1. In a triangle $ABC$, angle $A$ is twice angle $B$. Show that
\[ a^2 = b \cdot (b + c). \]

2. If $x$, $y$ and $z$ are three real numbers such that $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$, then show that each of $x$, $y$ and $z$ lies in the closed interval $[2/3, 2]$, that is $2/3 \leq x \leq 2$, $2/3 \leq y \leq 2$ and $2/3 \leq z \leq 2$. Can $x$ attain the extreme value 2/3 or 2?

3. Find the remainder when $19^{92}$ is divided by 92.

4. Find the number of permutations $(P_1, P_2, P_3, P_4, P_5, P_6)$ of $1, 2, 3, 4, 5, 6$ such that for any $k$, $1 \leq k \leq 5$, $(P_1, P_2, \ldots, P_k)$ does not form a permutation of $\{1, 2, \ldots, k\}$. That is $P_1 \neq 1$; $(P_1, P_2)$ is not a permutation of $\{1, 2\}$; $(P_1, P_2, P_3)$ is not a permutation of $\{1, 2, 3\}$, etc.

5. Show that $Y$ is also the mid-point of $XZ$.

6. Let $f(x)$ be a polynomial in $x$ with integer coefficients and suppose that for 5 distinct integers $a_1, a_2, a_3, a_4$ and $a_5$ one has
\[ f(a_1) = f(a_2) = f(a_3) = f(a_4) = f(a_5) = 2. \]
Show that there does not exist an integer $b$ such that $f(b) = 9$.

7. Find the number of ways in which one can place the numbers $1, 2, 3, \ldots, n^2$ on the $n \times n$ chessboard, one on each, such that the numbers in each row and each column are in arithmetic progression. (Assume $n \geq 3$).

8. Determine all pairs $(m, n)$ of positive integers for which
\[ 2^m + 3^n \]
is a perfect square.

9. Let $A_1A_2A_3 \ldots A_n$ be an $n$-sided regular polygon such that
\[ \frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}, \]
Determine $n$, the number of sides of the polygon.

10. Determine all functions $f : \mathbb{R} \setminus \{0, 1\} \rightarrow \mathbb{R}$ satisfying the functional relation
\[ f(x) + f \left( \frac{1}{1-x} \right) = \frac{2(1-2x)}{x(1-x)}, \]
where $x$ is a real number different from 0 and 1.
(Here $\mathbb{R}$ denotes the set of all real numbers.)