A Unified Characterization of Completeness and Triviality for Secure Function Evaluation

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A Unified Characterization of Completeness and Triviality for semi-honest Secure Function Evaluation

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Overview

Secure Function Evaluation

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  - They want to evaluate a function f
    - In general, f provides $f_A(x,y;R)$ to Alice and $f_B(x,y;R)$ to Bob
    - Special cases: $f_A = f_B$; $f_A = \text{const.}$; $|R| = 1$

Security Models

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- Active Adversaries
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  - For example: Standalone and UC

Hybrids

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- Reduction: If `f` can be securely implemented in g-hybrid then "`f` reduces to g"
  - Note: Needs an associated notion of security

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- An SFE `g` is complete if every `f` reduces to it
  - That is: Any `f` has secure protocol in `g`-hybrid
  - Example: Oblivious Transfer

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- Symmetric Randomized Functions: [KilianOO]

Our Aim for this Talk

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- Provide a characterization of complete (2-party) secure function evaluation

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- In this talk: We shall restrict to semi-honest security

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  - What is "`graph(g)`"?
  - What is "product graph"?
  - Rest of the talk: Understanding this result

Technical Content

Representation of SFE

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- A 2-party SFE is represented using a matrix
  - The (x,y)-th entry of the matrix contains the joint output distribution over Alice-Bob outputs corresponding to Alice input x and Bob input y

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  - Similarly, for Bob input `y` and his output `z`, the tuple `(y,z)` is "Bob View"
- Looking ahead: It will be interesting to consider joint views of parties

Example: OR

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	0	1
0	0	1
1	1	1

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	0	1
0	0	1
1	1	1

(0,0)

Example: OR

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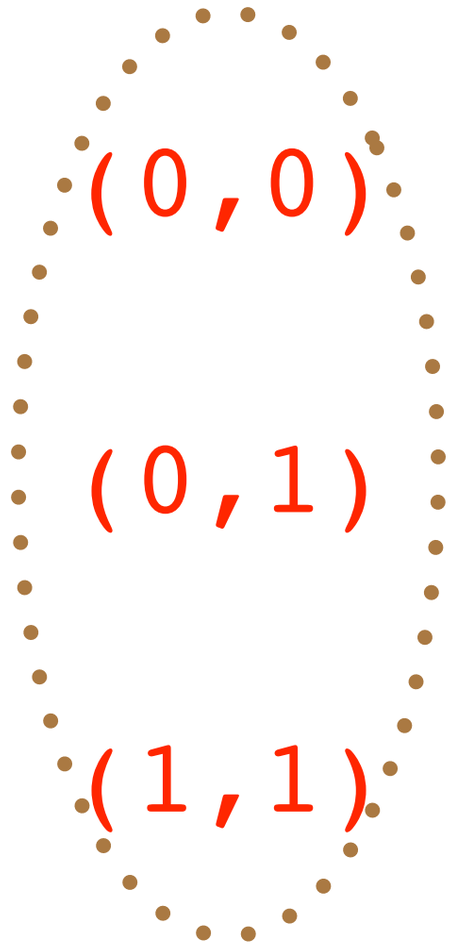

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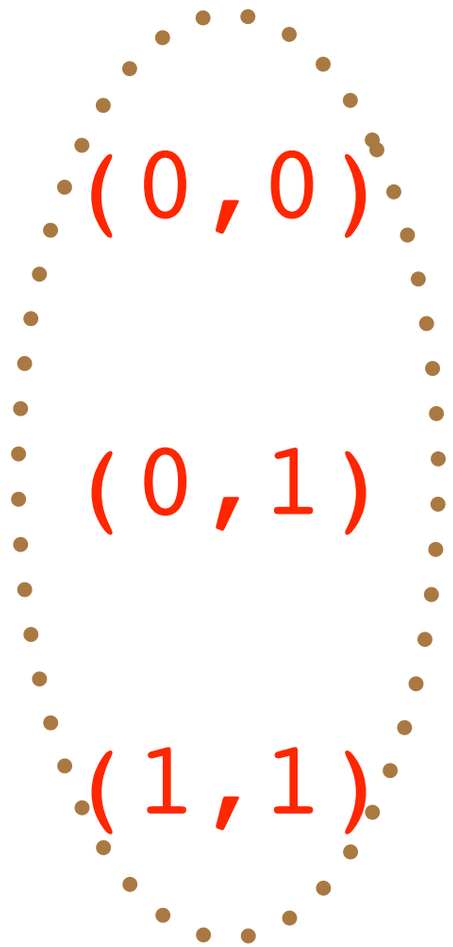

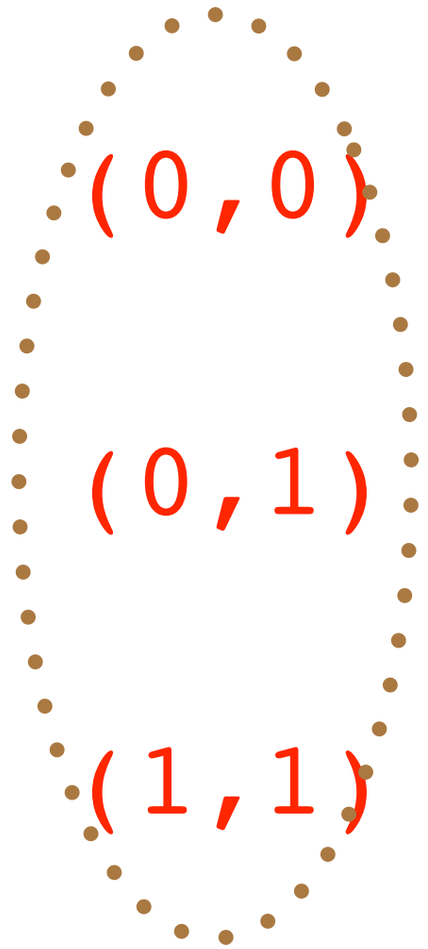

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```
(0,0)        (0,0)

(0,1)        (0,1)

(1,1)        (1,1)
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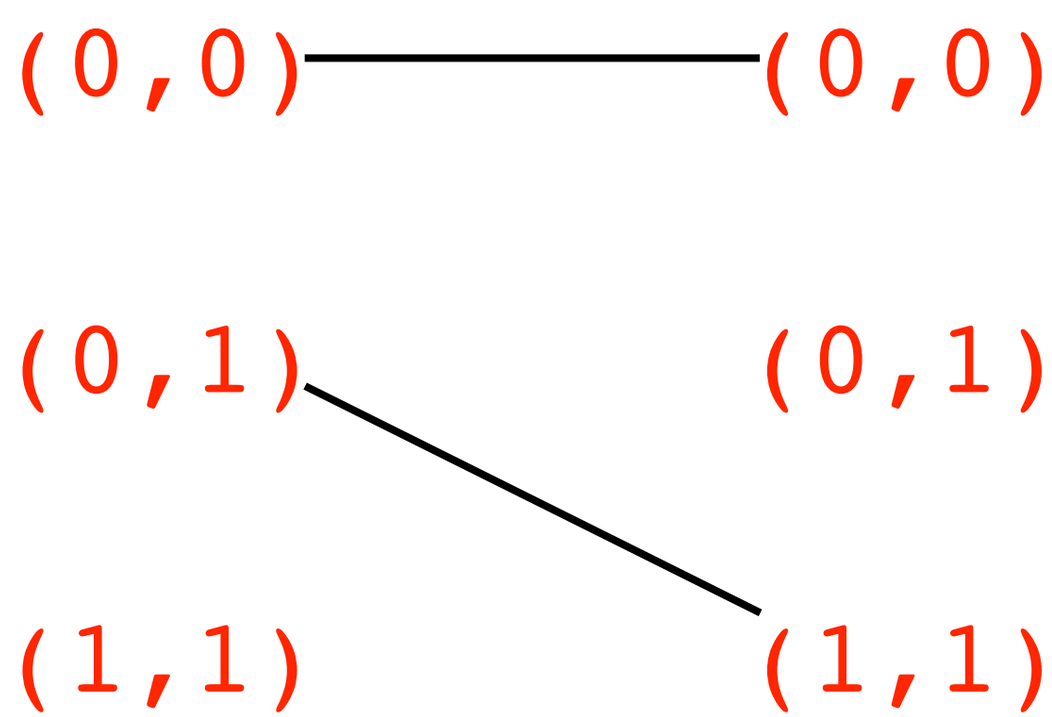

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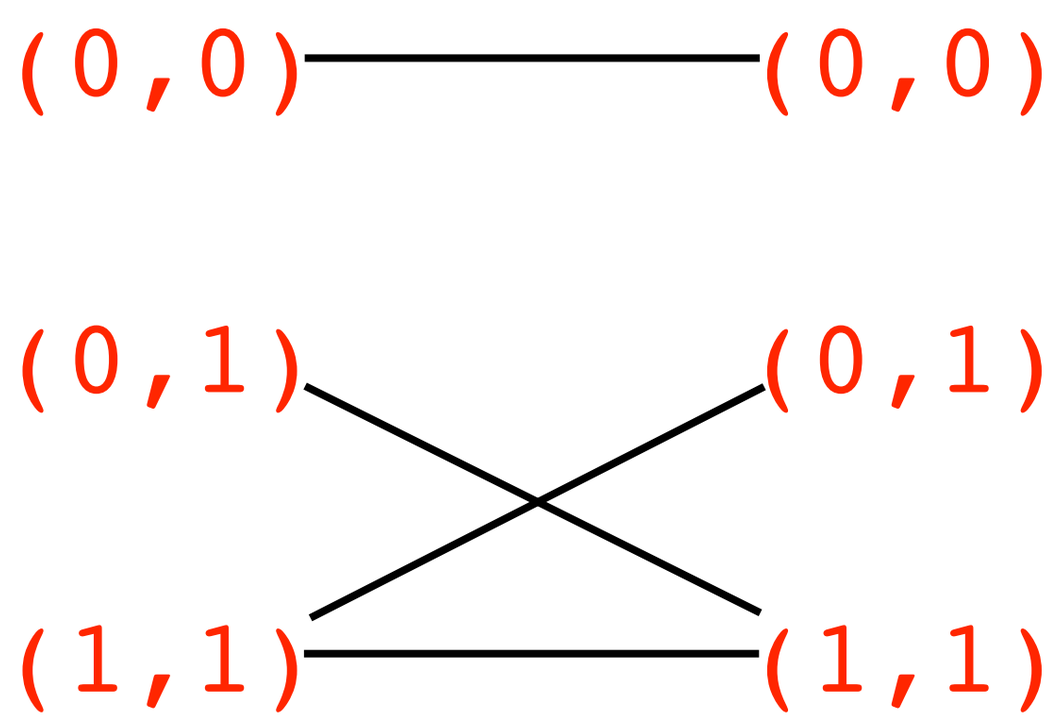

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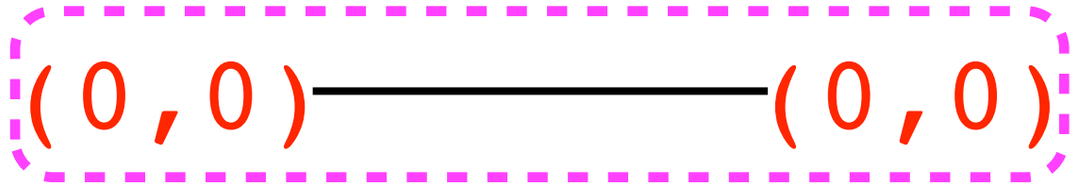

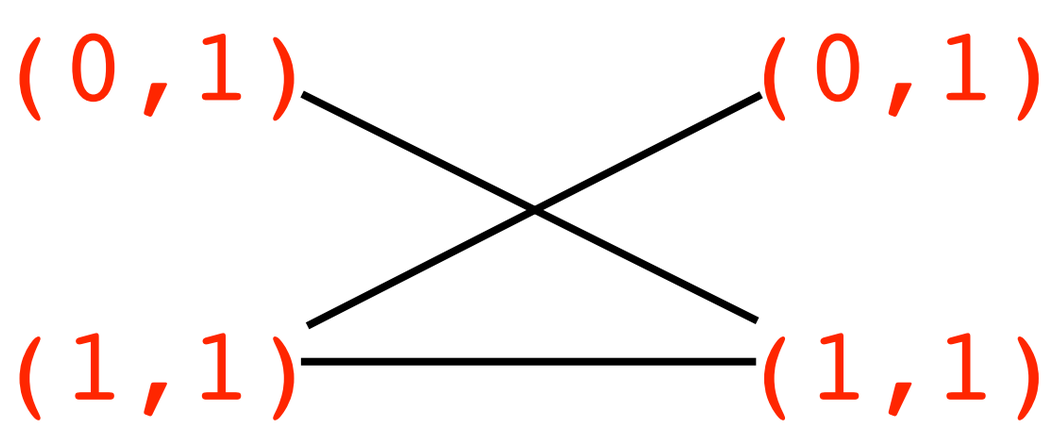

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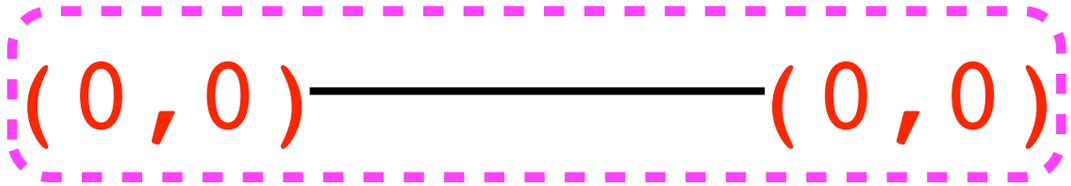

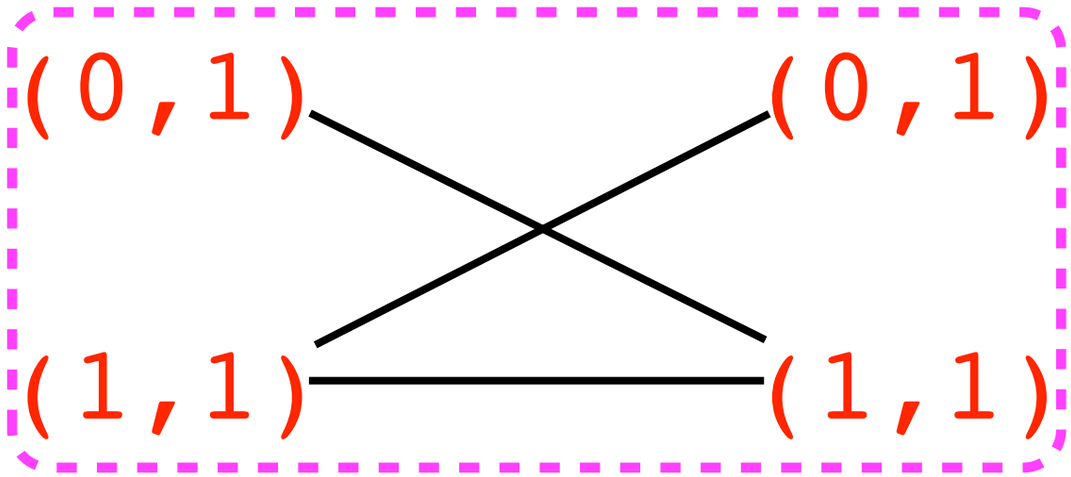

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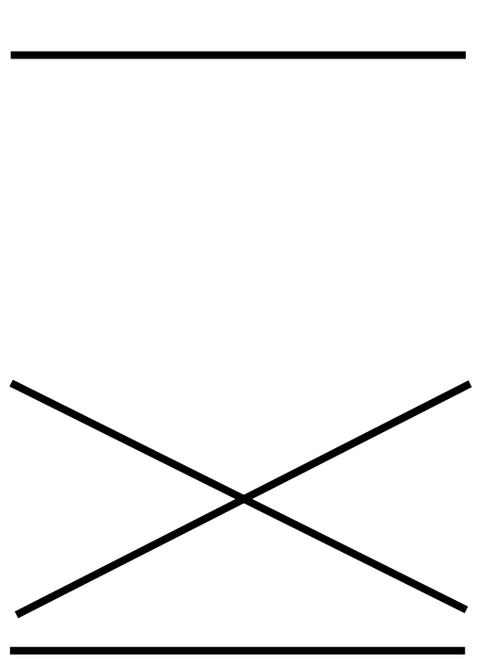

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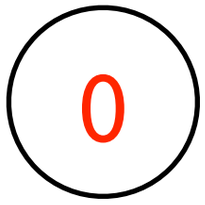

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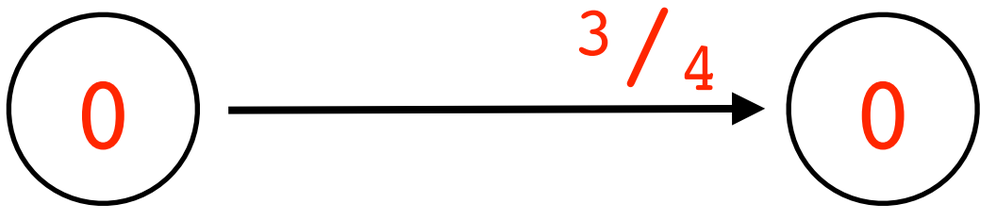

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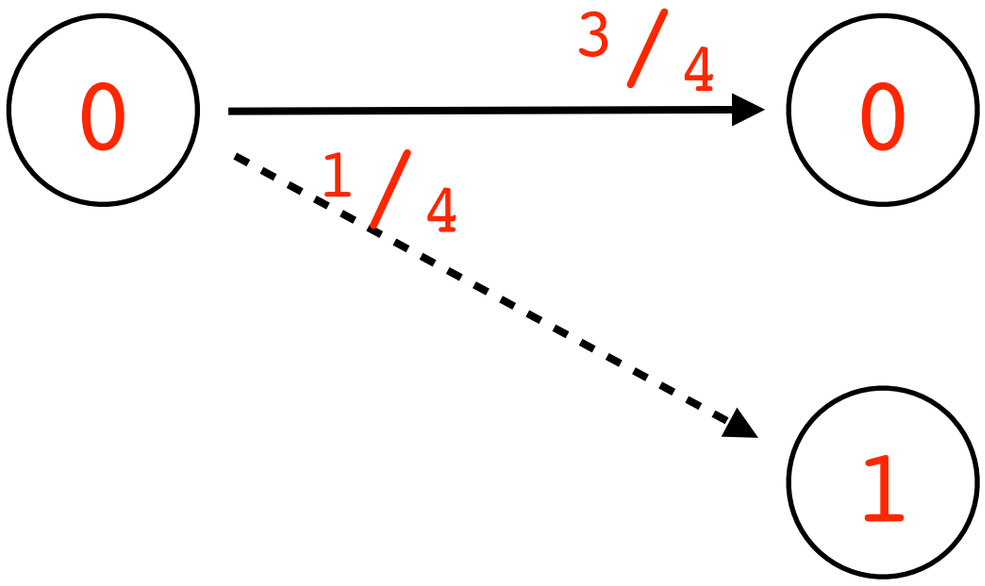

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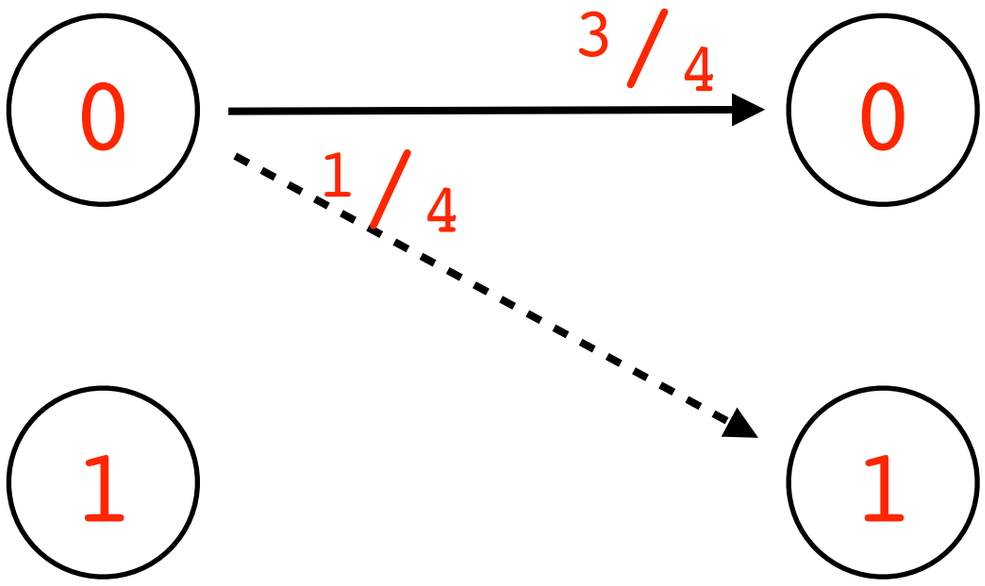

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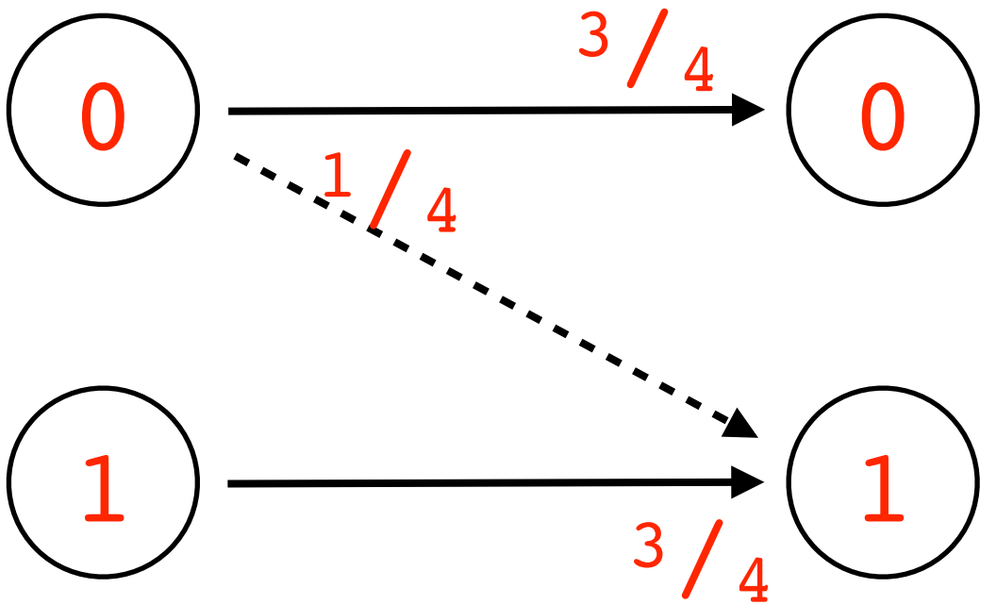

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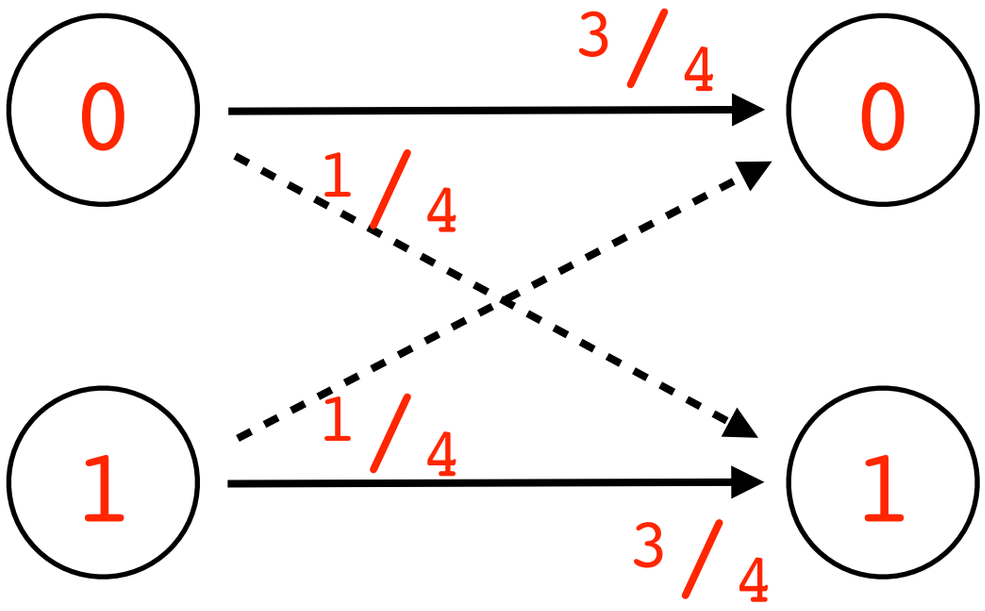

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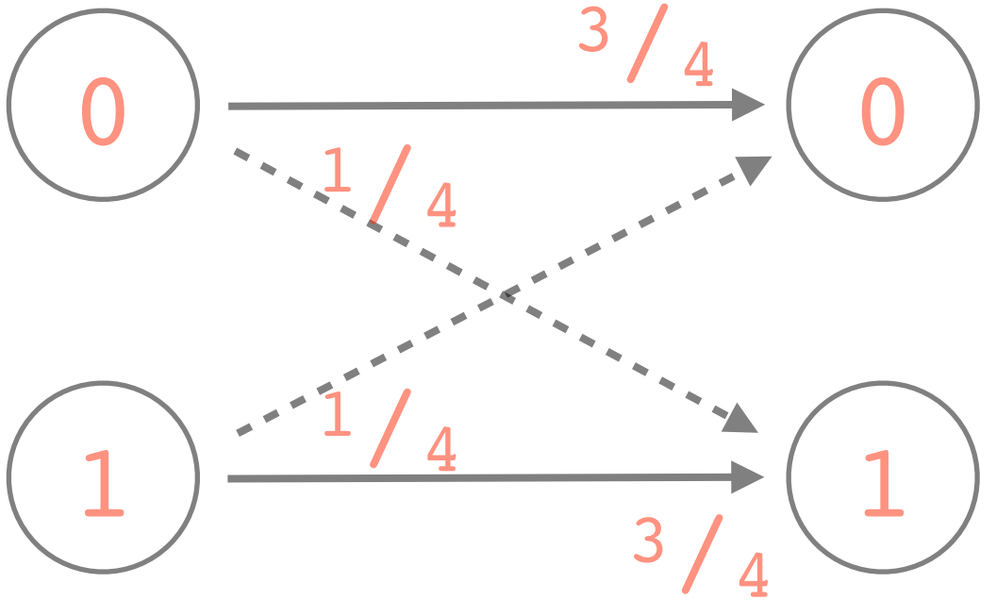

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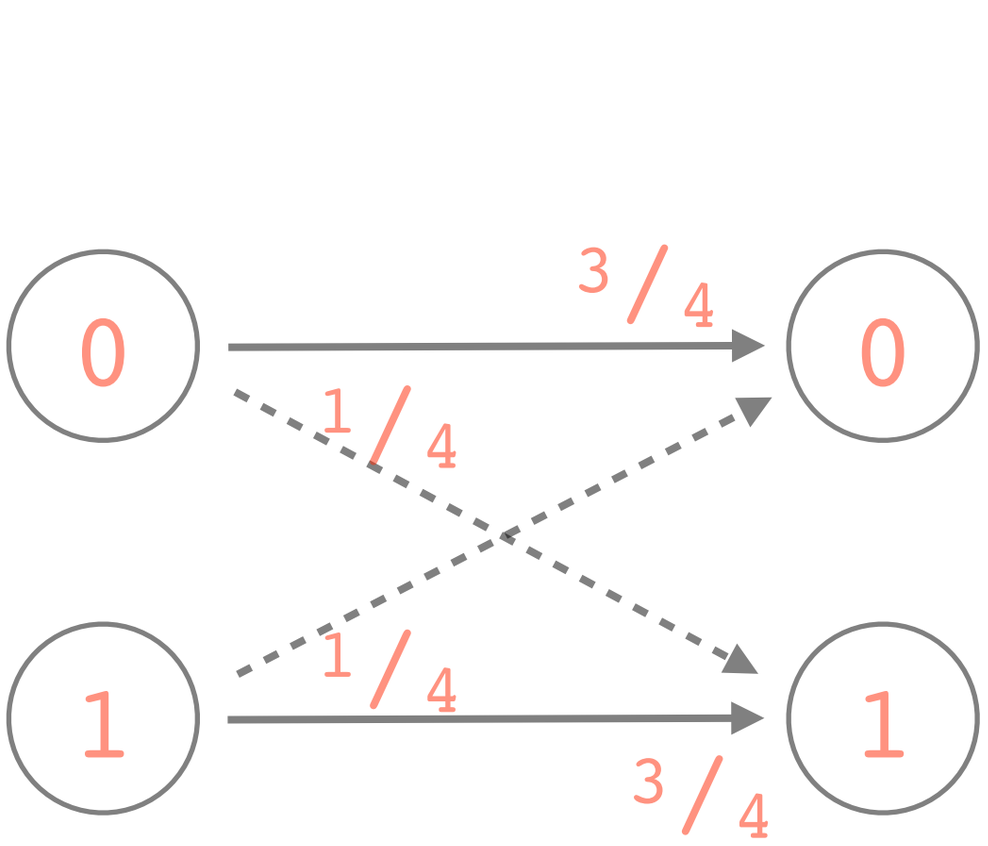

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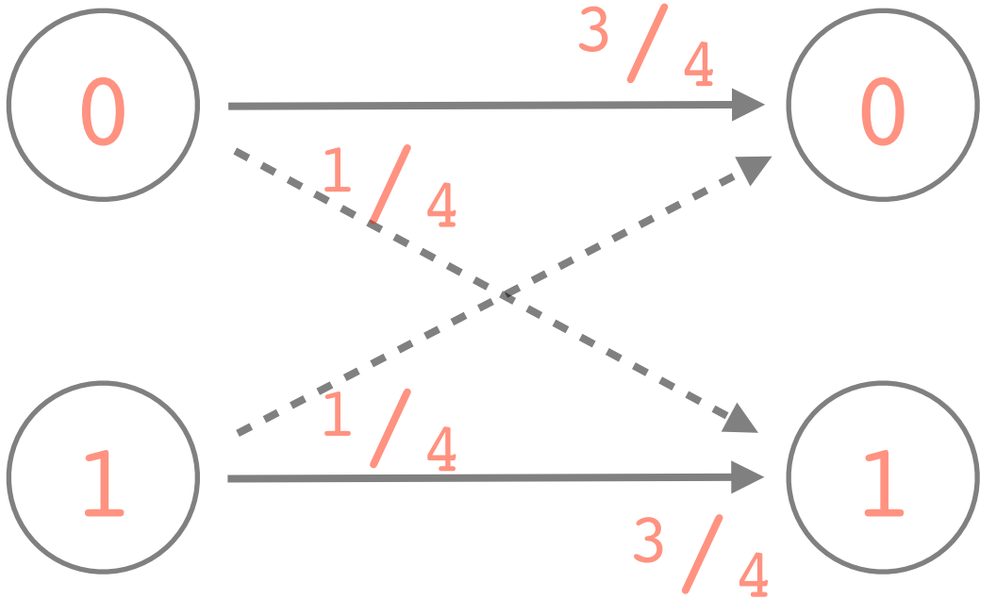

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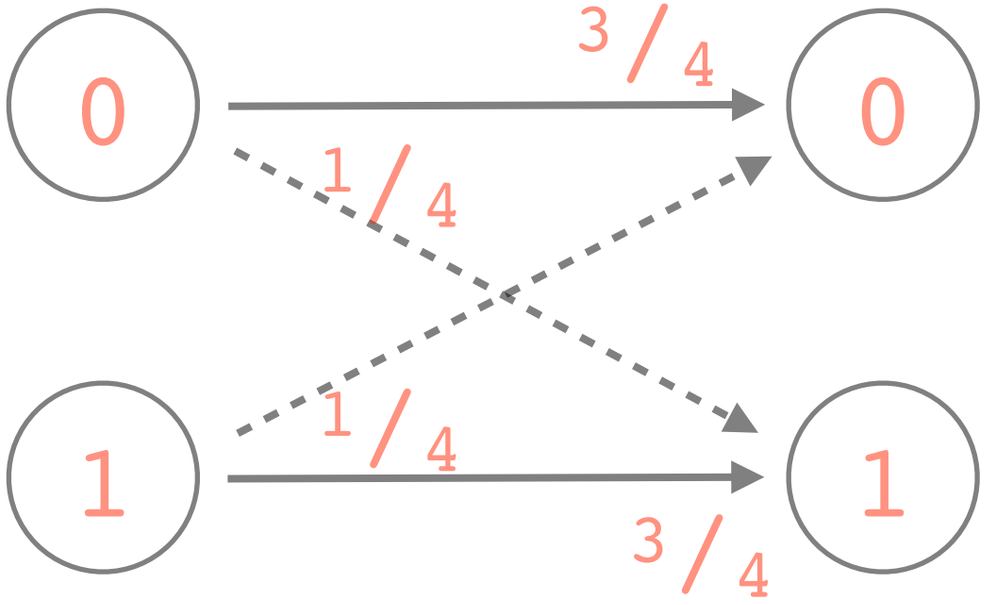

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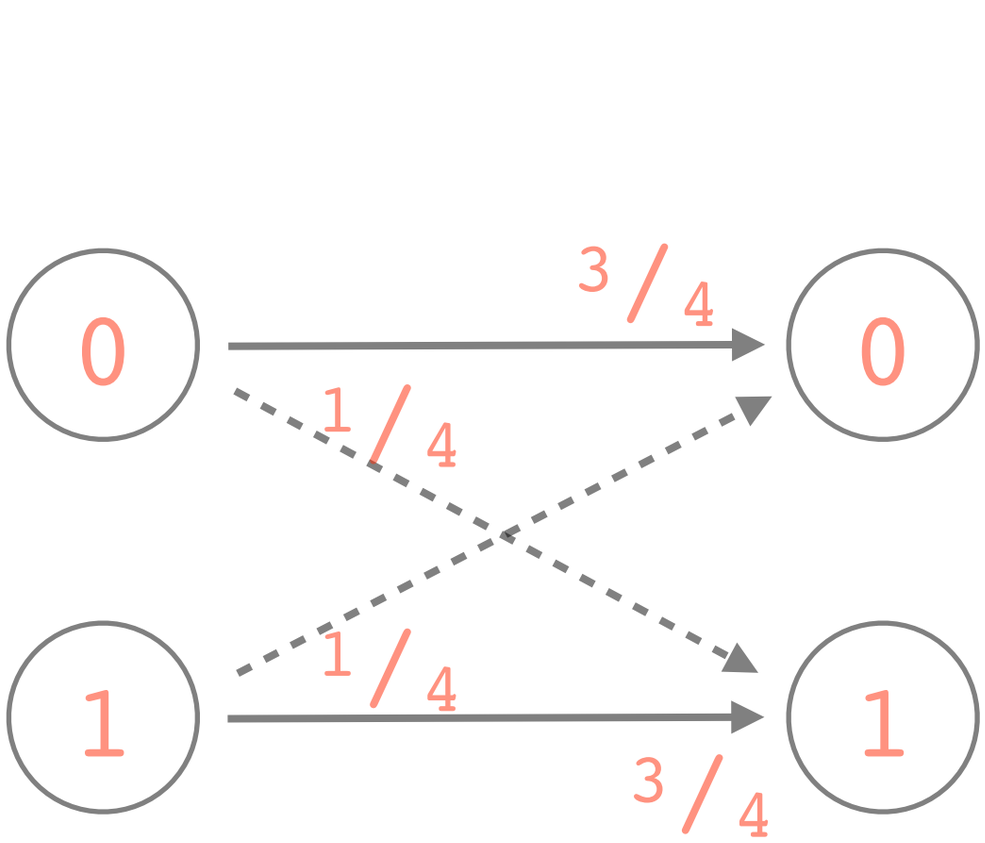

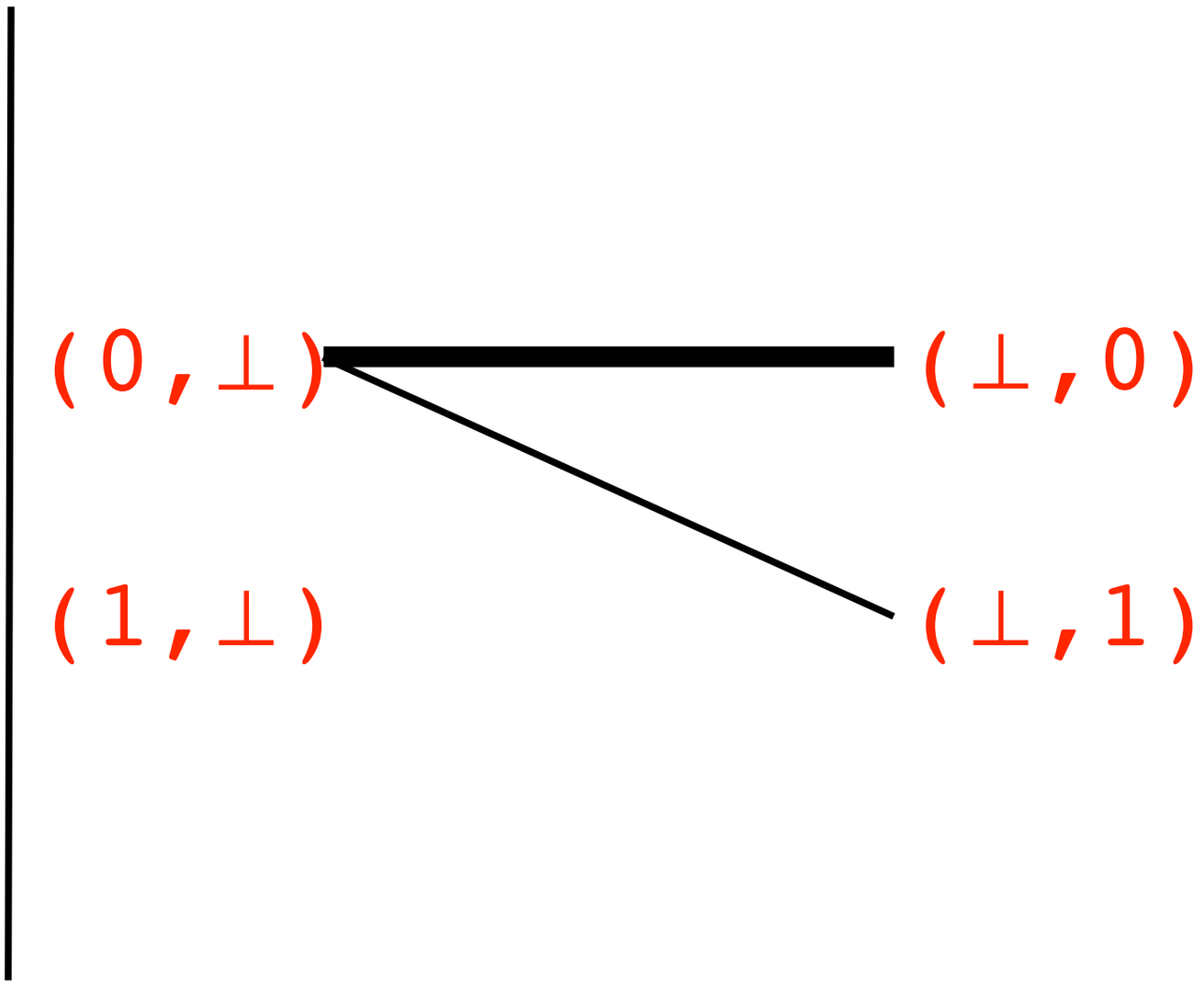

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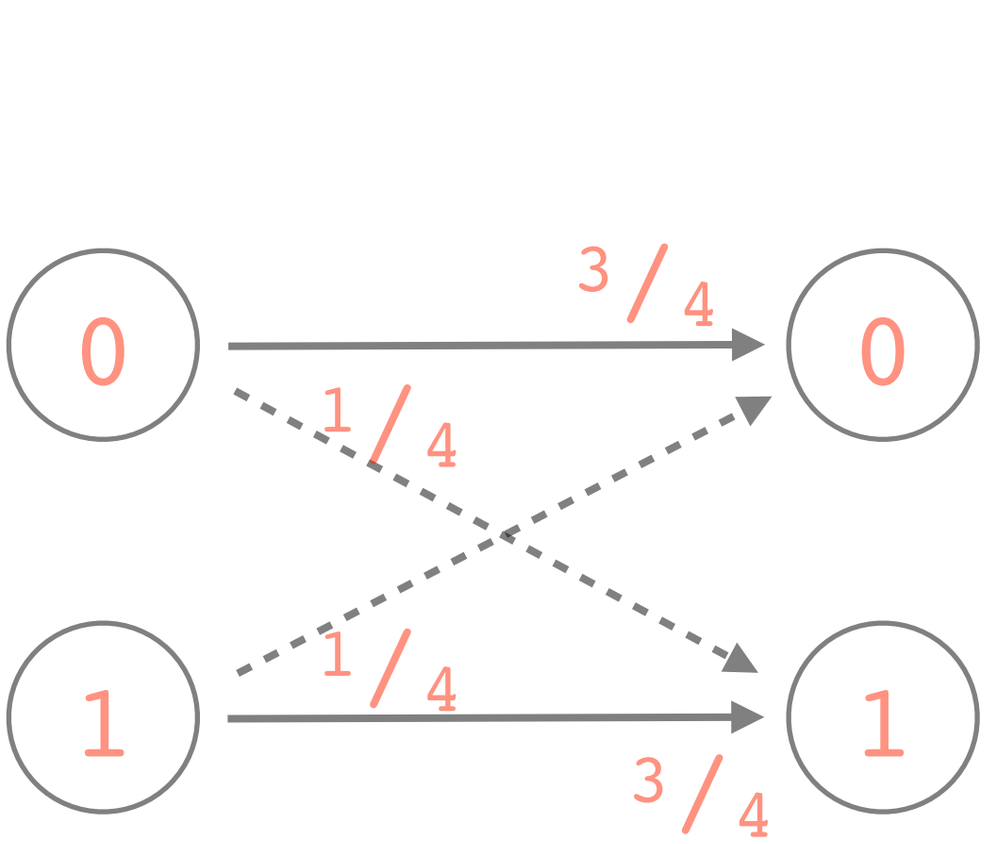

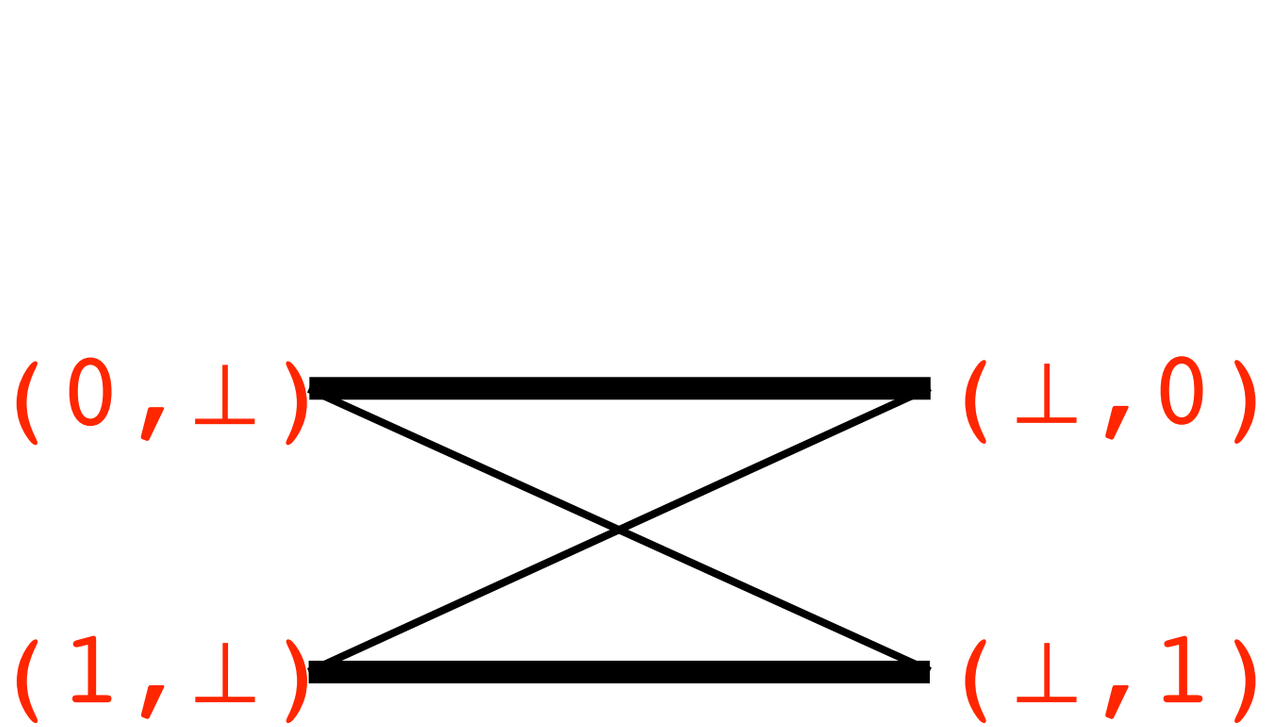

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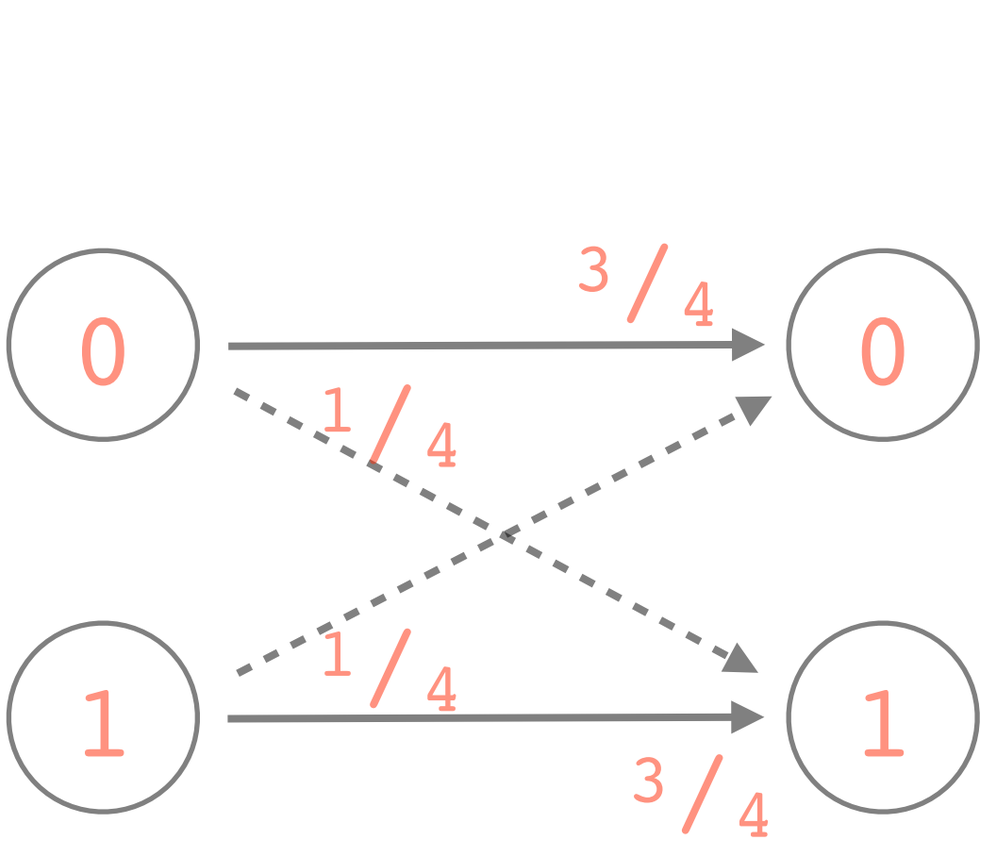

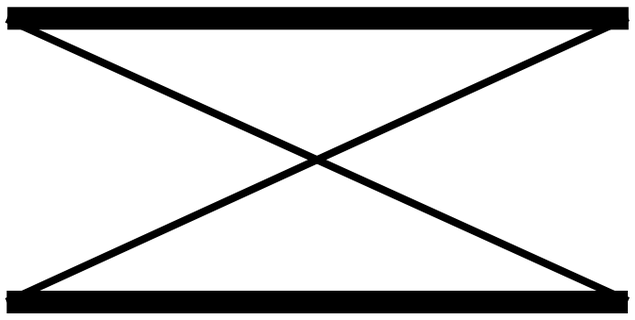

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- The edge connecting `(x,w)` to `(y,z)` has weight `Pr[ w,z | x,y]`

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  - Note: This is defined using uniform distribution over Bob's inputs

Product Graph Examples

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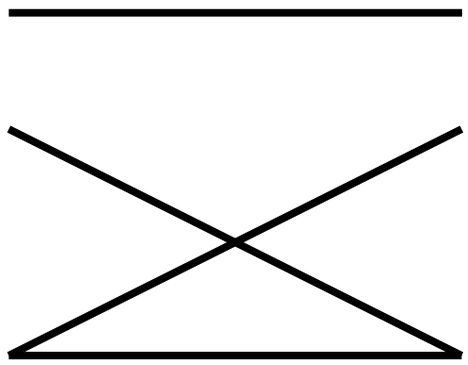
Symmetric OR

Product Graph Examples



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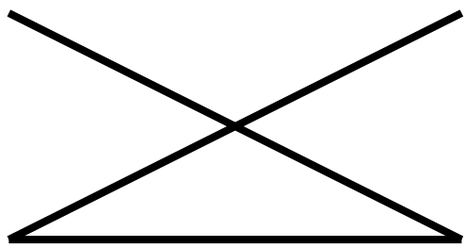

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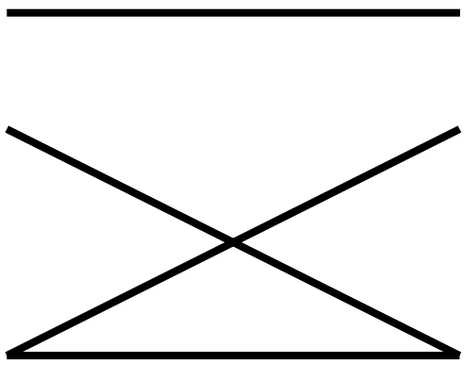

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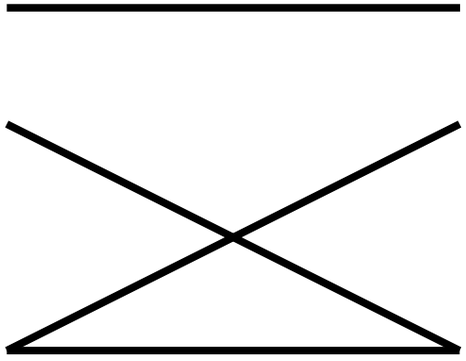

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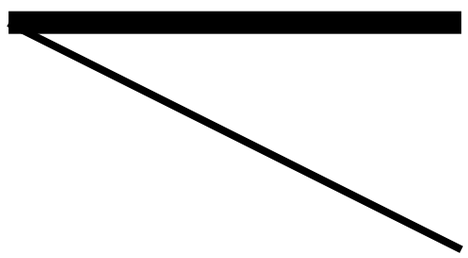

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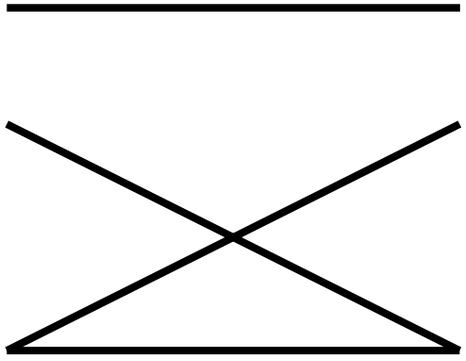

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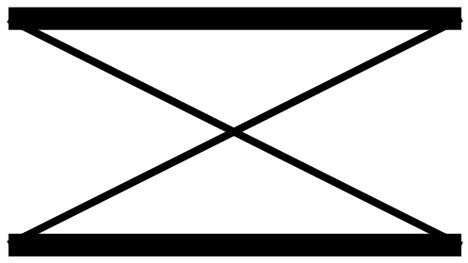

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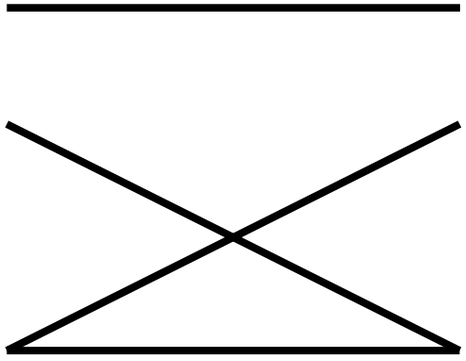

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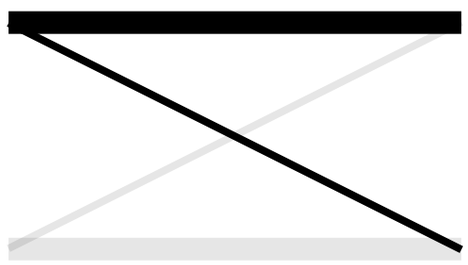

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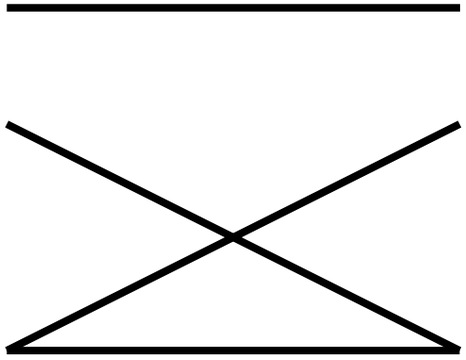
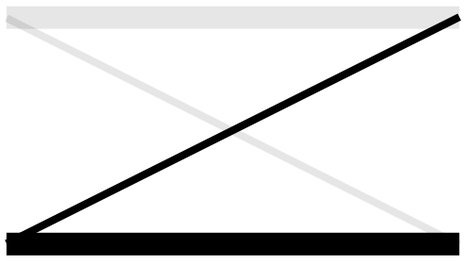
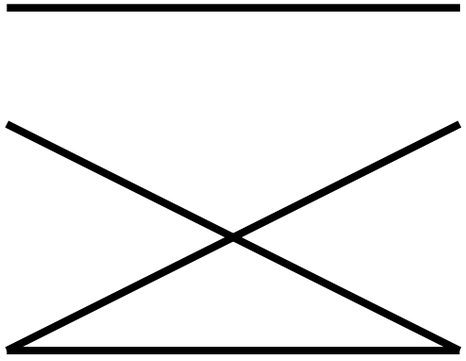


Symmetric OR



BSC

Product Graph Examples



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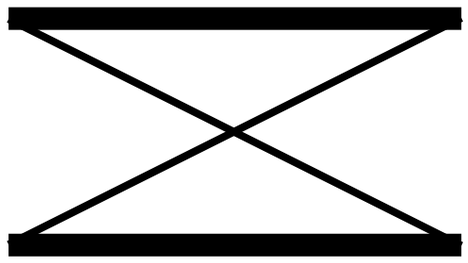

BSC

Inattentive channel

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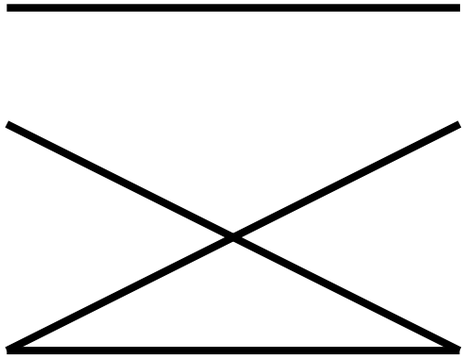

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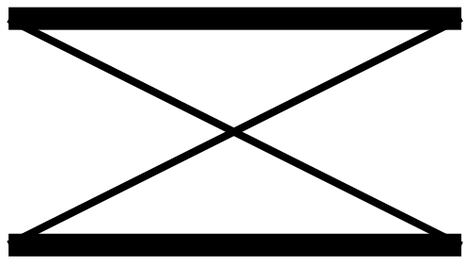

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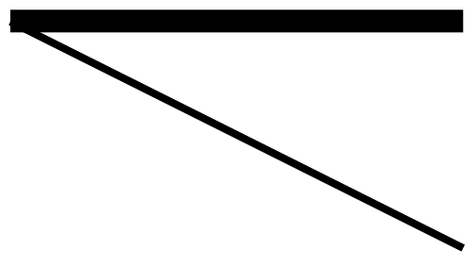

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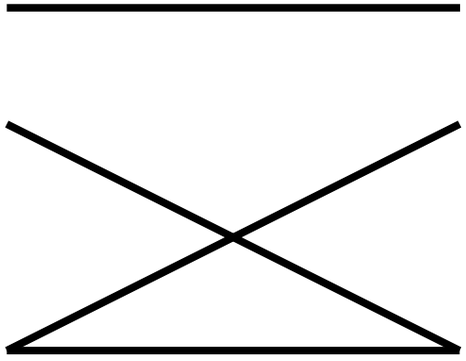

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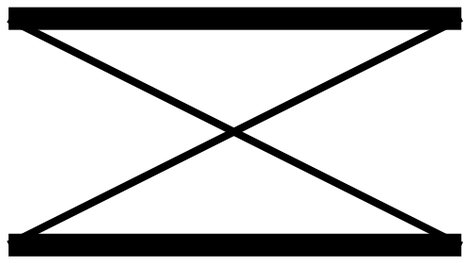

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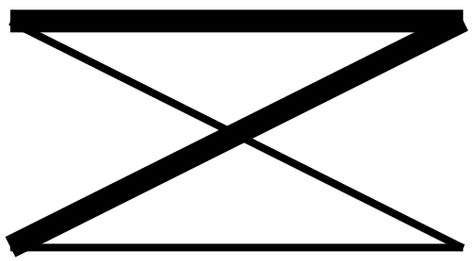

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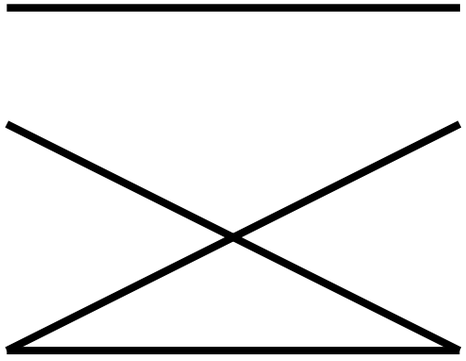

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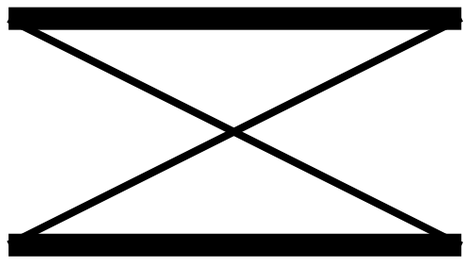

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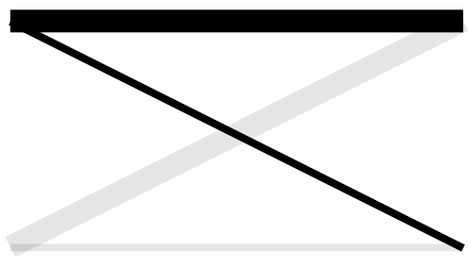

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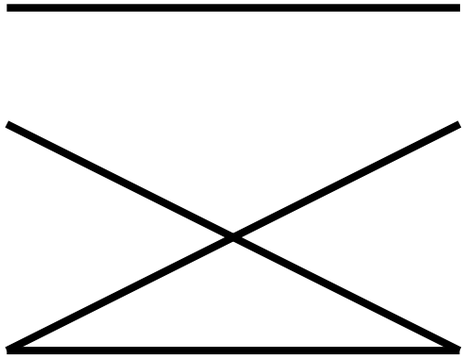

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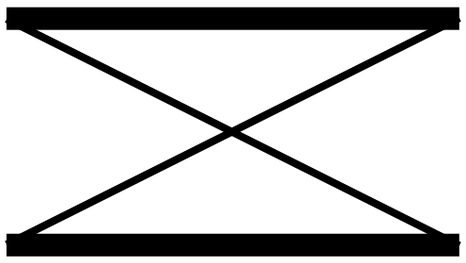

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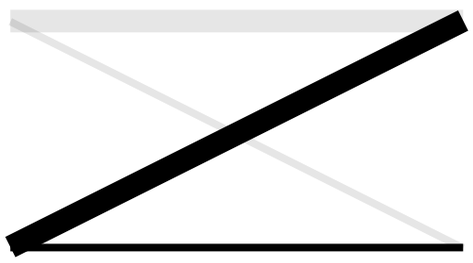

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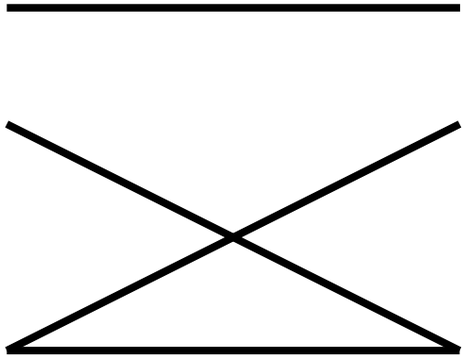

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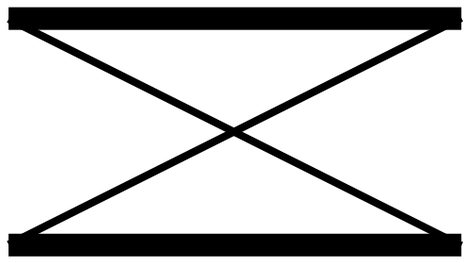

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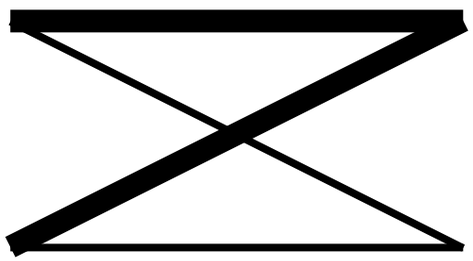

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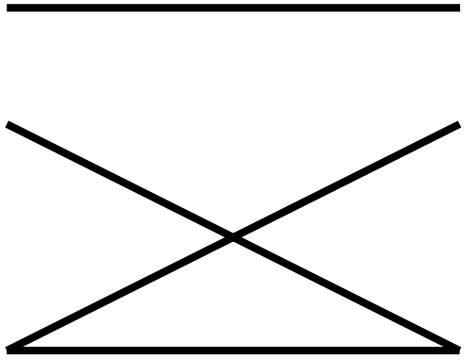

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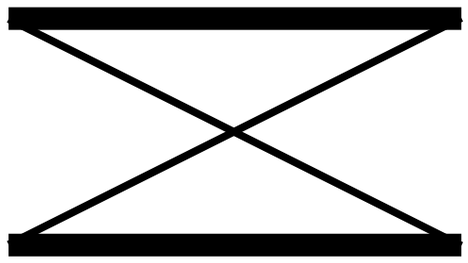

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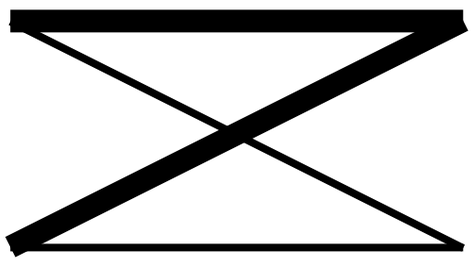

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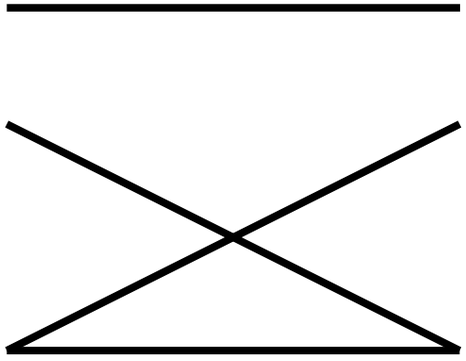

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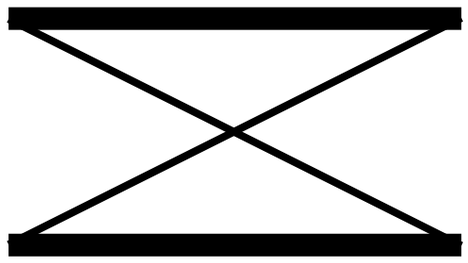

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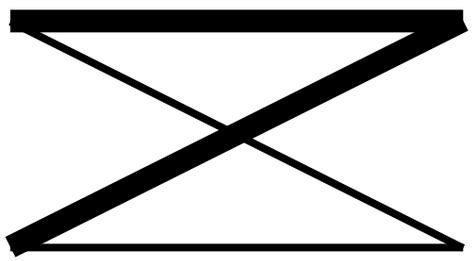

Inattentive channel

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  - This result resolves several open problems in semi-honest completeness characterization!

Malicious Case: Intro

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  - Recently proven by [Maji-Prabhakaran-12]

Open Problems

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Thanks