

Random-turn games, tug-of-war, and the infinity-Laplacian

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The game of Hex has two players who take turns placing stones of their respective colors on the hexagons of a rhombus-shaped hexagonal grid. Black wins by completing a crossing between two opposite edges, while White wins by completing a crossing between the other pair of opposite edges. Although ordinary Hex is famously difficult to analyze, Random-Turn Hex—in which players toss a coin before each turn to decide who gets to place the next stone—has a simple optimal strategy, which is related to percolation. Another random-turn game, tug-of-war, is closely related to the infinity-Laplacian, which is the L_∞ analogue of the usual Laplacian. A function is infinity-harmonic if its second derivative in the gradient direction is zero. The game theoretic description of infinity-harmonic functions yields new intuition and estimates; for instance, we prove power law bounds for infinity-harmonic functions in the unit disk with boundary values supported in a delta-neighborhood of a Cantor set on the unit circle.

This is a joint work with Yuval Peres, Oded Schramm, and Scott Sheffield.

List of invited speakers