

Grimm's Conjecture and Smooth numbers

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Abstract

Let $g(n)$ be the largest positive integer k such that there are distinct primes p_i for $1 \leq i \leq k$ so that $p_i | n + i$. This function is related to a celebrated conjecture of C.A. Grimm which is considered very difficult. In this talk, we present upper and lower bounds for $g(n)$ by relating its study to the distribution of smooth numbers. Standard conjectures concerning smooth numbers in short intervals imply $g(n) = O(n^\epsilon)$ for any $\epsilon > 0$. We also prove unconditionally that $g(n) = O(n^\alpha)$ with $0.45 < \alpha < 0.46$. The study of $g(n)$ has interesting implications for gaps between consecutive primes, results even better than those given by Riemann Hypothesis. This is a joint work with R. Murty.