Singular control with state constraints

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We study a stochastic control problem of the singular type with an infinite horizon discounted cost criterion. For a control to be admissible the corresponding controlled process must satisfy state constraints. We are concerned with the solvability and well-posedness of the associated HJB equation. It is well understood that an appropriate framework for second order degenerate elliptic equations, of which HJB equation for a singular control problem is a special case, is through the theory of viscosity solutions. However, typical comparison results in this theory which are used to argue uniqueness of solutions rely on a key coercivity property which is usually unavailable for PDEs corresponding to singular control problems. In this work we identify key conditions for uniqueness in two different settings. In the first, state space is a convex closed cone, the cost is allowed to be unbounded but required to be nonnegative. This framework covers a wide range of diffusion control problems that arise from queueing networks in heavy traffic. In the second setting state space is a closed, convex, bounded set and the nonnegativity requirement on the cost is relaxed. We identify a necessary and sufficient condition for uniqueness which is closely related to the "no arbitrage" condition of Harrison and Williams. This setup covers BCPs associated with a broad family of stochastic processing networks with finite buffers.

The first work is jointly with Rami Atar and the second is joint work with Rami Atar and Ruth Williams.

`List of invited speakers` `Schedule for December 11`