Effect of Channel Correlation on the Performance of Diversity Receivers

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Introduction

- Fading: random fluctuations of the received signal power level in a wireless communication environment owing to
  - multipath propagation
  - relative movement between transmitter and receiver
- Diversity at the receiver: combats the effects of signal fading in mobile wireless channels
- Diversity schemes:
  - <u>Space diversity</u>: use multiple antennas to receive same signal

– Frequency diversity: transmit the same signal over different frequencies
– Time diversity: Same signal is transmitted at multiple instants
– Path diversity: used in CDMA systems; resolve multipaths via code correlation and combine them in a rake receiver

- Different kinds of fading statistics of the amplitude of the signal of each diversity branch:
  – Rayleigh fading (diffuse component only, no specular or LOS component)
  – Rician fading (diffuse component and at most one specular component per branch)
  – Nakagami fading (superimposition of several Rayleigh faded diffuse components)

- Diversity combining techniques:
  - Maximal-ratio combining (MRC) (in presence of interference, use optimum combining (OC))
  - Predetection equal-gain combining (EGC)
  - Selection combining (SC)
  - Hybrid selection/maximal-ratio combining (H-S/MRC)
  - Postdetection equal-gain combining (PEGC)
- MRC, EGC, H-S/MRC need coherent reception
- SC can be used with coherent, noncoherent, or differentially coherent reception
- PEGC is used with noncoherent or differentially coherent reception

BNFSK and BDPSK with PEGC over Correlated Rician Channels

- Although independent fading between diversity branches is desirable, the fading can be correlated in many practical scenarios
- Consider a diversity reception system over a flat Rician fading channel with L correlated branches
- Receiver employs symbol-by-symbol detection
- Signal received over the kth diversity branch in a symbol interval of duration T_s:

$$r_k(t) = \mathrm{Re}\left\{\left[\alpha_k e^{-\jmath\phi_k} s(t) + n_k(t)\right] e^{\jmath 2\pi f_c t}\right\} ,$$
$$k = 1, \ldots, L, \quad 0 \leq t < T_s$$

- $s(t)$: complex baseband information-bearing signal with average symbol energy $2E_s$

  α_k, ϕ_k : random magnitude and random phase of kth diversity branch gain

  f_c : carrier frequency

  $n_k(t)$: additive noise (zero-mean complex white Gaussian random process with two-sided power spectral density $2N_0$)

- $\{n_k(t)\}$ are independent but $\{\alpha_k e^{-\jmath\phi_k}\}$ are correlated

- Each $n_k(t)$ is independent of the gains $\{\alpha_k e^{-\jmath\phi_k}\}$

- Complex channel gain vector:

$$\underline{g} \triangleq \left[\alpha_1 e^{-\jmath\phi_1}, \cdots, \alpha_L e^{-\jmath\phi_L}\right]^T = \underline{X}_c + \jmath\underline{X}_s$$

- $\underline{X}_c$: in-phase component
  $\underline{X}_s$: quadrature component
- $\underline{X}_c$ and $\underline{X}_s$ are real Gaussian vectors with mean vectors $\underline{\mu}_c$ and $\underline{\mu}_s$,
  covariance matrices $\underline{K}_{cc}$ and $\underline{K}_{ss}$,
  cross-covariance matrix $\underline{K}_{cs}$
- $\underline{K}_{cc}$ and $\underline{K}_{ss}$ have the same diagonal elements, and all diagonal elements of $\underline{K}_{cs}$ are zero
- $\mu_{c_k} + \jmath\mu_{s_k}$: specular component of g_k

- Now
$$\underline{X} \triangleq \begin{bmatrix} \underline{X}_c \\ \underline{X}_s \end{bmatrix} \sim \mathcal{N}\left(\underline{\mu}, \underline{K}\right) ,$$
where
$$\underline{K} \triangleq \begin{bmatrix} \underline{K}_{cc} & \underline{K}_{cs} \\ \underline{K}_{cs}^T & \underline{K}_{ss} \end{bmatrix}, \qquad \underline{\mu} \triangleq \begin{bmatrix} \underline{\mu}_c \\ \underline{\mu}_s \end{bmatrix}$$
- Let $\underline{K} = \underline{E}\,\underline{\Lambda}\,\underline{E}^T$, where $\underline{\Lambda} = \text{diag}(\lambda_1, \ldots, \lambda_{2L})$
- $\underline{\Lambda}$: diagonal matrix of eigenvalues of $\underline{K}$,
  $\underline{E}$: orthogonal matrix of eigenvectors of $\underline{K}$
- $\underline{\eta} = \left[\eta_1, \cdots, \eta_{2L}\right]^T = \underline{E}^T \underline{\mu}$
- For BNFSK, signal for symbol i :
$$s(t) = \sqrt{\frac{2E_s}{T_s}}\, e^{\jmath 2\pi (\Delta f_i) t}, \quad i = 0, 1$$

- $r_k(t)$ is passed through two narrowband bandpass filters (BPFs)
- u_{ki} : square of output envelope for BPF centered at $f_c + \Delta f_i$
- Decision variable : $D = \sum_{k=1}^{L} u_{k1} - \sum_{k=1}^{L} u_{k0}$
- When symbol 1 is transmitted:

$$u_{k1} = |2E_s g_k + N_{k1}|^2 \;, \qquad u_{k0} = |N_{k0}|^2$$

- N_{k1} and N_{k0} are i.i.d. $\mathcal{CN}(0, 4E_sN_0)$ random variables
- Similar approach for BDPSK

- $\gamma_k = \dfrac{E_s \alpha_k^2}{N_0}$: instantaneous SNR at branch k
- $\gamma_{tot} = \sum_{k=1}^{L} \gamma_k$: instantaneous SNR at combiner output
- By using results on quadratic forms in normal variables, we find the c.f. of γ_{tot} and then the c.f. of D
- SEP = Pr[$D < 0$ when symbol 1 is transmitted]
- The SEP is obtained directly from the c.f. of D using the inversion theorem (Gil-Pelaez lemma)

- Define

$$a_k \triangleq 2 + 4h\lambda_k \frac{E_s}{N_0}, \quad b_k \triangleq 2h\eta_k^2 \frac{E_s}{N_0},$$

where

$$h = \begin{cases} \frac{1}{2} \text{ for BNFSK,} \\ 1 \text{ for BDPSK} \end{cases}$$

- SEP:

$$P_e = \left[\prod_{k=1}^{2L} \frac{\exp\left\{-\frac{b_k}{a_k}\right\}}{a_k^{\frac{1}{2}}}\right] \times \sum_{\substack{(l_1,\ldots,l_{L-1}) \\ 0\le l_1,\ldots,l_{L-1}\le L-1 \\ l_1+2l_2+\cdots+(L-1)l_{L-1}=L-1}} \prod_{m=1}^{L-1} \frac{1}{l_m!}\left[\frac{1}{m} + \frac{1}{2m}\sum_{k=1}^{2L}\left(1-\frac{1}{a_k}\right)^m + \sum_{k=1}^{2L}\frac{b_k}{a_k^2}\left(1-\frac{1}{a_k}\right)^{m-1}\right]^{l_m}$$

- Numerical results for exponential correlation type of power balanced diversity:

$$\underline{K}_{cs} = \underline{0}\,, \quad (\underline{K}_{cc})_{ij} = (\underline{K}_{ss})_{ij} = \rho^{|i-j|}\sigma_Z^2$$

- ρ : correlation coefficient $(0 \leq \rho < 1)$
- Phase difference of ξ between specular components of kth and $(k+1)$th branches:

$$\left(\underline{\mu}_c + \jmath\underline{\mu}_s\right)_k = \mu_0 e^{\jmath(k-1)\xi}$$

- $\frac{E_s}{N_0}(\mu_0^2 + 2\sigma_Z^2)$: branch SNR
- $K = \frac{\mu_0^2}{2\sigma_Z^2}$: Rician factor

- For all ξ satisfying $0 \leq |\xi| < \xi_{max}$ (ξ_{max} depends on ρ, L, and branch SNR):
  - SEP versus K reaches a maximum at some $K = K_{max}$
  - K_{max} decreases as $|\xi|$ goes from 0 to ξ_{max}
  - For $|\xi| \geq \xi_{max}$, SEP attains a maximum at $K = 0$, which corresponds to Rayleigh fading
- In the case of correlated branches, for a certain range of values of ξ, the performance in Rayleigh fading is better than that in Rician fading over a range $0 < K < K_m$
- However, in the case of independent branches the performance in Rayleigh fading is worse than that in Rician fading

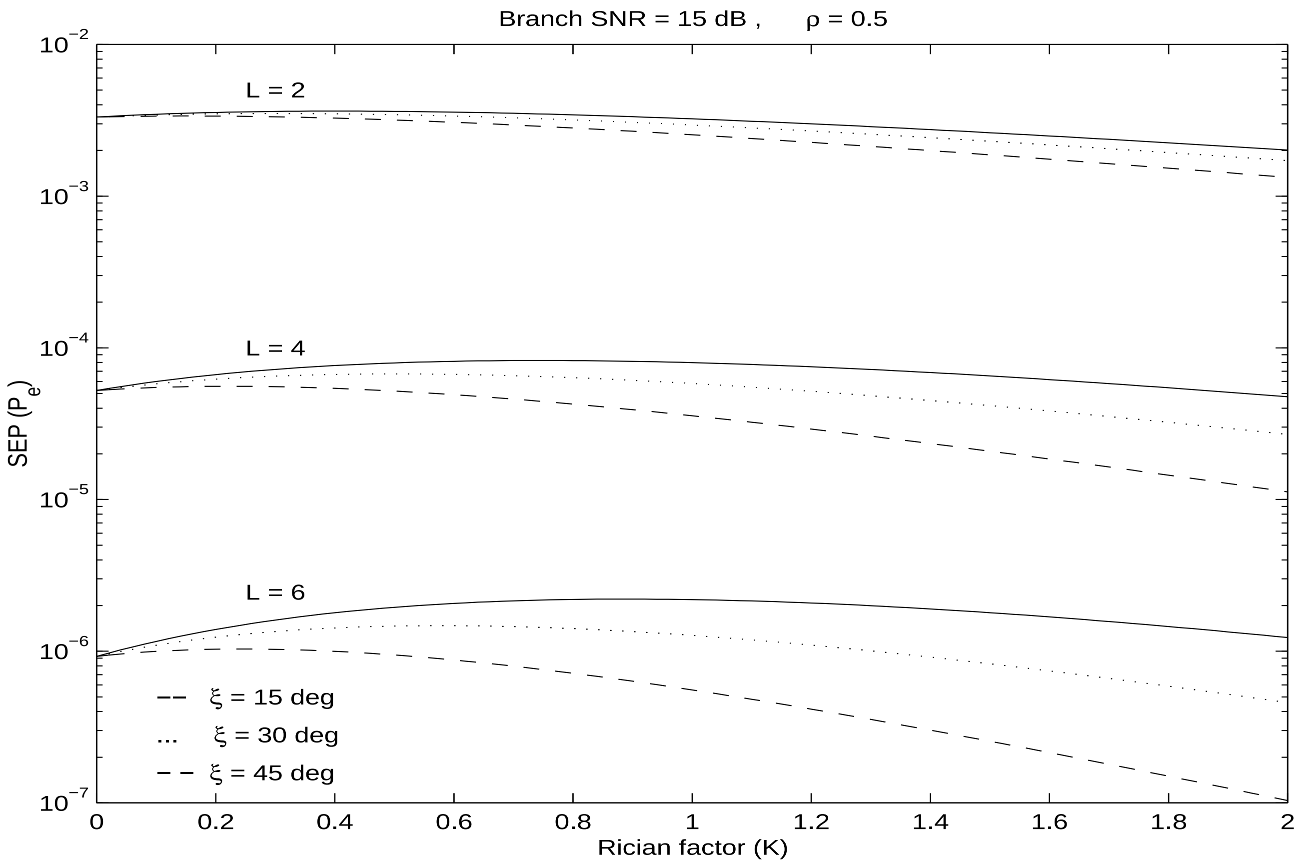

Figure 1: SEP versus Rician factor when branch SNR = 15 dB, $\rho = 0.5$ ($\xi_{max} \approx 60°$) for different values of L and ξ in case of BNFSK using exponential correlation type power balanced diversity

H-S/MRC in Correlated Nakagami Fading

- Nakagami fading model with evenly correlated diversity branches having the same m-parameter and same average SNR Γ
- Select a subset L of N available branches with highest instantaneous SNR
- From the joint c.f. of the instantaneous branch SNRs, an expression for the c.f. of the combiner output SNR is obtained
- This c.f. is used to obtain the SEP of coherent detection several M-ary modulation schemes
- Properties of order statistics are applied

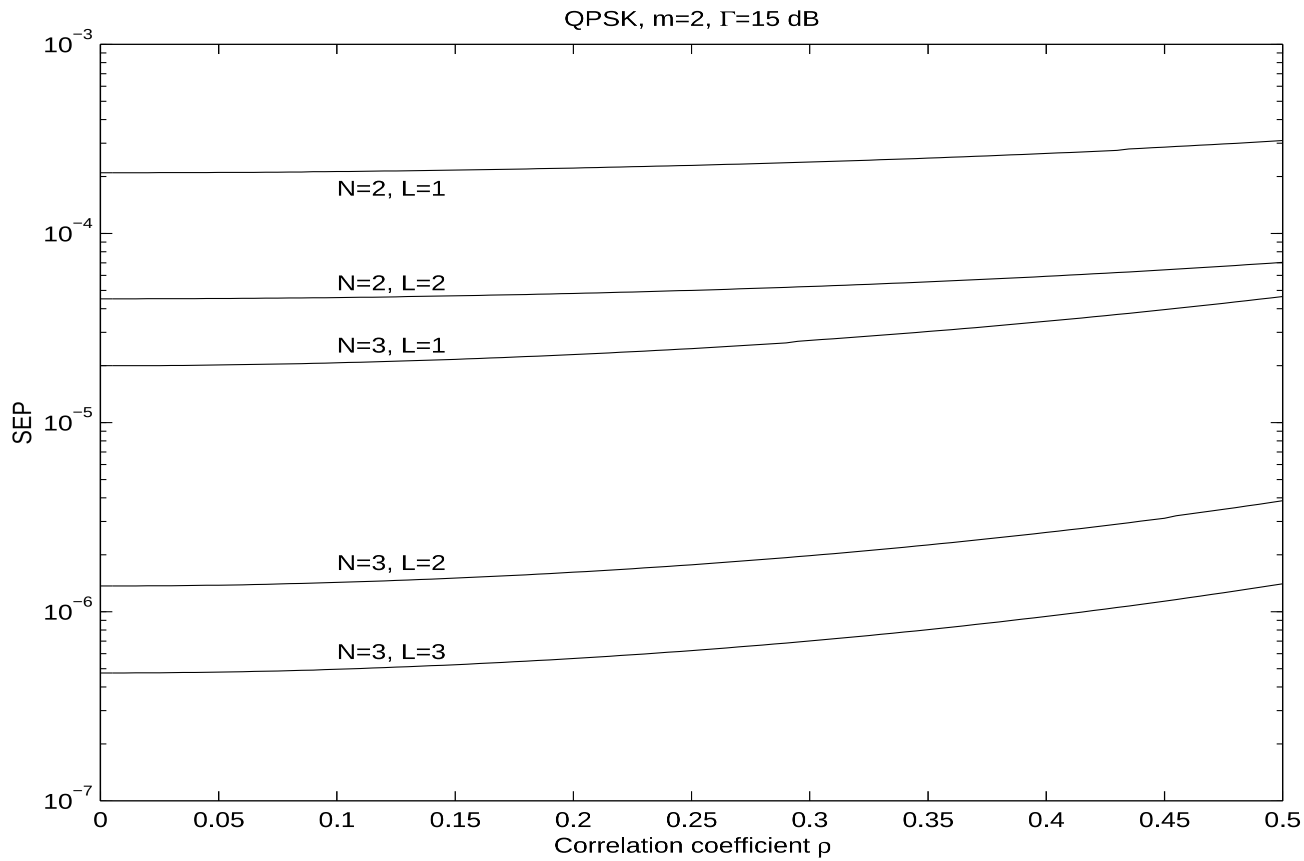

Figure 2: SEP as a function of correlation coefficient ρ for correlated Nakagami fading having parameter $m = 2$, average SNR per branch $\Gamma = 15$ dB, $1 \leq L \leq N$, $N = 2, 3$

Effect of Channel Correlation

- Correlation degrades the performance
- In case of PEGC, we do not estimate the channel; and we cannot do much to combat the effect of correlation
- In case of H-S/MRC, we have assumed that perfect estimates of $\{\alpha_k e^{-\jmath\phi_k}\}$ are available to the receiver
- In practice, there will be some estimation error, and this will further degrade the performance
- Schemes which need channel estimation can be modified to yield better performance in presence of channel correlation

Optimized Diversity Combining

- In the presence of CCI, OC is known to give the best error performance since it maximizes the instantaneous SINR of the combiner output
- This is based on the assumption that a perfect estimate of the channel is available
- With imperfect channel estimation, the receiver structure needs to be modified to combat the effect of channel correlation
- This results in what we call optimized diversity combining (ODC)
- Consider a pilot symbol based ML channel estimation method

- Correlated flat Rayleigh fading channel in the presence of CCI and additive noise
- The decision rule, which is optimum in the ML sense, is derived using concepts of Gaussian and Wishart statistics
- A system using receive diversity with L receive antennas, and subject to N cochannel interferers
- We use K pilot symbols to estimate the channel
- Complex baseband sampled received signal vector for kth pilot transmission:

$$\mathbf{r}_{p_k} = \mathbf{c}s_{p_k} + \sum_{i=1}^{N} \mathbf{b}_i q_{p_{i,k}} + \mathbf{n}_{p_k}\,, \quad k = 1, \ldots, K$$

- $\mathbf{c}$: propagation vector of desired user
  s_{p_k}: pilot symbol
  $\{\mathbf{b}_i\}$: propagation vectors of interferers
  $\{q_{p_{i,k}}\}$: their signal samples
  $\mathbf{n}_{p_k}$: AWGN vector
- $\left\{q_{p_{i,k}}\right\}_{i,k}$ are i.i.d. $\mathcal{CN}(0,1)$ random variables
- $\mathbf{n}_{p_1}, \ldots, \mathbf{n}_{p_K}$ are i.i.d. $\mathcal{CN}\left(\mathbf{0}, \sigma^2 \mathbf{I}_L\right)$ random vectors
- $\mathbf{c} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{K}_c\right)$
  $\mathbf{b}_1, \ldots, \mathbf{b}_N$ are i.i.d. $\mathcal{CN}\left(\mathbf{0}, \mathbf{K}_b\right)$
- $\mathbf{c}$, $\mathbf{b}_1, \ldots, \mathbf{b}_N$, $\left\{q_{p_{i,k}}\right\}_{i,k}$, and $\mathbf{n}_{p_1}, \ldots, \mathbf{n}_{p_K}$ are independent of each other

- Receiver has knowledge of $\mathbf{K}_c$, $\mathbf{K}_b$, and σ^2 which characterize the channel statistics
- Given the channel parameters $\mathbf{c}$, $\mathbf{b}_1, \ldots, \mathbf{b}_N$, the received signal vectors $\mathbf{r}_{p_1}, \ldots, \mathbf{r}_{p_K}$ are independent, $\mathbf{r}_{p_k}$ having a

$$\mathcal{CN}\left(\mathbf{c}s_{p_k},\ \sum_{i=1}^{N} \mathbf{b}_i\mathbf{b}_i^H + \sigma^2\mathbf{I}_L\right)$$

  distribution
- Interference correlation matrix: $\mathbf{A} = \sum_{i=1}^{N} \mathbf{b}_i\mathbf{b}_i^H$
- Pilot symbol vector:

$$\mathbf{s}_p = \begin{bmatrix} s_{p_1} \\ \vdots \\ s_{p_K} \end{bmatrix}$$

- Received pilot vector:

$$\mathbf{r}_p = \begin{bmatrix} \mathbf{r}_{p_1} \\ \vdots \\ \mathbf{r}_{p_K} \end{bmatrix}$$

- The distribution of $\mathbf{r}_p$, conditioned on the channel parameters $\mathbf{c}, \mathbf{A}$, is

$$\mathcal{CN}\left(\mathbf{s}_p \otimes \mathbf{c},\ \mathbf{I}_K \otimes \left(\mathbf{A} + \sigma^2 \mathbf{I}_L\right)\right)$$

- $\mathbf{A}$ is complex Wishart distributed when $N \geq L$, and complex pseudo-Wishart distributed when $N < L$
- Denote this distribution as $\mathcal{CW}_L(N, \mathbf{K}_b)$

- $\mathcal{M}_+(L)$: set of all $L \times L$ complex positive definite matrices
- $\mathcal{M}_+(L, N)$: set of all $L \times L$ complex Hermitian rank-N matrices each with positive definite $N \times N$ principal submatrices
- $\mathbf{A}_{[N]}$: first principal $N \times N$ submatrix of $\mathbf{A}$
- P.d.f. of $\mathbf{A}$:

  $\underline{N \geq L}$:

$$f(\mathbf{A}) = \frac{\{\det(\mathbf{A})\}^{N-L} \exp\left\{-\mathrm{tr}\left(\mathbf{K}_b^{-1}\mathbf{A}\right)\right\}}{\pi^{\frac{L(L-1)}{2}} \left\{\prod_{i=1}^{L} (N-i)!\right\} \{\det(\mathbf{K}_b)\}^N}, \quad \mathbf{A} \in \mathcal{M}_+(L)$$

  $\underline{N < L}$:

$$f(\mathbf{A}) = \frac{\left\{\det\left(\mathbf{A}_{[N]}\right)\right\}^{-(L-N)} \exp\left\{-\mathrm{tr}\left(\mathbf{K}_b^{-1}\mathbf{A}\right)\right\}}{\pi^{\frac{N(2L-N-1)}{2}} \left\{\prod_{i=1}^{N} (N-i)!\right\} \{\det(\mathbf{K}_b)\}^N}, \quad \mathbf{A} \in \mathcal{M}_+(L, N)$$

- ML estimate of $(\mathbf{c}, \mathbf{A})$:

$$(\hat{\mathbf{c}}, \hat{\mathbf{A}}) = \arg\left\{\max_{\mathbf{c},\mathbf{A}} \ln f(\mathbf{r}_p|\mathbf{c}, \mathbf{A})\right\}$$

- Maximization of the log likelihood function with respect to $\mathbf{c}$, $\mathbf{A}$ results in

$$\hat{\mathbf{c}} = \frac{1}{\left(\sum_{k=1}^{K} |s_{p_k}|^2\right)} \sum_{k=1}^{K} s_{p_k}^* \mathbf{r}_{p_k} \,,$$

$$\hat{\mathbf{A}} = \frac{1}{K} \sum_{k=1}^{K} \left(\mathbf{r}_{p_k} - \hat{\mathbf{c}} s_{p_k}\right) \left(\mathbf{r}_{p_k} - \hat{\mathbf{c}} s_{p_k}\right)^H - \sigma^2 \mathbf{I}_L$$

- It can be shown that $\hat{\mathbf{c}}$ and $\hat{\mathbf{A}}$, conditioned on $\mathbf{c}$, $\mathbf{A}$, are independent, and that

$$\hat{\mathbf{c}}\big|_{\mathbf{c},\mathbf{A}} \sim \mathcal{CN}\left(\mathbf{c},\, \frac{1}{\left(\mathbf{s}_p^H \mathbf{s}_p\right)} \left(\mathbf{A} + \sigma^2 \mathbf{I}_L\right)\right) ,$$

$$\left(\hat{\mathbf{A}} + \sigma^2 \mathbf{I}_L\right)\Big|_{\mathbf{c},\mathbf{A}} \sim \mathcal{CW}_L\left(K-1,\, \frac{1}{K}\left(\mathbf{A} + \sigma^2 \mathbf{I}_L\right)\right)$$

- Consider the reception of a transmitted data symbol s over the same channel after the reception of the K pilot symbols $s_{p_1}, \ldots, s_{p_K}$
- Complex baseband sampled received signal vector for data transmission:

$$\mathbf{r} = \mathbf{c}s + \sum_{i=1}^{N} \mathbf{b}_i q_i + \mathbf{n}$$

- Interfering signal samples $\{q_i\}_i$ are i.i.d. $\mathcal{CN}(0,1)$
- Additive noise vector $\mathbf{n}$ has a $\mathcal{CN}\left(\mathbf{0}, \sigma^2 \mathbf{I}_L\right)$ distribution
- $\mathbf{c}$, $\mathbf{b}_1, \ldots, \mathbf{b}_N$, $\{q_i\}_i$, and $\mathbf{n}$ are independent of each other, as well as of the interfering signal samples $\left\{q_{p_{i,k}}\right\}_{i,k}$, and the noise vectors $\{\mathbf{n}_{p_i}\}$ corresponding to pilot symbol reception
- s belongs to an M-ary signal constellation, and takes one of M complex values $S_1, \ldots, S_M$

- When perfect estimates of $\mathbf{c}$ and $\mathbf{A}$ are available, the optimum decision rule in the ML sense is given by

$$\hat{s} = \arg\left\{ \max_{s \in \{S_1, \ldots, S_M\}} f(\mathbf{r} | \mathbf{c}, \mathbf{A}, s) \right\}$$

- This results in

$$\hat{s} = \arg\left\{ \max_{s \in \{S_1, \ldots, S_M\}} \left[\mathrm{Re}\left\{ s^* \mathbf{w}_{oc}^H \mathbf{r} \right\} - \frac{|s|^2}{2} \mathbf{c}^H \left(\mathbf{A} + \sigma^2 \mathbf{I}_L \right)^{-1} \mathbf{c} \right] \right\}$$

where $\mathbf{w}_{oc} = \left(\mathbf{A} + \sigma^2 \mathbf{I}_L \right)^{-1} \mathbf{c}$ is the OC weight vector

- On the other hand, when we have imperfect estimates $\hat{\mathbf{c}}$ of $\mathbf{c}$ and $\hat{\mathbf{A}}$ of $\mathbf{A}$, the optimum decision rule in the ML sense is

$$\hat{s} = \arg\left\{ \max_{s \in \{S_1, \ldots, S_M\}} f(\mathbf{r} | \hat{\mathbf{c}}, \hat{\mathbf{A}}, s) \right\}$$

- We can express the conditional p.d.f. of $\mathbf{r}, \hat{\mathbf{c}}$ as

$$\begin{aligned} & f(\mathbf{r}, \hat{\mathbf{c}} | \mathbf{A}, s) \\ = & \frac{1}{\pi^{2L} \det\left(|s|^2 \mathbf{K}_c + \mathbf{A} + \sigma^2 \mathbf{I}_L\right) \det\left(\mathbf{M}_{22}^{-1}(\mathbf{A}, s)\right)} \\ & \times \exp\left\{ -\left[\mathbf{r}^H \mathbf{M}_{11}(\mathbf{A}, s)\mathbf{r} + \hat{\mathbf{c}}^H \mathbf{M}_{22}(\mathbf{A}, s)\hat{\mathbf{c}} + 2\,\mathrm{Re}\left(\hat{\mathbf{c}}^H \mathbf{M}_{12}^H(\mathbf{A}, s)\mathbf{r}\right)\right] \right\} \end{aligned}$$

- $\{M_{ij}(\mathbf{A}, s)\}$ are the blocks of the inverse of the covariance matrix of $\mathbf{r}, \hat{\mathbf{c}}$, and are given by

$$\mathbf{M}_{11}(\mathbf{A}, s) = \left(|s|^2 \mathbf{K}_c + \mathbf{A} + \sigma^2 \mathbf{I}_L - |s|^2 \mathbf{K}_c \left(\mathbf{K}_c + \frac{1}{\left(\mathbf{s}_p^H \mathbf{s}_p\right)} \left(\mathbf{A} + \sigma^2 \mathbf{I}_L\right) \right)^{-1} \mathbf{K}_c \right)^{-1},$$

$$\mathbf{M}_{22}(\mathbf{A}, s) = \left(\mathbf{K}_c + \frac{1}{\left(\mathbf{s}_p^H \mathbf{s}_p\right)} \left(\mathbf{A} + \sigma^2 \mathbf{I}_L\right) - |s|^2 \mathbf{K}_c \left(|s|^2 \mathbf{K}_c + \mathbf{A} + \sigma^2 \mathbf{I}_L \right)^{-1} \mathbf{K}_c \right)^{-1},$$

$$\mathbf{M}_{12}(\mathbf{A}, s) = -s \mathbf{M}_{11}(\mathbf{A}, s) \mathbf{K}_c \left(\mathbf{K}_c + \frac{1}{\left(\mathbf{s}_p^H \mathbf{s}_p\right)} \left(\mathbf{A} + \sigma^2 \mathbf{I}_L\right) \right)^{-1}$$

- The decision rule can be written as

$$\hat{s} = \arg \left\{ \max_{s \in \{S_1, \ldots, S_M\}} g(\mathbf{r}, \hat{\mathbf{c}}, \hat{\mathbf{A}} | s) \right\},$$

where the sufficient statistic

$$g(\mathbf{r}, \hat{\mathbf{c}}, \hat{\mathbf{A}} | s) = \int_{\mathbf{A}} f(\mathbf{r}, \hat{\mathbf{c}} | \mathbf{A}, s) f(\hat{\mathbf{A}} | \mathbf{A}) f(\mathbf{A}) d\mathbf{A}$$

- Numerical results for MPSK with $|s| = 1$,

$$(\mathbf{K}_c)_{k,l} = \begin{cases} P_S \rho^{|k-l|} & \text{if } \; k \neq l, \\ P_S & \text{if } \; k = l, \end{cases}$$

  P_S being the signal power, and

$$(\mathbf{K}_b)_{k,l} = \begin{cases} P_I \rho^{|k-l|} & \text{if } \; k \neq l, \\ P_I & \text{if } \; k = l \end{cases}$$

  P_I being the interference power

- Average SNR per branch: P_S/σ^2
  Average SIR per branch per interferer: P_S/P_I
  Pilot-to-noise ratio (PNR): $\mathbf{s}_p^H \mathbf{s}_p/\sigma^2$

- Comparison of ODC with OCI (OC using imperfect channel estimates), OC, and MRC

- In case of ODC, the sufficient statistic has been computed by Monte Carlo simulation
- Performance of ODC is in between that of OCI and OC
- At an SEP of 7×10^{-3}, ODC has an advantage of about 3 dB over OCI and a disadvantage of only about 1 dB compared to OC when $N = 2$
- OCI is about 2 times as complex as OC, while ODC is about 100 times as complex as OCI
- ODC can perform significantly better than OCI with appropriate choice of system parameters

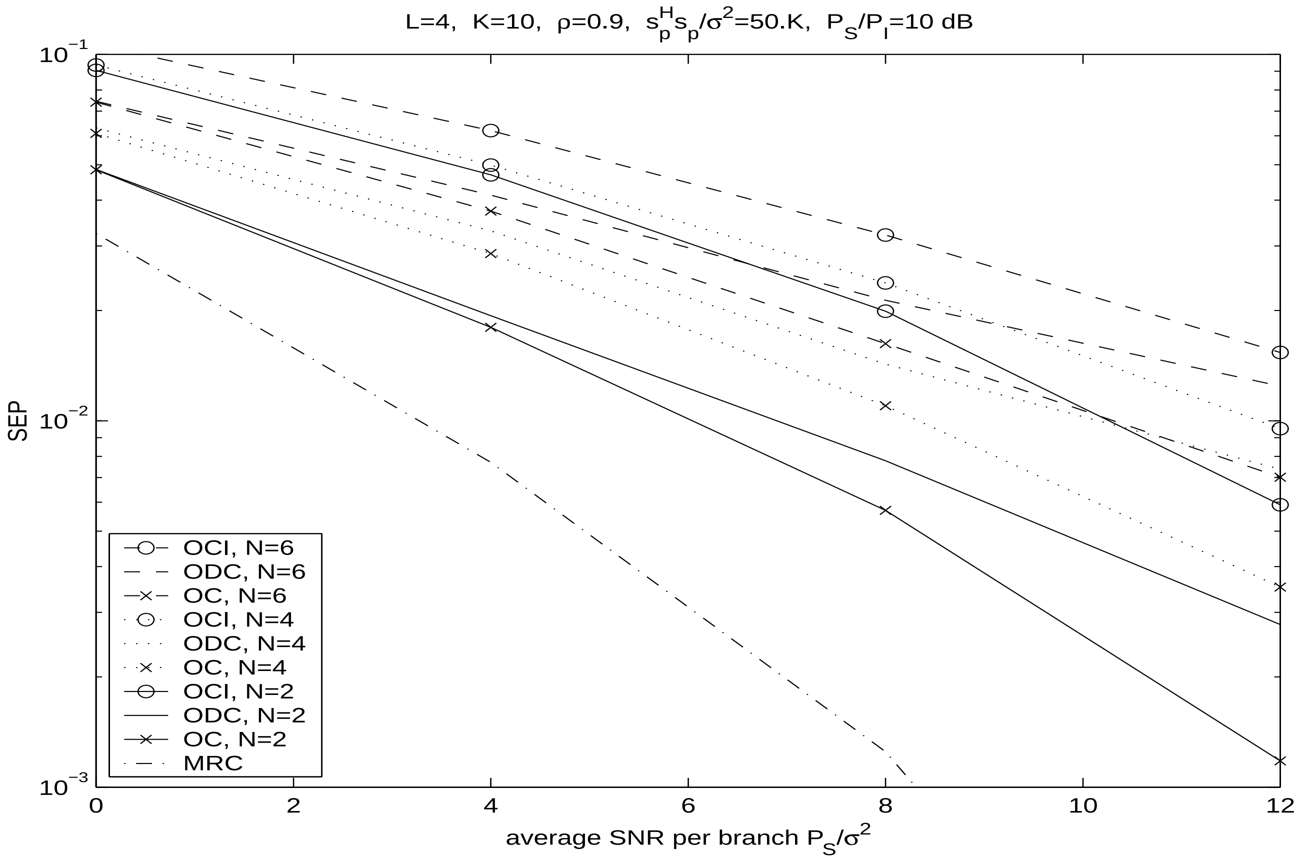

Figure 3: Comparison of error performance of ODC, OCI, and OC for BPSK with number of receive antennas $L = 4$, number of interferers $N = 2, 4, 6$, number of pilot symbols $K = 10$, exponentially correlated fading having $\rho = 0.9$, PNR $\mathbf{s}_p^H \mathbf{s}_p / \sigma^2 = 50 \cdot K$, and average SIR per branch per interferer $P_S/P_I = 10$ dB

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