1. Prove that the inradius of a right-angled triangle with integer sides is an integer.
2. Find the number of positive integers which divide $10^{999}$ but not $10^{998}$.
3. Let $ABCD$ be a square and $M, N$ points on sides $AB, BC$, respectively, such that $\angle MDN = 45^\circ$. If $R$ is the midpoint of $MN$ show that $RP = RQ$ where $P, Q$ are the points of intersection of $AC$ with the lines $MD, ND$.
4. If $p, q, r$ are the roots of the cubic equation $x^3 - 3px^2 + 3q^2x - r^3 = 0$, show that $p = q = r$.
5. If $a, b, c$ are the sides of a triangle prove the following inequality:
\[
\frac{a}{c + a - b} + \frac{b}{a + b - c} + \frac{c}{b + c - a} \geq 3.
\]
6. Find all solutions in integers $m, n$ of the equation
\[
(m - n)^2 = \frac{4mn}{m + n - 1}.
\]
7. Find the number of quadratic polynomials, $ax^2 + bx + c$, which satisfy the following conditions:
   (a) $a, b, c$ are distinct;
   (b) $a, b, c \in \{1, 2, 3, \ldots 1999\}$ and
   (c) $x + 1$ divides $ax^2 + bx + c$. 