A Neural Self-Organization based Optimization Technique for the Generation of Steiner Minimal Tree

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Overview

The interconnection problems :

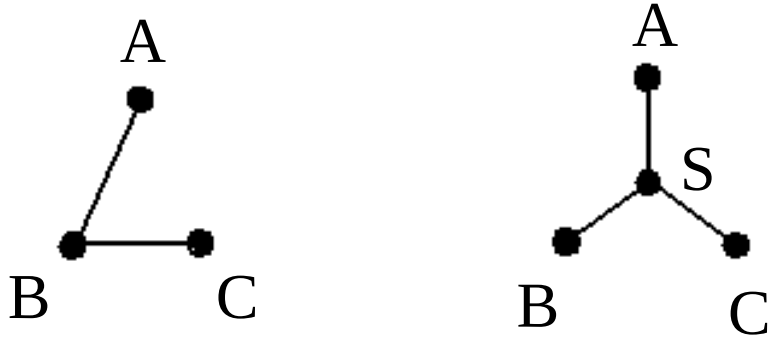
- VLSI Interconnection
- PCB layout
- Telephony
- Utility Connections at Home or Industry
- Connecting Oil Fields
- Laying Roads in Cities

Overview : The Problem

- N points $A = \{a_1, a_2, \Lambda\ , a_N\}$ in Euclidean plane are to be connected by the shortest network.

Shortest Path Interconnections

- Steiner Minimal Tree (SMT) : A set of points $A = \{a_1, a_2, \Lambda\ , a_N\}$ in the Euclidean plane results into a minimally connected network by adding some extra points $S = \{s_1, s_2, \Lambda\ , s_K, K \le N\}$, called Steiner Points.

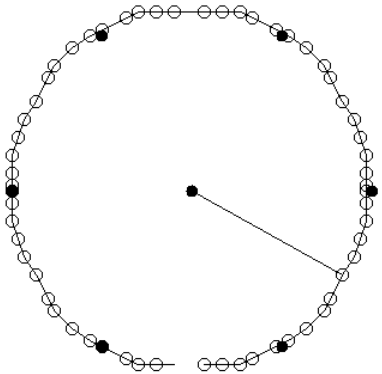


Shortest Path Interconnections

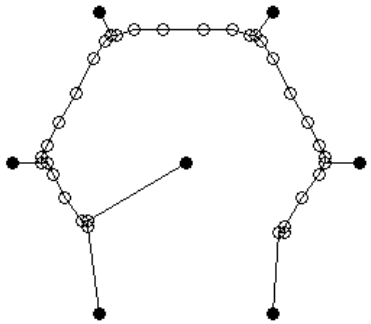
- Exact Method ： SMT consisting of FSTs
- Heuristic Method ： Voronoi diagram, Delaunay Triangulation, MST, Simulated Annealing, Soap filmsetc
- Neural Self-Organization Based Method

Neural Self-Organization

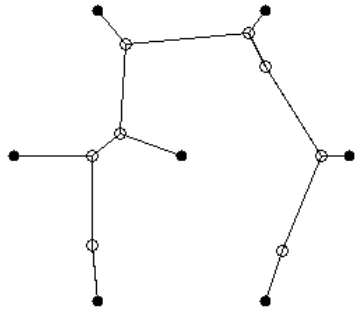
- **Jaydeva & Bhaumik (1994)**



(a)



(b)



(c)

$$E = \sum_{i=1}^{M} dist\left(2, x_{i+1}, x_i\right) + \sum_{i=1}^{N}\sum_{j=1}^{M} d_{ij} h\left(d_{ij}\right)$$

where

$$dist\left(2, x_{i+1}, x_i\right) = \left\| x_{i+1}, x_i \right\|_2, \qquad d_{ij} = dist\left(2, p_i, x_j\right)$$

$$h\left(d_{ij}\right) = \frac{\exp\left(-d_{ij}^2 / \beta^2(t)\right)}{\sum_{l=1}^{M} \exp\left(-d_{ij}^2 / \beta^2(t)\right)}$$

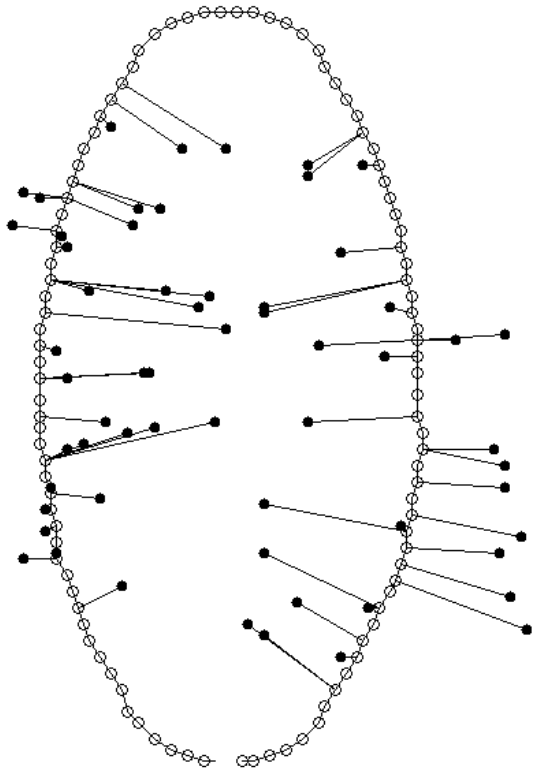
Dynamic Equation

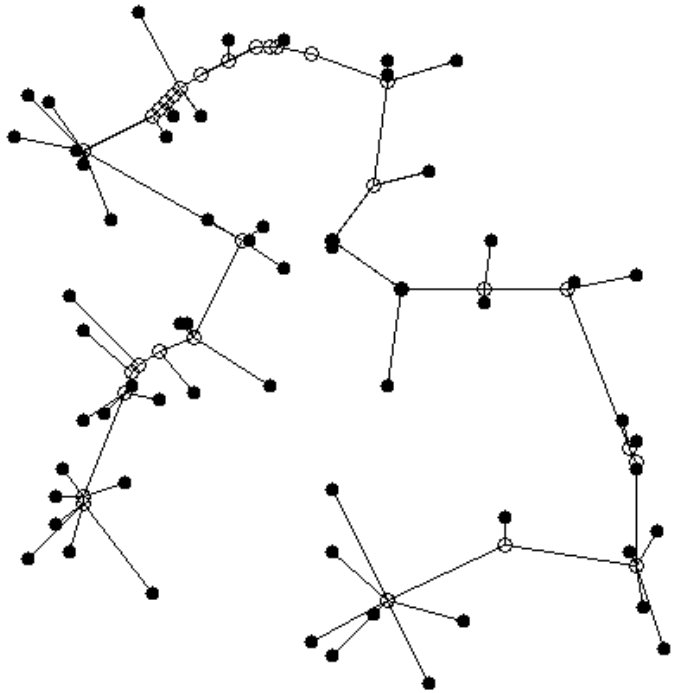
$$x_{ij}^{'} = -\frac{\partial E}{\partial x_{ij}} = \frac{\left(x_{i+1} - x_i\right)}{dist\left(2, x_{i+1}, x_i\right)} + \frac{\left(x_{i-1} - x_i\right)}{dist\left(2, x_i, x_{i-1}\right)}$$

$$+ \sum_{k=1}^{N} h\left(d_{ki}\right) * \frac{\left(p_{kj} - x_{ij}\right)}{d_{ki}}$$

$$+ \sum_{k=1}^{N} \left(x_{ij} - p_{kj}\right) \frac{1}{\beta^2(t)} h\left(d_{ki}\right)\left(h\left(d_{ki}\right) - 1\right)$$

Neural Self-Organization : Shortcomings

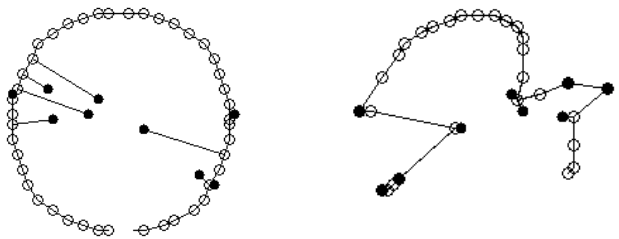




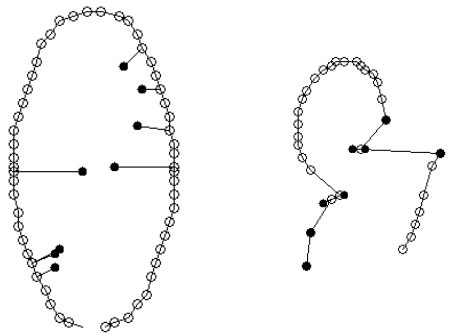
Features For Improving The Neural Self-Organization Method

- Shape of the Curve
- Partition Merging
- Number of Added Initial Extra Points
- Direction of Curve Opening
- Partitioning Methodology

Shape of the Curve



(a)



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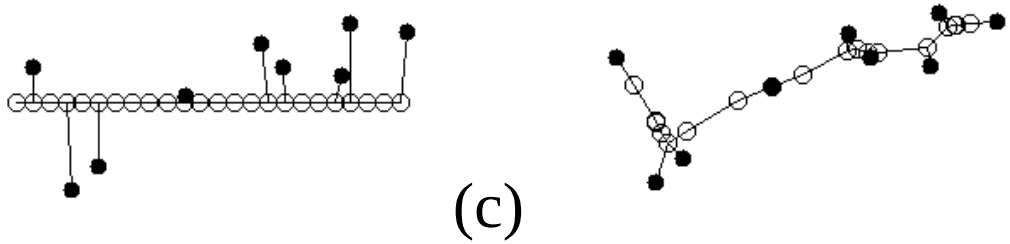
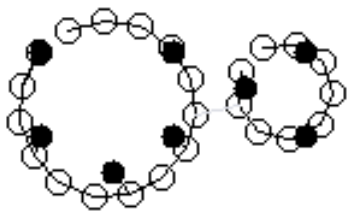
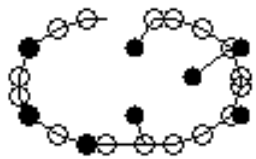
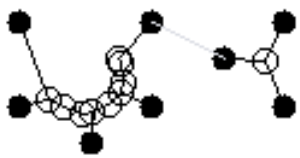


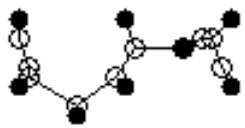
(c)

Partition Merging

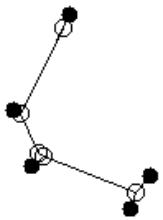




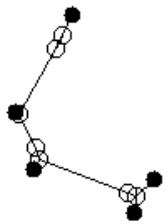




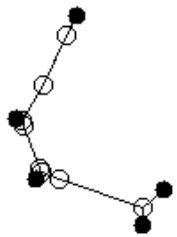
Number of Added Initial Extra Points



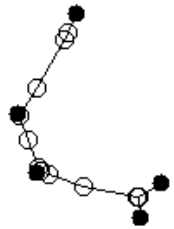
(a) 12



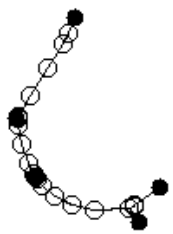
(b) 15



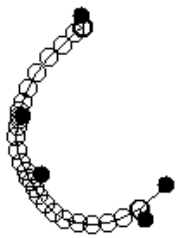
(c) 20



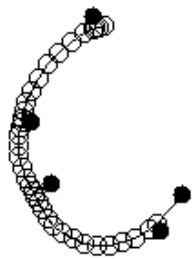
(d) 25



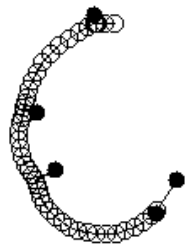
(e) 35



(f) 50

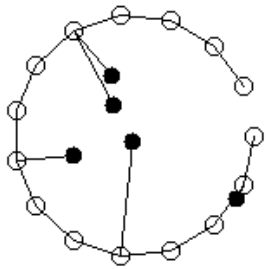


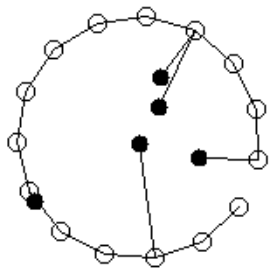
(g) 60

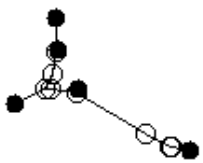


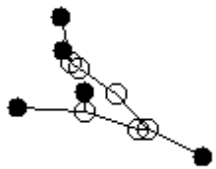
(h) 75

Direction of Curve Opening

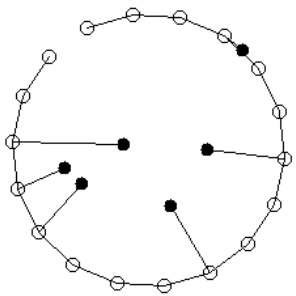


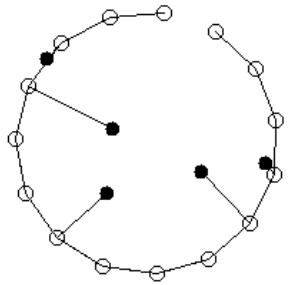


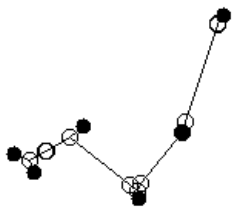


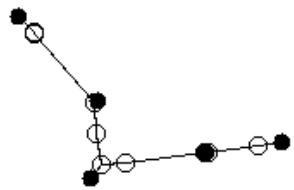


Direction of Curve Opening: More Results

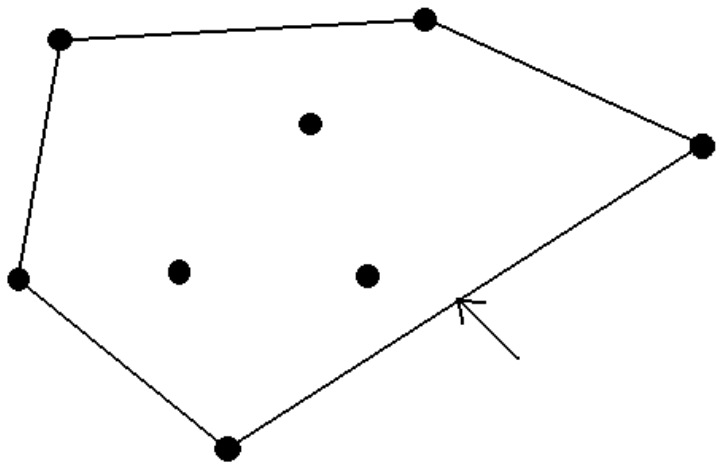




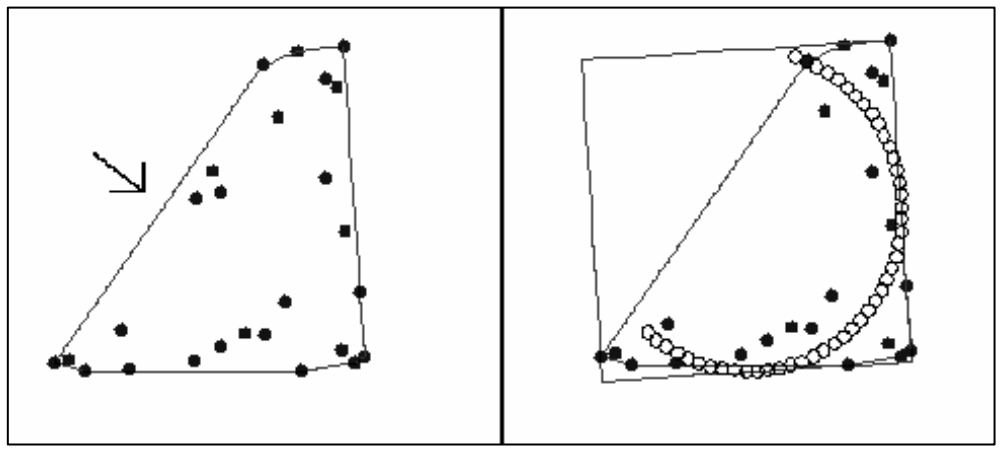




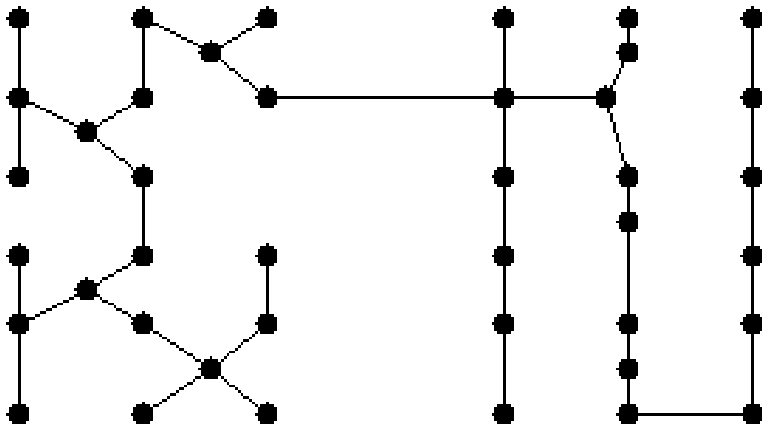
Direction of Curve Opening: Methodology



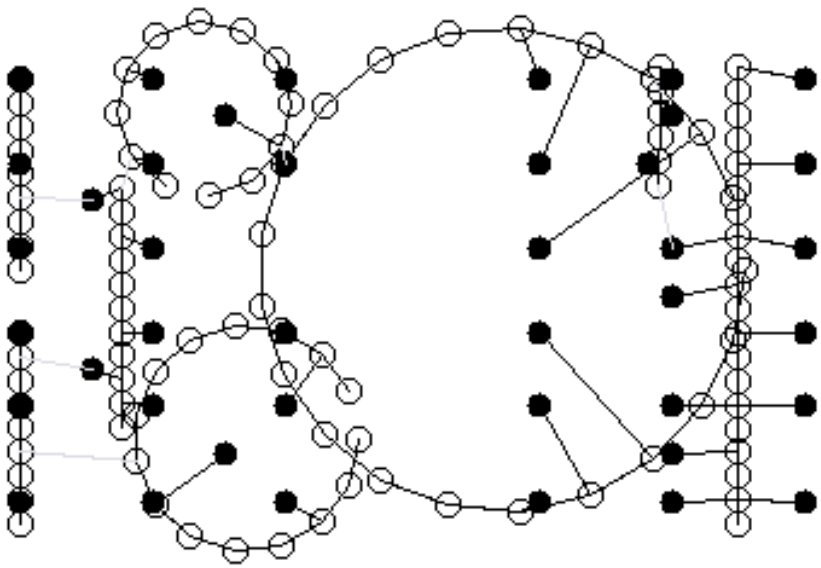
Direction of Curve Opening: Methodology



Partition Method



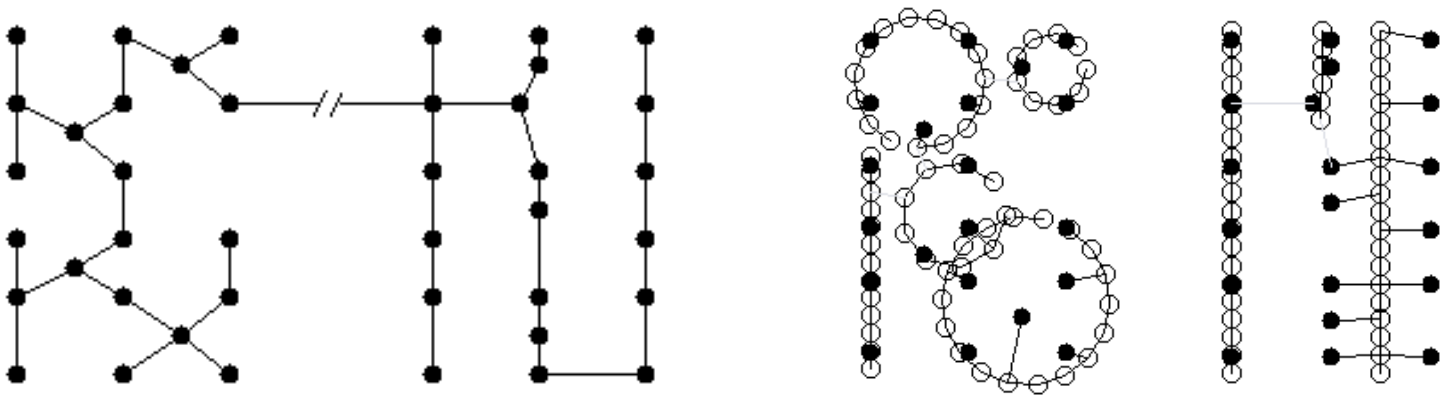
Partition Method: Multi Branch Point Partition



Partition Method: Hierarchical Partitioning

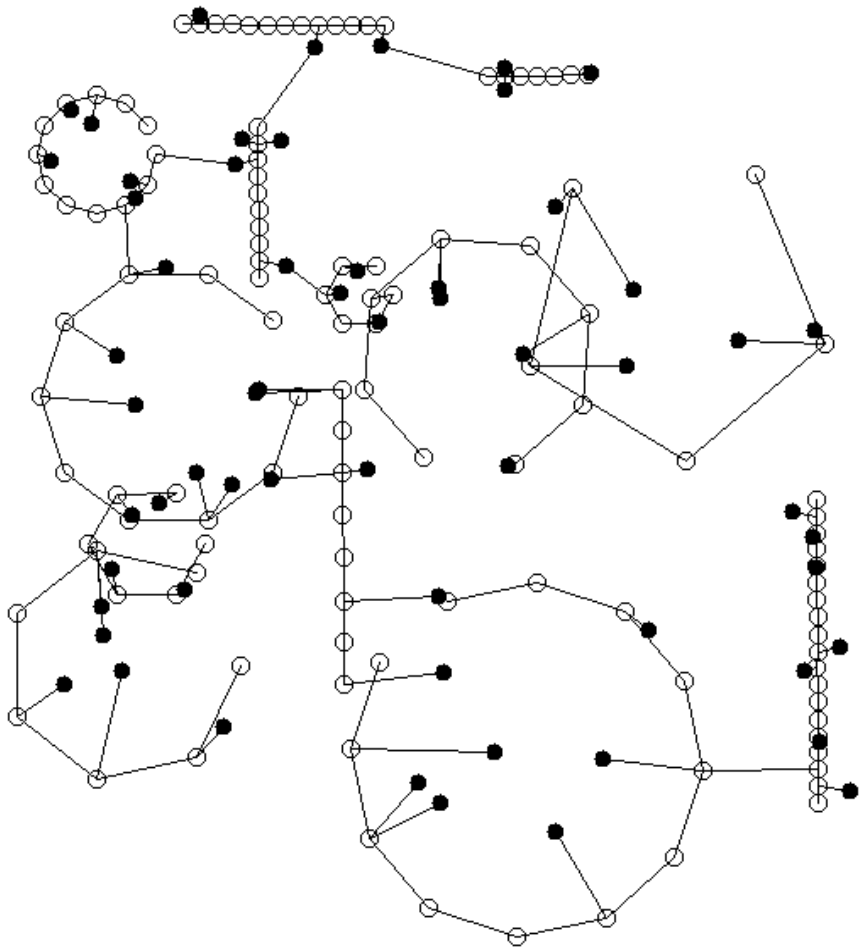
1. Top level partitioning using cluster finding by longest branch method
2. Within a cluster, partitioning using multi-branch point method

Partition Method: Hierarchical Partitioning

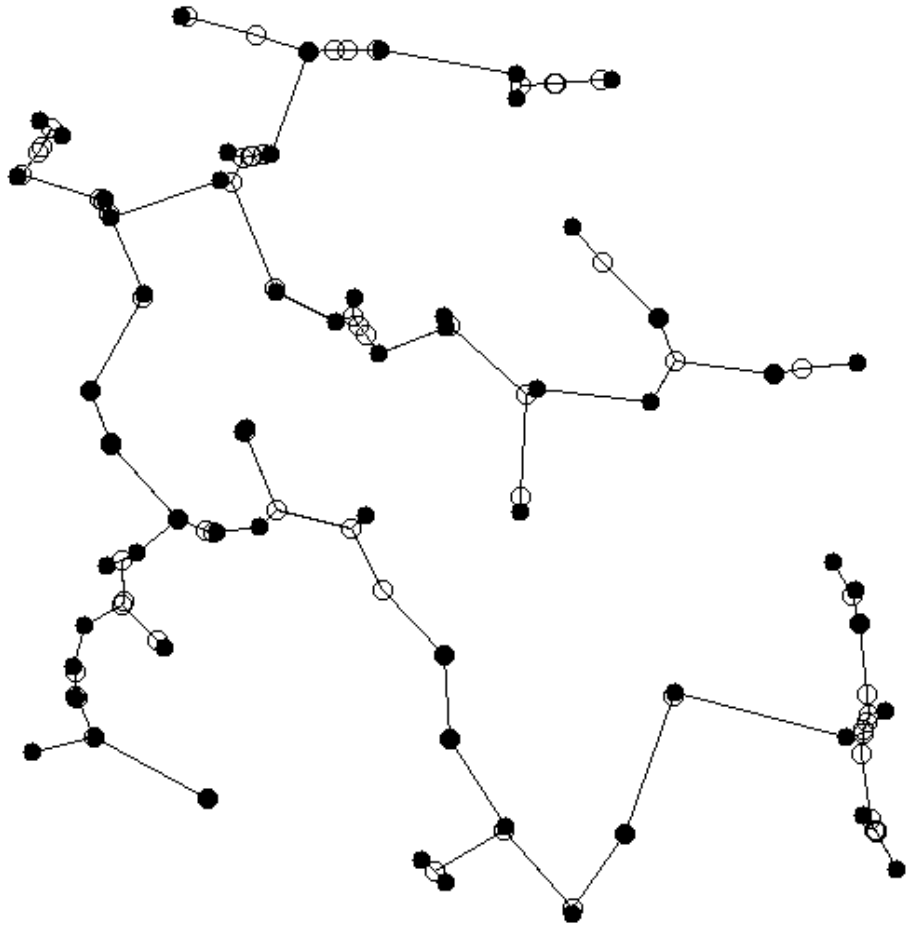


(a) (b)

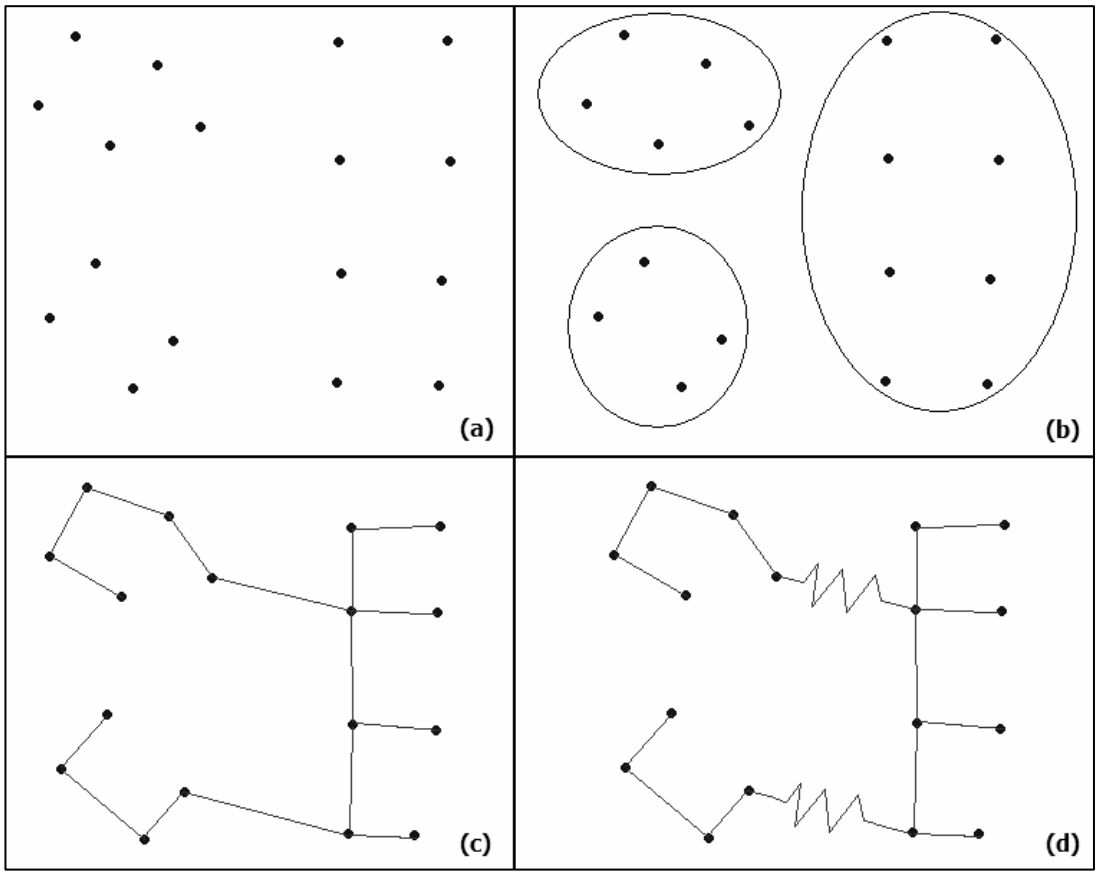
An Example: Initial Partitioning



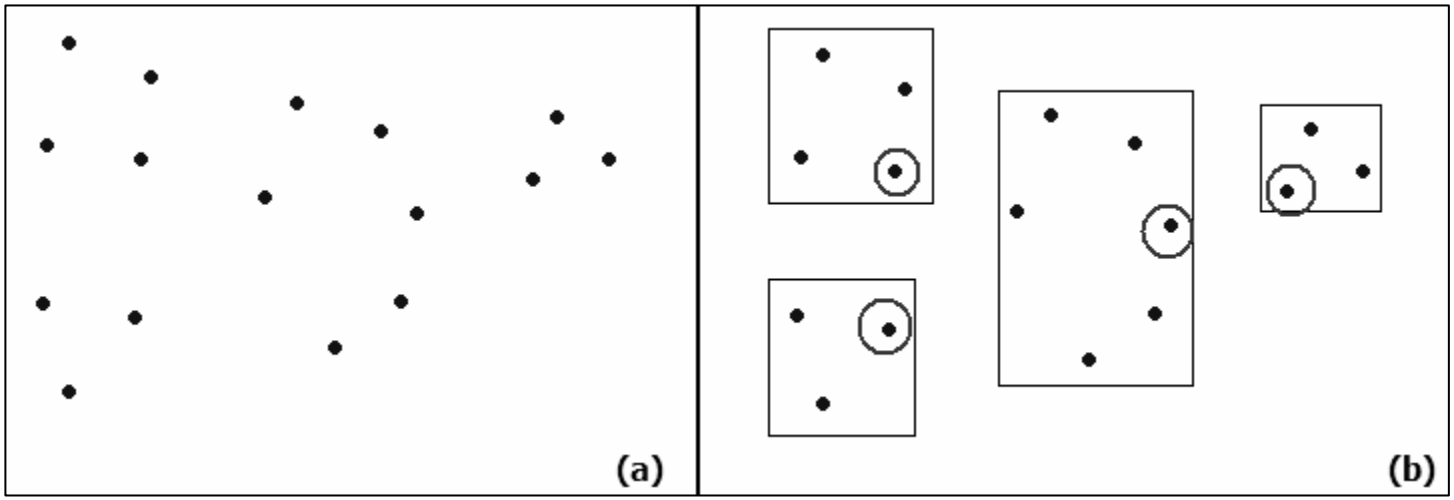
An Example: Final Result

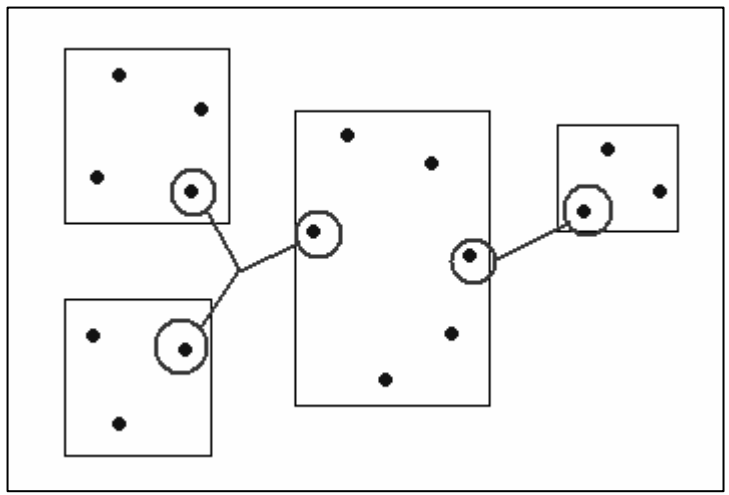


Gestalt type Clustering of points

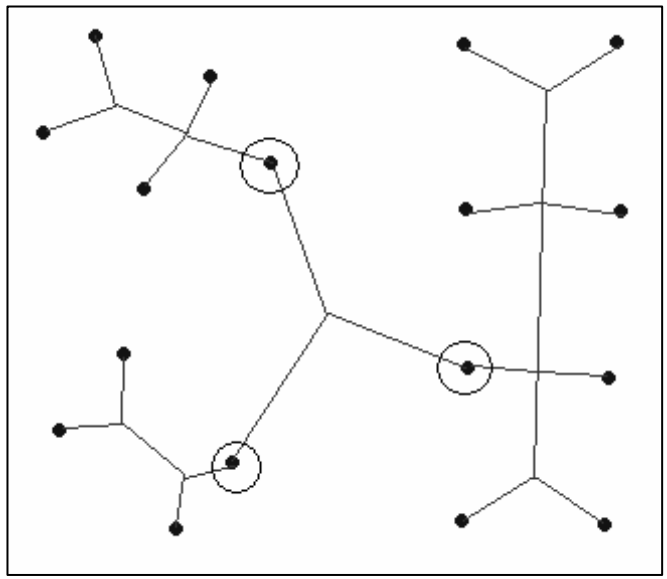


Overlap Issue





An Example:



Some More Results

Input File	N	Steiner Length	MST Length	Percent Decrease
tri.1	3	2.46003	2.75344	10.6564%
house.1	5	1.32224	1.36569	3.18117%
orlib/10/1.in	10	2.22118	2.11147	-5.19607%
pe.31	14	2.4473	2.52952	3.25057%
2.in	18	4.95028	5.04526	1.88259%
orlib/20/1.in	20	3.44595	3.21282	-7.25617%
3.in	36	10.5971	10.7905	1.79258%
orlib/50/3.in	50	5.40777	4.92898	-9.71379%

Conclusion & Future Work

- The features added have distinctly improved the performance of the Neural Self-Organization Technique.
- It can be used in VLSI design when rectilinear paths are used instead of the euclidean ones. All the features presented in this work will still be applicable there.
- To find the optimal network for a dynamic point set: e.g. in Sensor Network

References

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Thank You!