1. (a) Given any positive integer $n$, show that there exist distinct positive integers $x$ and $y$ such that $x+j$ divides $y+j$ for $j=1, 2, 3, \ldots, n$.

(b) If for some positive integers $x$ and $y$, $x+j$ divides $y+j$ for all positive integers $j$, prove that $x=y$.

2. Let $C_1$ and $C_2$ be two concentric circles in the plane with radii $R$ and $3R$ respectively. Show that the orthocentre of any triangle inscribed in circle $C_1$ lies in the interior of circle $C_2$. Conversely, show that also every point in the interior of $C_2$ is the orthocentre of some triangle inscribed in $C_1$.

3. Solve the following system of equations for real numbers $a, b, c, d, e$.

\[
3a = (b+c+d)^3,
3b = (c+d+e)^3,
3c = (d+e+a)^3,
3d = (e+a+b)^3,
3e = (a+b+c)^3.
\]

4. Let $X$ be a set containing $n$ elements. Find the number of all ordered triples $(A, B, C)$ of subsets of $X$ such that $A$ is a subset of $B$ and $B$ is a proper subset of $C$.

5. Define a sequence $(a_n)_{n \geq 1}$ by $a_1 = 1$, $a_2 = 2$ and $a_{n+2} = 2a_{n+1} - a_n + 2$ for $n \geq 1$. Prove that for any $m$, $a_m a_{m+1}$ is also a term in the sequence.

6. There is a $2n \times 2n$ array (matrix) consisting of 0’s and 1’s and there are exactly 3n zeros. Show that it is possible to remove all the zeros by deleting some $n$ rows and some $n$ columns.

[Note: A $m \times n$ array is a rectangular arrangement of $mn$ numbers in which there are $m$ horizontal rows and $n$ vertical columns.]