward as an explanation for empirical failure of Friedman-Ball hypothesis, Brunner and Hess (1993) have pointed out two other reasons. According to them, for testing the Friedman-Ball hypothesis directly, (i) conditional variance should include a lagged

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<sup>‡</sup>Golob (1993) and Davis and Kanago (2000) have compiled many of these earlier studies and found that the results are somewhat mixed, due to differences in (i) the sample periods, (ii) frequencies of data sets used, and (iii) the methodologies applied.

inflation term, and (ii) an asymmetric behaviour should be included in the conditional variance specification instead of a (symmetric) GARCH specification so as to allow for asymmetric news impact on inflation uncertainty. Subsequently, studies on inflation have used the exponential GARCH (EGARCH) by Nelson (1991) and threshold GARCH (TGARCH) by Glosten et al. (1993) to allow asymmetry in the conditional variance.

The Friedman-Ball hypothesis as also Cukierman-Meltzer hypothesis could be directly tested in a simultaneous approach where one can use the GARCH-in-mean (GARCH-M) model with the additional feature that a lagged term of inflation is included in the conditional variance equation. In particular, Brunner and Hess (1993) allowed asymmetric effects of inflation shocks on nominal uncertainty and found a weak link between inflation and its uncertainty for the US economy. Further, Caporale and Mckiernan (1997) observed a positive relationship between the level and variability of US inflation. Baillie et al. (1996) applied an ARFIMA-GARCH type model to describe the inflation dynamics for ten countries. They found that for six low inflation countries *viz.*, Canada, France, Germany, Italy, Japan and the U.S.A., there is no apparent relationship between inflation and inflation uncertainty. However, for the high inflation economics of Argentina, Brazil, Israel and the U.K., there is strong support for Friedman-Ball hypothesis. Hwang (2001) investigated the relationship for U.S monthly inflation from 1926 to 1992 with various ARFIMA-GARCH type models and concluded with the finding that inflation affects its uncertainty weakly and negatively while inflation uncertainty does not affect inflation significantly. Joyce (1995) applied the EGARCH and TGARCH models to the U.K. inflation data and found that inflation uncertainty is more responsive to positive inflation shocks than to negative shocks. Recently, Kontonikas (2004) has fitted a threshold GARCH (TGARCH) model

to capture asymmetry in the conditional variance and the component GARCH (CGARCH) model to capture short-run and long-run inflation uncertainties, to the U.K. inflation data.

Some studies have examined the link between nominal uncertainty and level of inflation using a two-step approach where an estimate of the conditional variance is first made from a GARCH-type model and then Granger causality test is used to test the bidirectional effects. In particular, by using GARCH and CGARCH models, Grier and Perry (1998) found that some of the G7 countries exhibit positive relations, while a few show negative relations and the remaining ones show no significant relations at all. For six European Union countries, Fountas et al. (2004) have employed an EGARCH model of conditional variance to investigate the relationship between inflation and uncertainty, and found that in five European countries inflation significantly raises its uncertainty. Conrad and Karanasos (2006) have used an ARFIMA-FIGARCH process to capture high degree of persistence in inflation and nominal uncertainty for ten European countries, and then employed Granger causality to test the bidirectional causal relationship between inflation and inflation uncertainty. Their study supports the Friedman-Ball hypothesis for all countries but provides a mixed result for the Cuckierman-Meltzer hypothesis. In fine, it may be concluded that the empirical evidence on the link between inflation and inflation uncertainty is mixed in nature.

Given the theoretical ambiguity about this relationship, it is not surprising that the statistical evidence is also mixed. The purpose of this paper is to take a re-examination of the Friedman-Ball hypothesis, and to this end, we raise two questions regarding the behaviour of inflation and inflation uncertainty. First, if a regime switching behaviour capturing asymmetry in the volatility process in response to positive and negative shocks is allowed in the conditional variance specification, then all the parameters including the coefficient which captures the link

between inflation and inflation uncertainty should be allowed to vary across the regimes. As argued by Friedman and Ball and also quite evident from the empirical results (see, Ungar and Zilberfarb (1993), Brunner and Hess (1993), Baillie et al. (1996), Davis and Kanago (1998) and Kim (1993) for relevant details), this relationship seems to be significant in the periods of high inflation only, and not in the periods of relatively moderate inflation. In a recent study, Chen et al. (2008) have observed that the effect of inflation on inflation uncertainty is asymmetric i.e., they have found a U-shaped pattern for four dragon economies of East Asia, based on a nonlinear flexible regression model of Hamilton (2001), that inflation uncertainty is more sensitive to inflation in an inflationary period than in a deflationary period. Keeping this in mind, it is quite obvious to attach regime specific behaviour to the coefficients which captures the link. Subsequently, the other question arises, namely, how high (low) is a high (low) inflation? In other words, does a threshold level of inflation exist? In their study, Ungar and Zilberfarb (1993) have pointed out that there may exist a threshold level of inflation above which the relationship between inflation and inflation uncertainty holds, whereas it does not exist when the inflation falls below the threshold. Based on a standard monetary growth model with informational asymmetry, Azariadis and Smith (1996) have also shown that if inflation exceeds a threshold, it leads to higher nominal uncertainty. Edmonds and So (1993) have found significant relationships for a group of high and low inflation countries, but not for a group of countries having moderate inflation in the range of six to ten per cent. Hess and Morris (1996), on the other hand, demonstrated that a significant positive relation exists for the countries with low and moderate inflation of less than 15 percent a year<sup>§</sup>. In recent years, most of the industrialized and emerging economies have been concerned with the benefits to be derived from reducing inflation from high levels to low levels, and hence, it is quite important for the policymakers to have some

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<sup>§</sup> See Fang et al. (2010), for further references on the existence of threshold level of inflation.

knowledge about the threshold level of inflation above which significant increase in inflation uncertainty may occur.

This study has attempted at addressing the first issue by adopting an asymmetric nonlinear smooth transition GARCH (ANSTGARCH) type model, where, apart from the usual GARCH coefficients, one lagged inflation term is explicitly incorporated in the conditional variance specification of inflation. One important advantage of these models is that the asymmetric behaviour of inflation uncertainty is reflected by the change in all parameters in the conditional variance specification, including the one which is attached to lagged inflation. It thus allows the relationship between inflation uncertainty and inflation to vary between the regimes. As regards the second issue, a lagged inflation term, instead of past shock, has been included in the smooth transition function, and regimes are defined based on lagged inflation being below or above a threshold level of inflation. We have applied this model to verify if the Friedman-Ball hypothesis holds for a group of 13 countries from the Euro area and G7 countries over the period 1960-2009. It is worth noting that an well known fact about almost all the countries in the Euro and G7 countries during the last five decades is that these countries have experienced several shocks in their economies, which started with the phenomenon of ‘great inflation’, which refers to the high and volatile inflation that occurred in the mid-1960s and lasted for almost twenty years. Since the early 1980s, several countries adopted anti inflationary policies to reduce these sustained price hike, which thus lead to a more stable inflation regime during 1990s. It is, therefore, relevant as well as important to examine whether the transition from the high inflation of the sixties and seventies to an era of low inflation during 1980s and 1990s affected the dynamic interaction between inflation and inflation uncertainty.

Apart from empirically verifying the Friedman-Ball link, this study takes a closer look at the issue of stationarity/ nonstationarity status of the time series on inflation. This issue has econometric relevance since the previous studies, especially those based on the inflation series of the U.K. and the U.S.A., provide mixed findings regarding the unit root status of the series (see Kontonikas (2004), Bhar and Hamori (2004), Fountas et al. (2004), Conrad and Karanasos (2006), Fountas et al. (2006), Daal et al. (2005) for details). It may be noted that the spans of the datasets used in those studies are quite large. Hence there is every likelihood of structural breaks occurring in the series. As a matter of fact, presence of at least one structural break in the trend of inflation seems quite probable, and hence this empirical issue needs to be checked applying the recent advances made in the literature on unit root test.

After the influential work by Perron (1989), the commonly used ADF test has been sharply criticised because of its bias towards non-rejection of the null hypothesis of a unit root against the alternative of trend stationarity in the presence of structural break, and also for its low power for near integrated process. On the other hand, the recent literature on tests for structural change in terms of intercept and / or slope of a trend function suggests that the performance of these tests depend on whether the noise component is stationary or nonstationary having unit roots. To deal with this problem, recently Perron and Yabu (2009) have proposed a novel test for structural change in the trend function of a univariate time series, which can be performed without any prior knowledge on whether the noise component is stationary or nonstationary containing unit roots. Since this test is very general in its approach insofar as the assumption on noise is concerned, we have performed this test to detect the presence of structural break, if any, in the trend function of a time series. In case this test suggests that there is one break in the deterministic trend, we have applied the testing procedure developed by Kim and Perron (2009)

to test the null hypothesis of unit root with a break against the alternative of stationarity with a break in the deterministic trend function. We have applied these two tests to each inflation series to find whether the shock really persists or it is transitory in nature around a break in the deterministic trend of the inflation series.

The format of the paper is as follows. The two most recent tests for structural break and unit roots are briefly discussed in the next section. The proposed model is described in Section 3. Empirical findings are discussed in Section 4. The paper ends with some concluding observations in Section 5.

## **2. Tests for Structural Break and Unit Roots**

In this section, we describe very briefly the two recent tests referred in Section 1, which are due to Perron and Yabu (2009) and Kim and Perron (2009).

### **2.1 Test for shift in trend with an integrated or stationary noise component**

To test for stationarity or otherwise of a time series, most often the augmented Dicky-Fuller (ADF) (1979) test is used. However, due to the influential work by Perron (1989), it is well-known that both the size and power of the ADF test are likely to be affected if there exists a break in the deterministic trend. In particular, Perron showed that the standard unit root test like the ADF test can produce a misleading result of the presence of unit roots in the time series when, in fact, that is indeed not the case in the sense that the alternative is that of a stationary noise component with a break in the intercept and/or slope of deterministic trend.

On the other hand, the recent literature on tests for structural change in terms of intercept and / or slope of a trend function suggests that the performance of these tests depend on whether

the noise component is stationary or nonstationary having unit roots. To deal with this problem, recently Perron and Yabu (2009) have proposed a novel test for structural change in the trend function of a univariate time series, which can be performed without any prior knowledge on whether the noise component is stationary or nonstationary containing unit roots.

Following Perron and Yabu (2009), let the data generating process of a time series variable  $y_t$  be written as  $y_t = x_t' \psi + u_t$  and  $u_t = \alpha u_{t-1} + e_t$ ,  $t = 1, 2, \dots, T$

where  $e_t \sim i.i.d. (0, \sigma^2)$ ,  $x_t$  is a  $(r \times 1)$  vector of deterministic components and  $\psi$  is a  $(r \times 1)$  vector of unknown parameters associate with  $x_t$ . Both the stationary and integrated error part are included in the model with the restriction on  $\alpha$  as  $-1 < \alpha \leq 1$ . The null hypothesis which is required to be tested is  $R\psi = \gamma$ , where  $R$  is a  $(q \times r)$  known matrix and  $\gamma$  is a  $(q \times 1)$  vector of restrictions. The restrictions incorporate the structural change in intercept and/ or slope in a trend function and the break date is denoted by  $T_1 = [\lambda_1 T]$  for some  $\lambda_1 \in (0, 1)$ . In the presence of a known break date, Perron and Yabu have derived the relevant test statistics for the following three models where a one-time change occurs in the intercept only, slope only, and both intercept and slope of the trend function, respectively.

*Model I:* Here  $x_t = (1, DU_t, t)'$  and  $\psi = (\mu_0, \mu_1, \beta_0)'$  where  $DU_t = 1(t > T_1)$ . This specification allows for an one-time change in the intercept. The null hypothesis of interest is  $\mu_1 = 0$ .

*Model II:* Here  $x_t = (1, t, DT_t)'$  and  $\psi = (\mu_0, \beta_0, \beta_1)'$  where  $DT_t = 1(T > T_1)(t - T_1)$ . This specification allows for a one-time change in the slope of the (linear) trend function without any change in the level. The relevant null hypothesis is  $\beta_1 = 0$ .

*Model III*: Here  $x_t = (1, DU_t, t, DT_t)'$  and  $\psi = (\mu_0, \mu_1, \beta_0, \beta_1)'$ . This specification allows for a simultaneous change in the intercept and slope parameters. The null hypothesis of interest in this model is  $\mu_1 = \beta_1 = 0$ .

The testing procedure is based on what is called the feasible generalized least square (FGLS) method that uses an estimate of, say  $\hat{\alpha}$ , where  $\hat{\alpha} = \sum_{t=2}^T \hat{u}_t \hat{u}_{t-1} / \sum_{t=2}^T \hat{u}_{t-1}^2$ , where  $\{\hat{u}_t\}$  are the OLS residuals from the regression of  $y_t$  on  $x_t$ . Perron and Yabu (2009) have used standard results on FGLS estimator to derive a testing procedure with the same limit distribution in both  $I(0)$  and  $I(1)$  cases. The procedure uses a superefficient estimate of the autoregressive

parameter  $\alpha$  when  $\alpha = 1$ , which is defined as  $\hat{\alpha}_s = \begin{cases} \hat{\alpha} & \text{if } T^\delta |\hat{\alpha} - 1| > d \\ 1 & \text{if } T^\delta |\hat{\alpha} - 1| \leq d \end{cases}$  for  $\delta \in (0, 1)$  and

$d > 0$ . Here the estimate of  $\alpha$  is the OLS estimate obtained from an autoregression applied to detrended data and is truncated to take a value 1 when the estimate is in a  $T^{-\delta}$  neighbourhood of 1. They have proposed the Wald statistic for testing the null hypothesis  $R\psi = \gamma$ . This test statistic is given by

$$W_{FS}(\lambda_1) = [R(\hat{\psi} - \psi)]' [s^2 R(X'X)^{-1} R']^{-1} [R(\hat{\psi} - \psi)]$$

where  $\hat{\psi}$  is the FGLS estimate of  $\psi$ ,  $X = \{x_t^{\hat{\alpha}_s}\}$ ,  $s^2 = T^{-1} \sum_{t=1}^T \hat{e}_t^2$ ,  $\hat{e}_t$  is the residual,  $x_1^{\hat{\alpha}_s} = x_1$ ,  $y_1^{\hat{\alpha}_s} = y_1$ ,  $x_t^{\hat{\alpha}_s} = (1 - \hat{\alpha}_s L)x_t$  and  $y_t^{\hat{\alpha}_s} = (1 - \hat{\alpha}_s L)y_t$ , for  $t = 2, \dots, T$ . Now, if  $|\alpha| < 1$ ,  $W_{FS}(\lambda_1) \xrightarrow{d} \chi^2(q)$  for all the three models. If, however,  $\alpha = 1$ , the limit distribution of  $W_{FS}(\lambda_1)$  is  $\chi^2(q)$  for *Model II* only. However, the same holds for *Models I* and *III* as well if the errors are normally distributed. In short, we note that in case of known break date, inferences on the intercept and slope parameters can be performed using the standard normal or chi-square distribution.

Now, in case of unknown break date, which is actually the case in reality, Perron and Yabu have considered three functionals of Wald statistics for different break dates, *viz.*, Mean- $W_{FS}$ , EXP- $W_{FS}$  and Sup- $W_{FS}$  (see, Andrews (1993) and Andrews and Ploberger (1994) for details on these test statistics). When break date is unknown, the limit distributions of the above test statistics are different in the  $I(0)$  and  $I(1)$  cases. However, among these three test statistics, EXP functional of the Wald statistic i.e., EXP- $W_{FS}$  yields a test with nearly identical limit distribution for the  $I(0)$  and  $I(1)$  cases and thus it can be used as a test to detect the structural break. This testing procedure is also extended to the case of multiple structural breaks where, the error term follows an  $AR(p)$  (see Perron and Yabu (2009) for details).

## **2.2 Unit root tests under trend break**

The modification of the ADF test by Perron (1989) allows for the possibility of a structural break in the deterministic trend function under both the null and alternative hypotheses. However, a serious limitation of this test is that the break date is assumed to be exogenously given. Following this influential paper, a number of studies such as Zivot and Andrews (1992), Perron (1997) and Vogelsang and Perron (1998) have considered the unit root test problem where the break date of the deterministic trend function is taken to be unknown. However, Kim and Perron (2009) have pointed out that all these studies which assume the break date as unknown do not allow for the possibility of a trend break under the null hypothesis; those tests consider break in the time series under the alternative only. Hence, the proposed test statistics are inferior in terms of size and power. Recently, Kim and Perron (2009) have developed a new test procedure on the line of Perron's (1989) original formulation of trend break being allowed under both the null and alternative hypotheses, and they have assumed that the break date is unknown.

According to Kim and Perron, any univariate time series can be generated by one of the three additive outlier models\*\* or two innovative outlier models. For each model the series consist of a deterministic trend and an error process, and the deterministic trend specification allows for a one-time break either in the intercept, or in the slope, or in both. The models are written as follows.  $y_t = z'_{t,1}\phi_1 + z'(T_1)_{t,2}\phi_2 + u_t$ , where  $z_{t,1} = (1, t)'$ ,  $\phi_1 = (\mu, \beta)'$ ,

$$z(T_1)_{t,2} = \begin{cases} DU_t & \text{for Model A1} \\ B_t & \text{for Model A2} \\ (DU_t, B_t)' & \text{for Model A3,} \end{cases} \quad \phi_2 = \begin{cases} \mu_b & \text{for Model A1} \\ \beta_b & \text{for Model A2} \\ (\mu_b, \beta_b)' & \text{for Model A3} \end{cases}$$

with  $DU_t = B_t = 0$  if  $t \leq T_1$  and  $DU_t = 1, B_t = t - T_1$ , if  $t > T_1$ . Here  $T_1 = \lambda^c T$  denotes the true break date, where  $\lambda^c$  with  $0 < \lambda^c < 1$  is the true break fraction. The noise  $\{u_t\}$  is such that  $A(L)u_t = B(L)\varepsilon_t$  where  $\varepsilon_t \sim i.i.d. (0, \sigma_\varepsilon^2)$ . By factorizing  $A(L)$  as  $(1 - \alpha L)A^*(L)$  and assuming that  $A^*(L)$  and  $B(L)$  have roots strictly outside the unit circle, the null and alternative hypotheses are stated as  $H_0: \alpha = 1$  and  $H_1: |\alpha| < 1$ .

It is to be noted that when the break date is known, the test statistic derived by Perron (1989) is the 'best' from consideration of allowing for a break under both the null and alternative hypotheses and hence the power of the test increases while maintaining the correct size. Kim and Perron exploited this fact and analyse unit root tests which use an estimate of the break fraction,  $\hat{\lambda}$ , instead of a true one  $\lambda^c$ , and derived the sufficient conditions for the limiting distributions of the test statistics,  $t_\alpha(\hat{\lambda})$ , which are same as those occurring when the break date is known. In the particular case where the available estimate of the break fraction does not satisfy the sufficient condition, a test statistic derived from trimming data around the estimated break date,  $t_\alpha(\hat{\lambda}_{tr})$ , is

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\*\*Kim and Perron (2009) have also considered the innovative outlier models. However, in this paper we have applied the additive models, and hence we describe briefly this approach.

a preferred one to test the hypothesis. Kim and Perron have also shown that in the absence of a trend break, the well-known ADF test is the appropriate test for testing the presence of unit roots in a time series.

### **3. Proposed Model for inflation**

In this section, we present the proposed model describing the relationship between inflation and inflation uncertainty along with two existing ones. With the introduction of the autoregressive conditional heteroskedasticity (ARCH) and generalised ARCH (GARCH) model by Engle (1982) and Bollerslev (1986), respectively, innumerable studies have used the GARCH or GARCH-type model as a measure of inflation uncertainty that underpin the dynamic nexus between inflation and inflation uncertainty. Among these, some studies (see, Brunner and Hess (1993), Baillie et al. (1996), Hwang (2001), Kontonikas (2004)) have used a simultaneous estimation technique to detect this link, while others like Grier and Perry (1998), Fountas et al. (2004), Conrad and Karanasos (2006) have relied on a method where the conditional variance is first estimated from a GARCH/GARCH-type model and then Granger causality technique is used to test for bi-directional effects. Pagan (1984) criticised the latter i.e., the two-step method because it suffers from a serious contradiction. This is so because in the first step the conditional variance is estimated from a model that implies that there is no theoretical cross-correlation between the inflation and its variance, and then in the second step this variance is used to check whether it Granger-causes the inflation or not. Using full information maximum likelihood (FIML) method, Pagan and Ullah (1988) addressed the issue that in case inflation affects inflation uncertainty, then the inflation variable should be included in the GARCH specification in the first step. Similarly, if the inflation uncertainty affects the inflation, then a measure of inflation uncertainty must be present in the first step of inflation specification. Keeping this

limitation in mind, inflation and inflation uncertainty are estimated jointly in this paper rather than following the two-step procedure.

Following Fountas et al. (2000) and Brunner and Hess (1993), the conditional variance specification of inflation explicitly includes a lag inflation term. Thus with the conditional mean specification being an AR( $k$ ) model, the model for describing inflation designated as AR( $k$ )-GARCH(1,1)<sup>††</sup>L(1)<sup>‡‡</sup>, consists of the following models for conditional mean and conditional variance.

$$\pi_t = \phi_0 + \phi_1\pi_{t-1} + \phi_2\pi_{t-2} + \dots + \phi_k\pi_{t-k} + \varepsilon_t, \varepsilon_t | \Psi_{t-1} \sim N(0, h_t) \quad (3.1)$$

$$h_t = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1h_{t-1} + \theta\pi_{t-1} \quad (3.2)$$

where  $\pi_t$  stands for the inflation at time ‘ $t$ ’,  $h_t$  for conditional variance at ‘ $t$ ’ and  $k$  is the optimal lag order in the mean specification. Here the coefficient  $\theta$  depicts the link between inflation uncertainty and inflation, and thus a statistically significant  $\theta$  establishes the Friedman-Ball hypothesis for a given time series on inflation. All the roots of the polynomial  $1 - \phi_1B - \dots - \phi_kB^k = 0$  are assumed to lie outside the unit circle for stationarity of  $\pi_t$ , and  $\omega > 0$ ,  $\alpha_1 \geq 0$ ,  $\beta_1 \geq 0$  for positivity of  $h_t$ .

However, Brunner and Hess (1993) have pointed out that since the GARCH model places a symmetric restriction on the conditional variance, it does not capture the asymmetric behaviour of an economic agent when there is more uncertainty about future inflation rising or falling unexpectedly. Hence, we have considered the original volatility model to be the threshold

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<sup>††</sup> In most empirical works with GARCH( $p,q$ ) specification for volatility, the orders  $p=q=1$  has been found to be adequate, and the same has been the case in this study as well. Hence, al throughout the paper, GARCH(1,1) has been considered.

<sup>‡‡</sup> ‘L(1)’ stands for the fact that the specification for  $h_t$  includes the first lag of inflation as a separate term.

GARCH (TGARCH) model which is due to Glosten et al. (1993), but an additional lag inflation term has been included in the volatility specification. The conditional variance specification, called the TGARCH(1,1)L(1) model, thus becomes

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 \varepsilon_{t-1}^2 I[\varepsilon_{t-1} < 0] + \beta_1 h_{t-1} + \theta \pi_{t-1} \quad (3.3)$$

where  $I[.]$  is the indicator function which takes the value 1 if  $\varepsilon_{t-1} < 0$  and 0 otherwise. The leverage effect is covered by the TGARCH model if  $\gamma_1 > 0$ .

To address the two issues raised in the introduction, *viz.*, consideration of (i) different values of  $\theta$  for the two different regimes in the model for inflation uncertainty as represented through time varying conditional variance of inflation and (ii) a threshold level of inflation, we propose, adopting the asymmetric nonlinear smooth transition GARCH (ANSTGARCH) model, as proposed by Anderson et al. (1999) and Nam et al. (2001), and then extending it to allow for the inclusion of a lag inflation term in the conditional variance equation<sup>§§</sup>. We extend this framework to a more general one, where smooth transition behaviour of regime changes is included in the conditional mean specification also. This model thus takes into account regime specific behaviour in both the conditional mean and conditional variance, and hence it has the advantage of measuring the effect of high inflation (deflation) on both the mean and variance.

The resultant model denoted as STAR( $k$ )-ANSTGARCH(1,1)L(1), is specified as follows:

$$\pi_t = (\phi_0^1 + \phi_1^1 \pi_{t-1} + \phi_2^1 \pi_{t-2} + \dots + \phi_k^1 \pi_{t-k}) [1 - F(Z_t; \lambda, c)] + (\phi_0^2 + \phi_1^2 \pi_{t-1} + \phi_2^2 \pi_{t-2} + \dots + \phi_k^2 \pi_{t-k}) F(Z_t; \lambda, c) + \varepsilon_t, \varepsilon_t | \Psi_{t-1} \sim N(0, h_t) \quad (3.4)$$

$$h_t = [\omega_1 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \theta_1 \pi_{t-1}] [1 - F(Z_t; \lambda, c)] + [\omega_2 + \alpha_2 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \theta_2 \pi_{t-1}] F(Z_t; \lambda, c) \quad (3.5)$$

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<sup>§§</sup> See Nam (1998) and Anderson et al. (1999) for an exposition of the ANSTGARCH models in finance.

where  $\phi^i = (\phi_0^i, \phi_1^i, \dots, \phi_k^i)'$  denotes the coefficient of lag term in the mean equation for the  $i^{\text{th}}$  regime, with  $i = 1, 2$ . Apart from the usual GARCH coefficients,  $\theta_1$  and  $\theta_2$  describe the link between inflation uncertainty and inflation in the two regimes, respectively. The regimes are governed by the transition function  $F(Z_t; \lambda, c)$ . A popular choice for the transition function is the logistic function which is defined as

$$F(Z_t; \lambda, c) = \frac{1}{1 + \exp\{-\lambda[Z_t - c]\}}$$

where  $Z_t$  is a threshold variable corresponding to a threshold value  $c$ . The parameter  $c$  can be interpreted as the threshold between the two regimes corresponding to  $F(Z_t; \lambda, c) = 0$  and  $F(Z_t; \lambda, c) = 1$ , in the sense that the logistic function changes monotonically from 0 to 1 as  $Z_t$  increases, while  $F(c; \lambda, c) = 0.5$ . The parameter  $\lambda$  determines the smoothness of the change in the value of logistic function, and thus the transition from one regime to the other. In this paper, as already stated, we have used a lag inflation term,  $\pi_{t-1}$ , as our threshold variable instead of past shock, and accordingly regimes are defined as the lag inflation is greater or smaller than an unknown threshold level of inflation. Thus, in the transition equation we replace  $Z_t$  by  $\pi_{t-1}$  as a probable candidate for the threshold variable. The proposed model for  $h_t$  in equation (3.5) is very similar to one of the models in Nam et al. (2001). A point of departure from their model is that the variance equation in our specification is augmented by a lagged inflation term, and the smooth transition function depends on  $\pi_{t-1}$ , instead of  $\varepsilon_{t-1}$ .

#### 4. Empirical analysis

In this empirical study, we have considered monthly time series on consumer price index from January 1960 to December 2009 for 13 different countries *viz.*, Canada, France, Germany,

Italy, Japan, the U.K., the U.S.A., Austria, Belgium, Finland, Greece, Luxembourg and Spain. The first 7 countries comprise the well-known G7 group, while the remaining countries are the countries from Euro zone<sup>\*\*\*</sup>. Among the G7 countries, France, Germany, and Italy are also included in the countries forming Euro Zone. All the time series have been downloaded from the official website of Federal Reserve Bank of St. Louis. Computations were done with Eviews, Gauss and Matlab.

To begin with, all the time series have been passed through the well-known X-12-ARIMA filter to make the series seasonally adjusted. The time series of inflation, denoted as  $\pi_t$ , has been obtained as  $\pi_t = 100[\ln(cpi)_t - \ln(cpi)_{t-1}]$ , where  $cpi$  is the seasonally adjusted consumer price index of a particular country. Figure 1 plots the inflation series for each of the above countries.

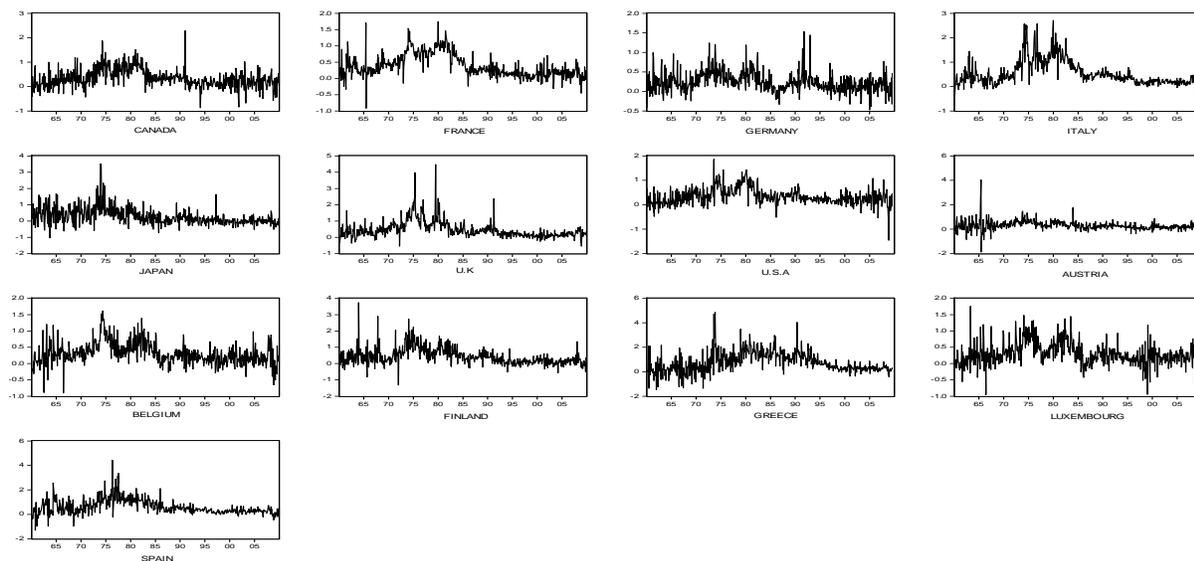


Figure 1: Plots of time series on inflation for the 13 countries

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<sup>\*\*\*</sup> See, among others, Daal et al. (2005) and Thornton (2007) for studies on the relationship between inflation and inflation uncertainty for developing countries.

Visual inspection of the above plots tend to support the view that the inflation of most countries stayed at a very high level during the period of ‘great inflation’, which was coupled with the first and second oil crises. The volatility appear to be more pronounced during that period while it remained low and stable at the latter half of the sample due to the adoption of anti inflationary policy by many countries to stabilize the inflationary hike. It is also evident from the plots that not only the volatility but also the trend component of inflation has decreased in the recent era for most of the series, suggesting, that the level and volatility for inflation have undergone regime shifts/switches.

In Table 1, we present the summary statistics on the time series of inflation of all the countries considered in this work.

Table 1: Descriptive statistics on inflation of 13 countries

<i>Country</i>	<i>Mean</i>	<i>Std. dev.</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>Jarque-Bera</i>	<i>Q(5)</i>	<i>Q(10)</i>	<i>Q<sup>2</sup>(5)</i>	<i>Q<sup>2</sup>(10)</i>
<i>Canada</i>	0.34	0.37	0.62	4.68	109.04*	503.6*	959.4*	307.0*	585.2*
<i>France</i>	0.38	0.35	0.75	3.67	67.87*	1440.8*	2730.7*	1355.3*	2413.6*
<i>Germany</i>	0.23	0.25	0.98	5.78	287.53*	212.9*	470.7*	71.0*	135.3*
<i>Italy</i>	0.53	0.49	1.59	5.84	455.16*	1687.7*	3188.0*	1123.7*	1984.8*
<i>Japan</i>	0.28	0.53	1.84	9.65	1442.10*	395.6*	807.9*	393.5*	660.2*
<i>The U.K.</i>	0.45	0.50	2.51	14.79	4099.36*	1061.3*	1917.1*	309.8*	518.8*
<i>The U.S.A.</i>	0.34	0.33	0.44	5.51	177.50*	579.1*	1108.7*	648.1*	1238.5*
<i>Austria</i>	0.29	0.37	1.97	24.44	11865.86*	57.8*	141.5*	117.6*	121.9*
<i>Belgium</i>	0.31	0.34	0.63	4.59	103.17*	592.7*	1084.9*	805.9*	1245.2*
<i>Finland</i>	0.42	0.49	1.52	8.45	972.94*	602.8*	1104.5*	180.3*	320.9*
<i>Greece</i>	0.75	0.82	0.86	5.25	201.43*	788.2*	1468.8*	473.1*	644.8*
<i>Luxembourg</i>	0.29	0.34	0.56	4.60	95.19*	362.7*	715.8*	239.7*	472.2*
<i>Spain</i>	0.59	0.59	1.38	7.76	755.42*	718.1*	1428.5*	352.5*	702.6*

[\* indicates significance at 1% level of significance.  $Q(\cdot)$  and  $Q^2(\cdot)$  represent the Ljung-Box test statistic values for linear and squared autocorrelation.]

From this table it is evident that among all countries, Greece exhibits the highest average inflation of 0.75 per cent with the largest standard deviation of 0.82 per cent. The relevant entries

of this table also indicate that all the 13 inflation series are positively skewed and have leptokurtic distribution. Therefore, as expectedly, results of the Jarque-Bera normality test clearly indicate that the null hypothesis of normality is rejected for all the inflation series. The Ljung-Box test results,  $Q(\cdot)$ , based on the inflation data indicate that all the 13 series are autocorrelated. Again, the values of  $Q^2(\cdot)$  test statistic for squared inflation series show a substantial evidence for the presence of volatility for all the 13 countries.

#### 4.1 Results of tests on presence on structural break and unit roots

The ADF test was carried out for testing stationarity of each of the 13 series. These test statistic values are reported in Table 2. The optimum number of lags for the ADF estimating equation has been chosen based on Schwarz's information criterion (SIC).

Table 2: Results of the ADF and PP tests on inflation of 13 countries

<i>Country</i>	<i>ADF test statistic</i>	<i>Decision at 5%</i>	<i>PP test statistic</i>	<i>Decision at 5%</i>
<i>Canada</i>	-3.112	<i>I(1)</i>	-23.804*	<i>I(0)</i>
<i>France</i>	-2.712	<i>I(1)</i>	-13.991*	<i>I(0)</i>
<i>Germany</i>	-3.469*	<i>I(0)</i>	-23.313*	<i>I(0)</i>
<i>Italy</i>	-2.635	<i>I(1)</i>	-7.379*	<i>I(0)</i>
<i>Japan</i>	-4.147*	<i>I(0)</i>	-22.446*	<i>I(0)</i>
<i>The U.K.</i>	-3.910*	<i>I(0)</i>	-14.228*	<i>I(0)</i>
<i>The U.S.A.</i>	-3.603*	<i>I(0)</i>	-19.007*	<i>I(0)</i>
<i>Austria</i>	-21.021*	<i>I(0)</i>	-21.630*	<i>I(0)</i>
<i>Belgium</i>	-3.500*	<i>I(0)</i>	-17.946*	<i>I(0)</i>
<i>Finland</i>	-4.268*	<i>I(0)</i>	-24.209*	<i>I(0)</i>
<i>Greece</i>	-3.441*	<i>I(0)</i>	-21.148*	<i>I(0)</i>
<i>Luxembourg</i>	-4.802*	<i>I(0)</i>	-23.038*	<i>I(0)</i>
<i>Spain</i>	-3.199	<i>I(1)</i>	-18.345*	<i>I(0)</i>

[An intercept and a deterministic linear trend term are included in the ADF and PP estimating equations. \* indicates significance at 5% level of significance.]

The computed value of the ADF test statistic for each of the time series on inflation is compared with the critical value of -3.417 at 5% level of significance. Accordingly, we conclude that out of

the 13 countries considered in this study, the unit root assumption can not be rejected for four countries, *viz.*, Canada, France, Italy and Spain. The Phillips-Perron (1988) test was also done, and its values are given in Table 2. It is evident from these values that all the 13 series are stationary.

As already discussed, in the presence of a structural break in the deterministic trend term, ADF test conclusion on unit root could be misleading. Since the span of the data sets is very large covering 50 years, one or more structural breaks in the series is(are) only very likely. Therefore, we first performed the test proposed by Perron and Yabu (2009) for testing the presence of a structural break in the series, which does not presume anything about stationarity/nonstationarity (i.e., having unit roots) of the series. Thereafter, we have applied the unit root test as proposed by Kim and Perron (2009) which allows the possibility of a break under both null and alternative hypotheses<sup>†††</sup>. As discussed in Section 2.1, three models - *Model I*, *Model II* and *Model III* - representing a single change in intercept only, a one-time change in the slope of the trend without a change in level, and a simultaneous change in the intercept and slope coefficients, respectively, are considered for the Perron-Yabu test. For this test, we have chosen the trimming parameter to be 0.15. Since the test statistic for *Model III* has the highest power against the alternatives, we have considered only *Model III* and the values of which are presented in Table 3.

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<sup>†††</sup> A GAUSS program code for Perron and Yabu (2009) structural break test and a MATLAB code for Kim and Perron (2009) unit root test in presence of structural break have been taken from the official website of Pierre Perron.

Table 3: Results of the Perron-Yabu test on structural break and the Kim-Perron test on unit roots

Country	Perron-Yabu structural break test		Kim-Perron unit root test	
	Statistic value for Model III	Estimated break date	Statistic value for Model A3	Statistic value for Model A3(trimmed)
Canada	144.93*	1982M06	-24.14*[0.45]	-21.66*[0.45]
France	109.61*	1983M09	-8.92*[0.47]	-8.19*[0.47]
Germany	19.52*	1982M06	-10.28*[0.45]	-9.01*[0.45]
Italy	15.02*	1982M12	-5.18*[0.46]	-4.24*[0.46]
Japan	24.74*	1976M12	-20.59*[0.39]	-18.74*[0.39]
The U.K.	64.24*	1981M04	-8.71*[0.43]	-5.91*[0.43]
The U.S.A.	84.99*	1981M09	-13.54*[0.43]	-11.46*[0.43]
Austria	17.16*	1976M08	-22.38*[0.33]	-19.71*[0.33]
Belgium	37.64*	1976M09	-7.98*[0.33]	-7.67*[0.33]
Finland	48.73*	1973M04	-7.54*[0.27]	-7.31*[0.27]
Greece	126.22*	1973M01	-12.32*[0.26]	-11.91*[0.26]
Luxembourg	81.24*	1984M03	-10.91*[0.48]	-9.74*[0.48]
Spain	39.52*	1973M03	-12.86*[0.26]	-10.65*[0.26]

[\* indicates significance at 5% level of significance. The fractions in parentheses denote the estimated break fraction.]

The Perron-Yabu test statistic values corresponding to *Model III* clearly establish that for all the 13 countries, at least one of the intercept and trend dummies are statistically significant. It is thus concluded that there is one break/change in the deterministic trend of inflation for each of these countries. Respective break dates, under *Model III* are shown in the 3<sup>rd</sup> column of this table. Thus, looking at the entries of the 3<sup>rd</sup> column, we note that all the estimated break dates are during the period of 1970s and 1980s. The findings thus provide empirical support to the world-wide observed fact that while inflation increased globally and became more volatile during the 1960s, it fell and became substantially less volatile since the 1980s.

After having estimated the break dates for the inflation series of each country, we have applied the third variant of additive outlier models, *Model A3*<sup>+++</sup>, as described in Kim and Perron's study, to each of the inflation series for testing the null hypothesis of unit root with a deterministic trend break against the alternative of stationarity with a break in the deterministic trend. Following, Kim and Perron the test statistics values for both non-trimmed and trimmed data of the Model A3 are reported in the 4<sup>th</sup> and 5<sup>th</sup> columns of Table 3. The computed values of *t*-statistic are compared with the appropriate critical values available in Perron (1989) and Perron and Vogelsang (1993). The findings clearly suggest that the inflation series of each of these countries is stationary when the presence of one structural break is taken into account. In each case, the null hypothesis of a unit root with a trend break against the alternative of a stationary process with a break is rejected. Thus, this test shows that the time series of inflation is stationary with a break in deterministic trend for all the 13 countries. This finding is somewhat in contrast with the one for the usual ADF (*cf.* Table 2) test, which concluded stationarity in the sense of having no unit root for 9 series.

We thus conclude that the underlying data generating process for each of the 13 inflation series can be treated as a trend stationary process (TSP) and an appropriate (deterministic) trend removal process would yield stationary series. We have now regressed inflation on an intercept, an intercept dummy, trend (linear) and trend (linear) dummy, and then collected the detrended series which is now stationary. This stationary time series on inflation is used for all subsequent analysis.

#### **4.2 Empirical findings on the relationship involving inflation and inflation uncertainty**

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<sup>+++</sup> The test was also done for Model A1 and A2 also, and the conclusion regarding stationarity with a break in the deterministic trend of the series were found to be the same.

We now discuss about the relationship involving inflation and inflation uncertainty. As already stated in the preceding section, we consider two competing models which are used in the extant empirical literature, *viz.*, AR( $k$ )-GARCH(1,1)L(1), AR( $k$ )-TGARCH(1,1)L(1). The performance of the proposed STAR( $k$ )-ANSTGARCH(1,1)L(1) is compared with these two existing models. Following, Fountas et al. (2000) and Kontonikas (2004), we first estimate the AR( $k$ )-GARCH(1,1)L(1) model which are specified by equation (3.1) and (3.2). The parameters of this model along with two other models *viz.*, AR( $k$ )-TGARCH(1,1)L(1) and STAR( $k$ )-ANSTGARCH(1,1)L(1) have been estimated by the maximum likelihood (ML) method under the assumption of normality. For the iterative optimization procedure involved, the well known BHHH algorithm has been used. The estimates of AR( $k$ )-GARCH(1,1)L(1) model are presented in Table 4. The optimal lag orders of the autoregressive terms for the 13 series were obtained by using the Akaike (1973) information criterion.

Table 4: Parameter estimates and residual diagnostic tests for the AR( $k$ )-GARCH(1,1)L(1) model

<i>Estimates</i>	<i>Canada</i>	<i>France</i>	<i>Germany</i>	<i>Italy</i>	<i>Japan</i>	<i>The U.K.</i>	<i>The U.S.A.</i>
$\phi_0$	0.004	0.003	0.004	-0.004	-0.004	-0.002	-0.004
$\phi_1$	-0.029	0.302*	0.078	0.283*	0.044	0.233*	0.135*
$\phi_2$	0.115**	0.137*	0.161*	0.219*	-0.007	0.237*	0.072
$\phi_3$	0.086*	0.126*	0.117*	0.118*	0.054	0.125**	0.046
$\phi_4$	0.052	-	0.011	0.056	-0.008	-0.084***	0.038
$\phi_5$	0.087**	-	-0.038	-0.027	0.114**	0.030	0.030
$\phi_6$	0.023	-	0.082**	0.097**	0.040	0.141*	0.022
$\phi_7$	0.017	-	0.038	0.137*	0.047	-	0.094**
$\phi_8$	-0.017	-	0.086*	0.062	0.060	-	0.003
$\phi_9$	0.094**	-	0.058***	0.037	0.129*	-	0.063
$\phi_{10}$	0.020	-	0.107*	0.030	0.047	-	0.091**
$\phi_{11}$	0.064	-	-	-0.085**	-	-	0.085**
$\phi_{12}$	-0.147*	-	-	-0.112*	-	-	-0.255*
$\omega$	0.014*	0.034*	0.015*	0.000***	0.005	0.006**	0.001

$\alpha_1$	0.191*	0.192*	0.458*	0.150*	0.104**	0.418*	0.102*
$\beta_1$	0.665*	0.000	0.324*	0.848*	0.867*	0.636*	0.897*
$\theta$	0.016	0.007	0.026***	0.000	-0.004	0.002	0.001
<i>MLLV</i>	-87.22	119.29	90.38	166.80	-244.91	-88.95	35.70
<i>Autocorrelation in standardized residual</i>							
$Q(1)$	0.781	0.683	0.040	0.486	0.013	0.713	0.047
$Q(5)$	1.469	1.413	0.850	0.860	0.512	2.537	1.119
$Q(10)$	1.836	8.667	1.071	1.894	1.607	8.345	4.119
<i>Autocorrelation in squared standardized residual</i>							
$Q^2(1)$	0.060	1.332	0.379	1.802	0.323	0.037	3.450
$Q^2(5)$	1.206	2.056	2.442	2.159	2.303	2.836	6.333
$Q^2(10)$	3.228	2.458	4.797	7.203	3.020	4.888	9.879

[\*, \*\* and \*\*\* indicate significance at 1%, 5% and 10% level of significance, respectively. *MLLV* is the maximum log likelihood value.]

<i>Estimates</i>	<i>Austria</i>	<i>Belgium</i>	<i>Finland</i>	<i>Greece</i>	<i>Luxembourg</i>	<i>Spain</i>
$\phi_0$	0.001	-0.002	-0.002	-0.011	-0.001	-0.005
$\phi_1$	0.013	0.247*	0.045	-0.003	-0.003	0.209*
$\phi_2$	0.096**	-0.002	0.163*	0.203*	0.128*	0.097**
$\phi_3$	0.036	0.035	0.165*	0.066	0.088**	0.051
$\phi_4$	0.017	0.142*	0.138*	0.085***	0.062	-0.013
$\phi_5$	0.046	0.015	0.090**	0.077***	-0.000	0.134*
$\phi_6$	0.140*	0.046	0.099**	0.104**	0.109*	0.074**
$\phi_7$	0.147*	0.021	0.017	0.042	0.052	0.093*
$\phi_8$	0.026	0.002	0.026	0.052	0.016	0.132*
$\phi_9$	-0.010	0.199*	0.058	0.070***	0.116*	0.068**
$\phi_{10}$	0.068***	0.016	-	0.034	0.058	0.128*
$\phi_{11}$	0.063	0.110**	-	0.045	0.096**	-0.028
$\phi_{12}$	-0.071***	-0.137*	-	-0.095*	-0.098**	-0.250*
$\phi_{13}$	-	-	-	-	-0.039	0.108*
$\phi_{14}$	-	-	-	-	-	0.026
$\phi_{15}$	-	-	-	-	-	0.062**
$\omega$	0.004*	0.007*	0.008*	0.002	0.048*	0.018*
$\alpha_1$	0.197*	0.045	0.224*	0.089**	0.480*	0.764*
$\beta_1$	0.766*	0.845*	0.750*	0.910*	0.000	0.381*

$\theta$	0.005	0.021*	0.012	0.003	0.006	0.044*
<i>MLLV</i>	-40.87	8.84	-188.39	-407.63	-60.07	-182.28
<i>Autocorrelation in standardized residual</i>						
$Q(1)$	2.012	0.057	0.000	0.005	0.139	0.097
$Q(5)$	2.692	1.594	1.840	0.640	0.440	3.418
$Q(10)$	6.044	2.698	3.324	1.587	1.630	4.992
<i>Autocorrelation in squared standardized residual</i>						
$Q^2(1)$	6.285**	0.179	0.719	3.360	0.224	0.019
$Q^2(5)$	8.618	1.922	3.294	4.355	1.047	1.879
$Q^2(10)$	12.252	11.187	15.601	7.679	19.608**	3.279

[\*, \*\* and \*\*\* indicate significance at 1%, 5% and 10% level of significance, respectively. *MLLV* is the maximum log likelihood value.]

It is observed from this table that the GARCH parameters are statistically significant for all series. Apart from the usual GARCH coefficients, it is also evident that for the countries like, Germany, Belgium and Spain, a significant positive link exists between inflation uncertainty and inflation, thus supporting the Friedman-Ball hypothesis. As for instance, in case of Germany the estimate of parameter depicting the relationship i.e.,  $\theta$  is 0.026 and this is statistically significant at 10% level of significance. In terms of residual diagnostics, the Ljung-Box test,  $Q(\cdot)$ , for testing the presence of autocorrelation in standardized residuals, it is found that the null hypothesis of ‘no autocorrelation’ can not be rejected for all the countries. However, the same conclusion was obtained for the squared standardized residuals,  $Q^2(\cdot)$ , in all but Austria and Luxembourg.

Empirical findings on the next model i.e., AR( $k$ )-TGARCH(1,1)L(1) are presented in the next table i.e., Table 5. Since GARCH model places a symmetric restriction on the conditional variance and consequently the resultant model does not incorporate the asymmetric news effect in the conditional variance specification, this model uses TGARCH given in equation (3.3), as

the volatility specification, so that the impact of asymmetric news on inflation uncertainty could also be studied. The computational results are reported in Table 5.

Table 5: Parameter estimates and residual diagnostic test for the AR( $k$ )-TGARCH(1,1)L(1) model

<i>Estimates</i>	<i>Canada</i>	<i>France</i>	<i>Germany</i>	<i>Italy</i>	<i>Japan</i>	<i>The U.K.</i>	<i>The U.S.A.</i>
$\phi_0$	0.005	0.003	0.002	-0.000	-0.002	-0.010	0.003
$\phi_1$	-0.008	0.302*	0.087***	0.268*	0.041	0.230*	0.087***
$\phi_2$	0.140*	0.137*	0.186*	0.221*	-0.001	0.221*	0.086**
$\phi_3$	0.091**	0.126*	0.122*	0.116**	0.061	0.104**	0.107**
$\phi_4$	0.058	-	0.007	0.045	-0.006	-0.070	0.113**
$\phi_5$	0.081**	-	-0.044	-0.024	0.116**	0.015	0.026
$\phi_6$	0.026	-	0.083**	0.087***	0.042	0.145*	0.032
$\phi_7$	0.024	-	0.024	0.152*	0.059	-	0.098**
$\phi_8$	-0.010	-	0.085*	0.067	0.059	-	-0.013
$\phi_9$	0.101**	-	0.068**	0.039	0.139*	-	0.054
$\phi_{10}$	0.034	-	0.094**	0.036	0.056	-	0.071
$\phi_{11}$	0.057	-	-	-0.073***	-	-	0.013
$\phi_{12}$	-0.156*	-	-	-0.114*	-	-	-0.209*
$\omega$	0.018**	0.033*	0.019*	0.000***	0.005***	0.005**	0.006**
$\alpha_1$	0.344**	0.201	0.726*	0.040	0.054	0.575*	0.528*
$\gamma_1$	0.070	0.187**	0.200	0.198*	0.156**	0.232*	0.082
$\beta_1$	0.602*	0.000	0.251**	0.871*	0.860*	0.672*	0.639*
$\theta$	0.047***	0.008	0.063**	-0.003	-0.018	0.019***	0.038*
<i>MLLV</i>	-86.68	119.30	92.28	171.33	-244.10	-85.76	37.56
<i>Autocorrelation in standardized residual</i>							
$Q(1)$	0.148	0.685	0.003	1.109	0.040	0.711	0.235
$Q(5)$	1.409	1.389	2.730	2.204	0.453	2.027	8.216
$Q(10)$	2.039	8.671	3.115	2.787	0.911	8.305	11.990
<i>Autocorrelation in squared standardized residual</i>							
$Q^2(1)$	0.037	1.375	0.264	3.090	0.129	0.001	0.200
$Q^2(5)$	0.741	2.085	2.831	4.484	2.239	2.365	4.984
$Q^2(10)$	2.433	2.490	6.033	9.095	2.805	4.232	8.338

[\*, \*\* and \*\*\* indicate significance at 1%, 5% and 10% level of significance, respectively. MLLV is the maximum log likelihood value.]

<i>Estimates</i>	<i>Austria</i>	<i>Belgium</i>	<i>Finland</i>	<i>Greece</i>	<i>Luxembourg</i>	<i>Spain</i>
$\phi_0$	0.005	-0.002	0.004	-0.008	-0.001	0.003
$\phi_1$	0.027	0.249*	0.012	-0.009	0.039	0.234*
$\phi_2$	0.111**	-0.003	0.131*	0.202*	0.112**	0.128*
$\phi_3$	0.039	0.033	0.150*	0.069	0.087***	0.032
$\phi_4$	0.019	0.145*	0.119**	0.087***	0.070	0.068
$\phi_5$	0.051	0.015	0.097**	0.080***	0.032	0.038
$\phi_6$	0.162*	0.047	0.126*	0.106**	0.125*	0.067
$\phi_7$	0.132*	0.020	0.037	0.042	0.017	0.077***
$\phi_8$	0.006	0.003	0.039	0.053	0.007	0.067
$\phi_9$	-0.021	0.164*	0.079**	0.077	0.151*	0.053
$\phi_{10}$	0.052	0.017	-	0.036	0.068	0.117*
$\phi_{11}$	0.064***	0.112**	-	0.044	0.063	-0.040
$\phi_{12}$	-0.093**	-0.137*	-	-0.091**	-0.100**	-0.143*
$\phi_{13}$	-	-	-	-	-0.076***	-0.020
$\phi_{14}$	-	-	-	-	-	0.012
$\phi_{15}$	-	-	-	-	-	0.130*
$\omega$	0.004*	0.006	0.006*	0.002	0.008***	0.001***
$\alpha_1$	0.039	0.026	0.119**	0.073***	0.340**	0.072**
$\gamma_1$	0.294*	0.055	0.327*	0.106**	0.060	0.090**
$\beta_1$	0.787*	0.862*	0.764*	0.909*	0.722*	0.920*
$\theta$	-0.016	0.017	0.002	0.001	0.026***	0.004***
<i>MLLV</i>	-36.17	8.86	-186.57	-407.43	-58.69	-173.60
<i>Autocorrelation in standardized residual</i>						
<i>Q(1)</i>	1.242	0.034	0.346	0.001	0.024	0.591
<i>Q(5)</i>	2.300	1.408	1.094	0.697	1.138	1.356
<i>Q(10)</i>	5.743	2.364	3.300	2.042	4.873	1.607
<i>Autocorrelation in squared standardized residual</i>						
<i>Q<sup>2</sup>(1)</i>	4.055**	0.226	0.672	3.164	3.004	3.391
<i>Q<sup>2</sup>(5)</i>	7.095	1.965	3.990	4.435	9.767	5.114
<i>Q<sup>2</sup>(10)</i>	10.633	11.333	14.767	7.793	17.469***	7.907

[\*, \*\* and \*\*\* indicate significance at 1%, 5% and 10% level of significance, respectively. MLLV is the maximum log likelihood value.]

It is noted from Table 5 that insofar as the model for conditional mean is concerned the chosen lags are adequate in capturing the respective autocorrelations prevalent in the time series of inflation. This is clearly evident from the Ljung-Box test statistic  $Q(\cdot)$  values, where none of these values for any reported country has been found to be significant.

As regards the model for inflation uncertainty, we note that the estimate of coefficient  $\gamma_1$ , the parameter of relevance for TGARCH volatility specification, are significant for 8 countries *viz.*, France, Italy, Japan, the U.K., Austria, Finland, Greece and Spain, and the signs are also as expected, suggesting the significant presence of leverage effect in these inflation series. Finally, estimates of  $\theta$ , the parameter linking the inflation to inflation uncertainty, show that this crucial parameter is significant in 6 countries only *viz.*, Canada, Germany, the U.K., the U.S.A., Luxembourg and Spain. The signs are all positive for these countries which give support to Friedman-Ball hypothesis that high level of inflation raises inflation uncertainty. For the remaining seven countries  $\theta$  is insignificant suggesting no direct relationship between inflation and inflation uncertainty. The values of  $Q(\cdot)$  test statistic with squared standardized residuals, *i.e.*,  $Q^2(\cdot)$  values, show that for all countries the uncertainty in autocorrelation is captured completely by this model since except for  $Q^2(1)$  for Austria and  $Q^2(10)$  for Luxembourg, all other  $Q^2(\cdot)$  values are found to be insignificant.

Finally we discuss the empirical findings on the proposed STAR( $k$ )-ANSTGARH(1,1)L(1) model as specified in equations (3.4), (3.5), which includes asymmetry in both inflation and inflation uncertainty models, from consideration of regime switching behaviour wherein, *inter alia*, the coefficient attached to the inflation term in inflation uncertainty model is also taken to be different in the two regimes of inflation characterised by a threshold value of inflation, which is now taken to be a parameter to be estimated. This model being highly

nonlinear in nature involved substantial volume of computations<sup>§§§</sup>. The computational figures of this model are presented in Table 6.

Table 6: Parameter estimates and residual diagnostic test for the STAR(k)-ANSTGARCH(1,1)L(1) model

<i>Estimates</i>	<i>Canada</i>	<i>France</i>	<i>Germany</i>	<i>Italy</i>	<i>Japan</i>	<i>The U.K.</i>	<i>The U.S.A.</i>
$\phi_0^1$	0.004	-0.004	0.001	-0.007	0.016	-0.111	-0.005
$\phi_1^1$	0.019	0.267*	0.129**	0.227*	0.115	-0.109	0.183*
$\phi_2^1$	0.030	0.140*	0.148*	0.251*	-0.023	0.101	0.061
$\phi_3^1$	0.177*	0.073	0.095**	0.133**	0.044	0.266*	0.029
$\phi_4^1$	0.122**	-	0.030	0.006	-0.016	-0.012	0.052
$\phi_5^1$	0.140**	-	-0.040	0.018	0.081	-0.133	0.046
$\phi_6^1$	-0.010	-	0.093*	0.038	0.008	0.145*	0.012
$\phi_7^1$	0.010	-	0.046	0.224*	0.010	-	0.062
$\phi_8^1$	0.008	-	0.078**	0.054	0.044	-	0.002
$\phi_9^1$	0.056	-	0.044	0.049	0.117**	-	0.050
$\phi_{10}^1$	0.001	-	0.081**	-0.008	0.083	-	0.110**
$\phi_{11}^1$	0.005	-	-	-0.013	-	-	0.014
$\phi_{12}^1$	-0.199*	-	-	-0.158*	-	-	-0.251*
$\phi_{13}^1$	-	-	-	-	-	-	-
$\phi_{14}^1$	-	-	-	-	-	-	-
$\phi_{15}^1$	-	-	-	-	-	-	-
$\phi_0^2$	0.016	0.019	0.147*	0.157	-0.074	0.216	0.152
$\phi_1^2$	-0.078	0.181**	-0.450*	0.442	0.212***	-0.037	-0.471**
$\phi_2^2$	0.277*	0.246**	-0.015	-0.235	0.029	0.362*	0.145
$\phi_3^2$	0.018	0.210*	0.064	-0.149	0.079	-0.099	-0.028
$\phi_4^2$	-0.027	-	-0.042	0.285	-0.051	-0.154	-0.014
$\phi_5^2$	0.051	-	0.020	-0.487***	0.221**	0.352***	0.021
$\phi_6^2$	0.009	-	-0.184***	0.626***	0.073	0.098	0.265***
$\phi_7^2$	0.103***	-	0.010	-0.187	0.030	-	0.574*
$\phi_8^2$	-0.083	-	0.125	-0.041	0.080	-	0.178
$\phi_9^2$	0.159*	-	0.511*	-0.216	0.054	-	0.149

<sup>§§§</sup> For writing the GAUSS codes of STAR(k)-ANSTGARCH(1,1)L(1) model, we have made use of codes from the GAUSS programs developed by Philip Hans Franses and Dick Van Dijk on different volatility models, which were downloaded from <http://www.few.eur.nl/few/few/people/frances>.

$\phi_{10}^2$	0.082	-	0.164	0.802	0.016	-	0.173
$\phi_{11}^2$	0.066	-	-	-0.797*	-	-	0.434**
$\phi_{12}^2$	-0.092	-	-	0.286	-	-	-0.247***
$\phi_{13}^2$	-	-	-	-	-	-	-
$\phi_{14}^2$	-	-	-	-	-	-	-
$\phi_{15}^2$	-	-	-	-	-	-	-
$\omega_1$	0.037**	0.023*	0.011*	0.000	0.010	0.002	0.000
$\alpha_1$	0.559*	0.067	0.636*	0.000	0.063	0.045*	0.050***
$\beta_1$	0.221	0.000	0.380*	0.973*	0.459*	0.606*	0.914*
$\omega_2$	0.005	0.053*	0.000	0.005	0.000	0.000	0.022**
$\alpha_2$	0.000	0.301*	0.000	0.491*	0.142	0.000	0.000
$\beta_2$	0.657*	0.255	0.000	0.215	1.035*	1.013*	1.145*
$\theta_1$	0.047	-0.075*	0.020	-0.002	-0.185**	-0.034***	-0.012**
$\theta_2$	0.099**	-0.109*	0.211*	0.029	-0.010	0.035	-0.046
$\lambda^{****}$	$3.34 \times 10^{26}$	$5.75 \times 10^{64}$	$6.53 \times 10^{22}$	3.84	29.75	5.72	8.99
$c$	0.015	0.077	0.185**	0.533*	0.056**	0.134**	0.273*
$MLLV$	-71.56	142.94	113.08	186.25	-234.65	-59.78	60.19
<i>Autocorrelation in standardized residual</i>							
$Q(1)$	0.341	0.118	0.170	0.539	0.211	0.122	0.125
$Q(5)$	2.263	2.213	0.773	1.148	0.738	1.646	0.500
$Q(10)$	3.166	10.798	2.519	1.832	2.635	8.038	2.881
<i>Autocorrelation in squared standardized residual</i>							
$Q^2(1)$	0.215	0.560	0.597	1.588	0.017	0.016	2.126
$Q^2(5)$	0.644	1.00	4.357	2.711	1.806	2.174	6.312
$Q^2(10)$	2.270	1.877	7.335	6.688	2.094	4.441	8.350

[\*, \*\* and \*\*\* indicate significance at 1%, 5% and 10% level of significance, respectively. MLLV is the maximum log likelihood value.]

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\*\*\*\* As stated in Franses and Dijk (2004), the standard interpretation of the t-ratio as a test of hypothesis  $\lambda = 0$  is invalid in STAR-type models because of a 'specific numerical problem', and hence the significance of  $\lambda$  is not indicated

<i>Estimates</i>	<i>Austria</i>	<i>Belgium</i>	<i>Finland</i>	<i>Greece</i>	<i>Luxembourg</i>	<i>Spain</i>
$\phi_0^1$	0.012	0.012	-0.035	-0.279	-0.019	0.003
$\phi_1^1$	0.144**	0.227*	-0.023	-0.358	0.021	0.378*
$\phi_2^1$	0.142*	0.062	0.140**	0.180***	0.093**	0.089
$\phi_3^1$	0.066	0.055	0.185*	0.196***	0.018	0.030
$\phi_4^1$	0.040	0.139*	0.154**	0.071	0.058	-0.008
$\phi_5^1$	0.050	0.021	0.144**	0.050	0.082**	0.076***
$\phi_6^1$	0.052	0.037	0.060	0.027	0.147*	0.062***
$\phi_7^1$	0.055	0.060	-0.027	0.061	0.049	0.077**
$\phi_8^1$	0.001	0.004	-0.006	0.263**	0.048	0.099*
$\phi_9^1$	-0.010	0.182*	0.086	0.201***	0.135*	0.044
$\phi_{10}^1$	0.047	0.053	-	-0.114	-0.037	0.112*
$\phi_{11}^1$	0.064	0.075***	-	-0.053	0.127*	-0.029
$\phi_{12}^1$	-0.185*	-0.126*	-	0.008	-0.140*	-0.244*
$\phi_{13}^1$	-	-	-	-	0.002	0.154*
$\phi_{14}^1$	-	-	-	-	-	0.080*
$\phi_{15}^1$	-	-	-	-	-	0.039
$\phi_0^2$	-0.116***	-0.090**	-0.001	0.268	0.011	-0.143
$\phi_1^2$	0.146	0.511*	0.176***	-0.095	-0.018	0.074
$\phi_2^2$	0.081	-0.112	0.289*	0.165	0.048	-0.129
$\phi_3^2$	0.038	0.060	0.129***	-0.045	0.006	0.374**
$\phi_4^2$	-0.118	0.177**	0.131**	0.147	0.010	-0.162
$\phi_5^2$	0.338*	0.239*	-0.018	0.063	0.072	0.216
$\phi_6^2$	0.231*	-0.128**	0.023	0.195***	0.004	0.423*
$\phi_7^2$	0.109	-0.134	-0.060	0.050	-0.006	0.464*
$\phi_8^2$	0.196**	0.073	-0.014	-0.109	0.005	-0.417*
$\phi_9^2$	-0.402*	0.145***	-0.037	-0.119	0.165*	-0.021
$\phi_{10}^2$	0.357*	-0.038	-	0.155	0.100**	0.143
$\phi_{11}^2$	-0.010	0.065	-	0.145	0.083***	-0.350**
$\phi_{12}^2$	0.260**	-0.095	-	-0.192***	0.093	0.190
$\phi_{13}^2$	-	-	-	-	-0.120***	-0.255***
$\phi_{14}^2$	-	-	-	-	-	-0.002
$\phi_{15}^2$	-	-	-	-	-	0.152
$\omega_1$	0.010*	0.036*	0.010	0.000	0.027*	0.011*
$\alpha_1$	0.630*	0.584*	0.283**	0.046	1.327*	0.841*

$\beta_1$	0.472*	0.000	0.786*	0.898*	0.043	0.409*
$\omega_2$	0.000	0.087*	0.000	0.005	0.054*	0.000
$\alpha_2$	0.245*	0.000	0.000	0.000	0.000	0.059
$\beta_2$	0.561**	0.000	0.654*	0.884*	0.000	0.000
$\theta_1$	0.030	0.067**	0.030	-0.064***	0.039	0.025**
$\theta_2$	0.027	0.007	0.149*	0.115**	0.124**	0.311*
$\lambda$	$8.77 \times 10^5$	$2.24 \times 10^{17}$	$1.07 \times 10^6$	3.34	$5.02 \times 10^{19}$	29.61
$c$	0.255*	0.098	0.012***	0.000	0.055	0.541*
<i>MLLV</i>	-15.30	21.62	-180.77	-395.14	-45.25	-139.51
<i>Autocorrelation in standardized residual</i>						
$Q(1)$	1.062	0.332	1.811	5.759	0.157	0.359
$Q(5)$	2.163	1.944	7.834	6.757	4.457	2.517
$Q(10)$	3.690	6.556	10.544	9.977	8.574	4.089
<i>Autocorrelation in squared standardized residual</i>						
$Q^2(1)$	0.774	1.023	0.580	4.904	0.193	0.348
$Q^2(5)$	5.380	1.345	2.169	6.679	1.602	2.594
$Q^2(10)$	12.321	6.901	15.701	8.913	15.982	6.731

[\*, \*\* and \*\*\* indicate significance at 1%, 5% and 10% level of significance, respectively. *MLLV* is the maximum log likelihood value.]

It is observed from this table that the lag order in the conditional mean equation is quite high in both the regimes and for most of the countries. For instance, in case of Canada these lag values are 12 and 9, for the first and second regime, respectively. It is also evident from these coefficient estimates that the same lag values in both the regimes are not statistically significant in most countries. As for example, in case of Canada, there is no such lag at all, while for the U.S.A., the first and twelfth lags are significant in both regimes of inflation although the estimates are of different signs for first lag (0.183 and -0.471) but of same sign for the twelfth lag (-0.251 and -0.247). These findings show the usefulness and efficacy of consideration of two regimes for modelling inflation where the regimes are characterised by the preceding inflation value i.e.,  $\pi_{t-1}$  exceeding or not exceeding a certain threshold level of inflation,  $c$ , which is determined endogenously.

As regards the smoothness parameter,  $\lambda$ , it is found that its estimate is very large for most of the countries indicating that the transition from one regime of inflation to the other is not smooth at all; it is rather sharp and almost instantaneous at  $\pi_{t-1} = c$ . It is interesting as well as important to note that in 8 out of 13 countries considered in this study, the estimates of threshold value of inflation have been found to be significant. These countries are Germany, Italy, Japan, the U.K., the U.S.A., Austria, Finland and Spain. Further the estimates of this threshold inflation for these countries are quite different, For instance,  $\hat{c}$  is 0.533 for Italy while it is 0.056 for Japan. While this difference is only likely since the prevailing level of inflation and its management and control for any country depends, inter alia, on the size, strength and stability of the economy.

Our modelling approach of consideration of regime for both the conditional mean and variance, where regime is defined based on a threshold value of inflation is thus found to be very useful. Even for the countries  $c$  is not statistically significant i.e., for Canada, France, Belgium, Greece and Luxembourg, this finding namely that  $c = 0$  is important since it means that the relationship between inflation and inflation uncertainty is determined based on whether inflation in the preceding prime point i.e.,  $\pi_{t-1}$  is positive or not positive.

Now looking at the parameters in the conditional variance in the two inflation regimes, we note that either or both the coefficients of  $\varepsilon_{t-1}^2$  in the two regimes i.e., either or both  $\alpha_1$  and  $\alpha_2$ , are significant. Additionally,  $\beta_1$  and/or  $\beta_2$  are also so. It is thus found that consideration of regimes by a threshold level of inflation for conditional variance, i.e., inflation uncertainty in this study is also relevant and statistically meaningful. Regime-specific behaviour of inflation is found to be prevalent in all the 13 countries. The coefficients capturing the link between inflation uncertainty and inflation in the two regimes i.e.,  $\theta_1$  and  $\theta_2$  are very important in the proposed

model. We find from the estimated values of these two parameters, that both are significant in 3 countries only, namely, in France, Greece and Spain. Of the remaining 10, at least one of  $\theta_1$  and  $\theta_2$  is significant in 8 countries. It is only in Italy and Austria that both  $\theta_1$  and  $\theta_2$  are insignificant suggesting that inflation does not affect inflation uncertainty. In the first regime,  $\theta_1$  is significant and positive for Belgium and Spain only, the values being 0.067 and 0.025, respectively. This indicates that at the first regime where inflation is low in the sense of being below the threshold, inflation reduces inflation uncertainty. The conclusion based on  $\hat{\theta}_1$  for France, Japan, the U.K., the U.S.A. and Greece is, however, somewhat unusual although not exceptional. For these 5 countries,  $\hat{\theta}_1$  is negative and this parameter is significant too. This means that lower inflation increases inflation uncertainty, and it is concluded that Friedman-Ball hypothesis fails to hold for these countries in the low inflation period. Such evidence has been found by a few other studies as well. For instance, based on a study on 12 European Monetary Union countries, Caporale and Kontonikas (2009) have found that during the Euro period when average inflation is low, a further reduction in inflation tends to increase rather than decrease uncertainty in inflation of a number of countries in Euro Zone. In a recent study based on monthly data on US inflation over the period 1926 to 1992, Hwang (2001) has found that inflation affects its uncertainty weakly negatively during the period of both high and low inflation. Chen et al. (2008) has observed a U-shaped pattern in some Asian economies, strongly implying that a high rate of inflation or deflation results in high inflation uncertainty in both high and low inflationary periods.

In the second regime where inflation is above the threshold value, we find that a positive and significant relationship exists between inflation and inflation uncertainty for countries like Canada, Germany, Finland, Greece, Luxembourg and Spain. Thus we can conclude that the Friedman-Ball hypothesis holds for these countries in the second regime. On the other hand for

France,  $\hat{\theta}_2$  is negative (-0.019) and significant and this suggests that an increase in inflation reduces inflation uncertainty in the high inflation regime and thus supporting the view of Pourgerami and Maskus (1987). Combing all these findings on the relationship between inflation and inflation uncertainty across low and high inflation periods, and consequently empirical validity or otherwise of Friedman-Ball hypothesis, we may state that our findings, on the whole, suggest that the relationship between inflation and inflation uncertainty depends on a threshold level of inflation and it is also regime specific.

Finally we report on the Ljung-Box test statistic values based on residuals of this model for all the 13 countries. This test statistic has been computed for both the standardized and squared standardized residuals. It is evident from the relevant entries that that none of them are significant, and hence it can be concluded that the estimates with their choice of lag values in both conditional mean and conditional variance specification for all the 13 countries are adequate.

Lastly, we make a comparison between the benchmark model i.e., AR( $k$ )-GARCH(1,1)-L(1) model and the proposed STAR( $k$ )-ANSTGARCH(1,1)L(1) by means of likelihood ratio (LR) test to find if introduction of regimes based on low and high inflation in both conditional mean and conditional variance has led to any statistical gain in understanding the relationship involving inflation and inflation uncertainty<sup>††††</sup>. The likelihood ratio test is as follows:

$$LR = -2(L(H_0) - L(H_1)) \sim \chi_{m-p}^2 \text{ under } H_0.$$

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<sup>††††</sup> Although all the maximized log likelihood values for the proposed model are uniformly higher than the ones for the AR( $k$ )TGARCH(1,1)L(1) model, no formal test could be carried out since the two models are not nested.

where  $L(H_0)$  and  $L(H_1)$  stand for the maximized log likelihood values under the restricted and unrestricted hypotheses, respectively. The test statistic follows  $\chi^2$  distribution with  $m - p$  degrees of freedom, where  $m$  is the number of parameters of the unrestricted model and  $p$  is the number of parameters of the restricted model. In Table 7 we report the LR test statistic values.

Table 7: LR test statistic values

<i>Country</i>	Model I Vs Model II
<i>Canada</i>	31.32**
<i>France</i>	47.3*
<i>Germany</i>	45.4*
<i>Italy</i>	38.9*
<i>Japan</i>	20.52
<i>The U.K.</i>	58.34*
<i>The U.S.A.</i>	48.98*
<i>Austria</i>	51.14*
<i>Belgium</i>	25.56
<i>Finland</i>	15.24
<i>Greece</i>	24.98
<i>Luxembourg</i>	29.64***
<i>Spain</i>	85.54*

[ Model I:  $AR(k)$ -GARCH(1,1)L(1) and Model II: STAR(k)-ANSTGARCH(1,1)L(1). \*, \*\* and \*\*\* indicate the significance at 1%, 5% and 10% level of significance, respectively.]

We find from Table 7 that the LR test statistics values are highly significant for 9 out of 13 countries and hence we conclude that the proposed model on inflation and inflation uncertainty are statistically ‘better’ than the benchmark model for these countries.

## 5. Concluding Remarks

In the literature on the relationship between inflation and inflation uncertainty, the Friedman-Ball hypothesis is an important hypothesis and its empirical validity has been found to

be mixed. The paper has revisited this hypothesis by proposing a new model where the conditional mean as well as the conditional variance for inflation are based on consideration of two regimes of inflation – below and above a certain level of inflation- and which is determined endogenously.

The time series data on consumer price indices from some countries belonging to Euro Zone and G7 have been used in this study. The test for stationarity of this series has been carried out by the newly developed tests by Perron and Yabu (2009) and Kim and Perron (2009), where the possibility of existence of structural break in the trend function of a series are also duly considered. All the series were found to be stationary with a break in their respective trend functions.

The empirical results of application of the proposed model to the inflation data for 13 countries are summarised as follows.

- (i) There exists a positive threshold level of inflation for 8 out of 13 countries considered. This means that the control of inflation would crucially depend on this threshold level of inflation which is estimated endogenously.
- (ii) There are two regimes of inflation characterising the conditional mean of inflation for most of the countries.
- (iii) The parameter, capturing the effect of inflation on inflation uncertainty in the specification of conditional variance clearly shows that this coefficient is different in the two inflation regimes.

The improvement in terms of maximum log likelihood values for the proposed model is significant as compared to one of the existing models like the  $AR(k)$ -GARCH(1,1)L(1)

model. By likelihood ratio test, it is found that in countries like Canada, France, Germany, Italy, the U.K., the U.S.A., Austria, Luxembourg and Spain the proposed model performs better.

Finally, we summarize the findings on the empirical validity of the Friedman-Ball hypothesis based on  $AR(k)$ -GARCH(1,1)L(1),  $AR(k)$ -TGARCH(1,1)L(1) and the proposed model. From the results on the  $AR(k)$ -GARCH(1,1)L(1) model, it is found that the Friedman-Ball hypothesis holds for three countries only *viz.*, Germany, Belgium and Spain, while the number of countries satisfying this hypothesis increases to six - Canada, Germany, the U.K., the U.S.A., Luxembourg and Spain- if asymmetry in inflation uncertainty is captured through the TGARCH specification.

In case of our model where regimes based on a threshold value of inflation has been introduced and all parameters of the models for conditional mean and conditional variance have been allowed to vary across the two regimes, the data generating process of inflation is expected to be more appropriate, and this is indeed borne out by the findings on the model. It is observed from the results that the Friedman-Ball hypothesis holds for five countries *viz.*, Canada, Germany, Finland, Greece and Luxembourg in the high inflation regime, one country (Belgium) in the low inflation regime and one country (Spain) in both the regimes. For France, the relationship is negative in the high inflation regime, suggesting that high inflation reduces inflation uncertainty and thus it supports the view of Pourgerami and Maskus (1987). Further, the relationship is negative in the low inflation regime for five countries *viz.*, France, Japan, the U.K., the U.S.A. and Greece, which suggests that in case of deflation, deflation uncertainty also increases, and this goes counter to the Friedman-Ball hypothesis.

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