Ergodic theory, Abelian groups, and point processes associated with stable random fields

Parthanil Roy

ETH, Zurich

We consider the point process sequence $\left\{ \sum_{\|t\|_\infty \le n} \delta_{b_n^{-1} X_t} : n \ge 1 \right\}$ induced by a stationary symmetric α-stable $(0 < \alpha < 2)$ discrete parameter random field $\{X_t\}_{t \in \mathbb{Z}^d}$ for a suitable choice of scaling sequence $b_n \uparrow \infty$. It is easy to prove, following the arguments in the one-dimensional case, Resnick and Samorodnitsky (2004), that if the random field is generated by a dissipative $\mathbb{Z}^d$-action then $b_n = n^{d/\alpha}$ is appropriate and with this choice the above point process sequence converges weakly to a cluster Poisson process. For the conservative case, no general result is known even when $d = 1$. In this part of the talk, we look at a specific class of stable random fields generated by conservative actions for which the effective dimension $p \le d$ can be computed using the structure theorem of finitely generated abelian groups and some basic counting techniques. For this class of random fields, in order to incorporate the clustering effect of extreme observations due to longer memory, we need to normalize the point process itself in addition to using a scaling sequence $b_n = n^{p/\alpha}$.The weak limit of this normalized point process happens to be a random measure but not a point process. A number of limit theorems for various functionals of the random field can be obtained by continuous mapping arguments from these weak convergence results.

This is based on a joint work with Gennady Samorodnitsky.

`List of invited speakers`   `Schedule for December 14`