
Contents

1	Statistical Preliminaries	1
1.1	Common Models	1
1.1.1	Exponential Families	4
1.1.2	Location-Scale Families	5
1.1.3	Regular Family	6
1.2	Likelihood Function	7
1.3	Sufficient Statistics and Ancillary Statistics	9
1.4	Three Basic Problems of Inference in Classical Statistics	11
1.4.1	Point Estimates	11
1.4.2	Testing Hypotheses	16
1.4.3	Interval Estimation	20
1.5	Inference as a Statistical Decision Problem	21
1.6	The Changing Face of Classical Inference	23
1.7	Exercises	24
2	Bayesian Inference and Decision Theory	29
2.1	Subjective and Frequentist Probability	29
2.2	Bayesian Inference	30
2.3	Advantages of Being a Bayesian	35
2.4	Paradoxes in Classical Statistics	37
2.5	Elements of Bayesian Decision Theory	38
2.6	Improper Priors	40
2.7	Common Problems of Bayesian Inference	41
2.7.1	Point Estimates	41
2.7.2	Testing	42
2.7.3	Credible Intervals	48
2.7.4	Testing of a Sharp Null Hypothesis Through Credible Intervals	49
2.8	Prediction of a Future Observation	50
2.9	Examples of Cox and Welch Revisited	51
2.10	Elimination of Nuisance Parameters	51

2.11	A High-dimensional Example	53
2.12	Exchangeability	54
2.13	Normative and Descriptive Aspects of Bayesian Analysis, Elicitation of Probability	55
2.14	Objective Priors and Objective Bayesian Analysis	55
2.15	Other Paradigms	57
2.16	Remarks	57
2.17	Exercises	58
3	Utility, Prior, and Bayesian Robustness	65
3.1	Utility, Prior, and Rational Preference	65
3.2	Utility and Loss	67
3.3	Rationality Axioms Leading to the Bayesian Approach	68
3.4	Coherence	70
3.5	Bayesian Analysis with Subjective Prior	71
3.6	Robustness and Sensitivity	72
3.7	Classes of Priors	74
3.7.1	Conjugate Class	74
3.7.2	Neighborhood Class	75
3.7.3	Density Ratio Class	75
3.8	Posterior Robustness: Measures and Techniques	76
3.8.1	Global Measures of Sensitivity	76
3.8.2	Belief Functions	81
3.8.3	Interactive Robust Bayesian Analysis	83
3.8.4	Other Global Measures	84
3.8.5	Local Measures of Sensitivity	84
3.9	Inherently Robust Procedures	91
3.10	Loss Robustness	92
3.11	Model Robustness	93
3.12	Exercises	94
4	Large Sample Methods	99
4.1	Limit of Posterior Distribution	100
4.1.1	Consistency of Posterior Distribution	100
4.1.2	Asymptotic Normality of Posterior Distribution	101
4.2	Asymptotic Expansion of Posterior Distribution	107
4.2.1	Determination of Sample Size in Testing	109
4.3	Laplace Approximation	113
4.3.1	Laplace's Method	113
4.3.2	Tierney-Kadane-Kass Refinements	115
4.4	Exercises	119

5	Choice of Priors for Low-dimensional Parameters	121
5.1	Different Methods of Construction of Objective Priors	122
5.1.1	Uniform Distribution and Its Criticisms	123
5.1.2	Jeffreys Prior as a Uniform Distribution	125
5.1.3	Jeffreys Prior as a Minimizer of Information	126
5.1.4	Jeffreys Prior as a Probability Matching Prior	129
5.1.5	Conjugate Priors and Mixtures	132
5.1.6	Invariant Objective Priors for Location-Scale Families	135
5.1.7	Left and Right Invariant Priors	136
5.1.8	Properties of the Right Invariant Prior for Location-Scale Families	138
5.1.9	General Group Families	139
5.1.10	Reference Priors	140
5.1.11	Reference Priors Without Entropy Maximization	145
5.1.12	Objective Priors with Partial Information	146
5.2	Discussion of Objective Priors	147
5.3	Exchangeability	149
5.4	Elicitation of Hyperparameters for Prior	149
5.5	A New Objective Bayes Methodology Using Correlation	155
5.6	Exercises	156
6	Hypothesis Testing and Model Selection	159
6.1	Preliminaries	159
6.1.1	BIC Revisited	161
6.2	P-value and Posterior Probability of H_0 as Measures of Evidence Against the Null	163
6.3	Bounds on Bayes Factors and Posterior Probabilities	164
6.3.1	Introduction	164
6.3.2	Choice of Classes of Priors	165
6.3.3	Multiparameter Problems	168
6.3.4	Invariant Tests	172
6.3.5	Interval Null Hypotheses and One-sided Tests	176
6.4	Role of the Choice of an Asymptotic Framework	176
6.4.1	Comparison of Decisions via P-values and Bayes Factors in Bahadur's Asymptotics	178
6.4.2	Pitman Alternative and Rescaled Priors	179
6.5	Bayesian P-value	179
6.6	Robust Bayesian Outlier Detection	185
6.7	Nonsubjective Bayes Factors	188
6.7.1	The Intrinsic Bayes Factor	190
6.7.2	The Fractional Bayes Factor	191
6.7.3	Intrinsic Priors	194
6.8	Exercises	199

7	Bayesian Computations	205
7.1	Analytic Approximation	207
7.2	The E-M Algorithm	208
7.3	Monte Carlo Sampling	211
7.4	Markov Chain Monte Carlo Methods	215
7.4.1	Introduction	215
7.4.2	Markov Chains in MCMC	216
7.4.3	Metropolis-Hastings Algorithm	218
7.4.4	Gibbs Sampling	220
7.4.5	Rao-Blackwellization	223
7.4.6	Examples	225
7.4.7	Convergence Issues	231
7.5	Exercises	233
8	Some Common Problems in Inference	239
8.1	Comparing Two Normal Means	239
8.2	Linear Regression	241
8.3	Logit Model, Probit Model, and Logistic Regression	245
8.3.1	The Logit Model	246
8.3.2	The Probit Model	251
8.4	Exercises	252
9	High-dimensional Problems	255
9.1	Exchangeability, Hierarchical Priors, Approximation to Posterior for Large p , and MCMC	256
9.1.1	MCMC and E-M Algorithm	259
9.2	Parametric Empirical Bayes	260
9.2.1	PEB and HB Interval Estimates	262
9.3	Linear Models for High-dimensional Parameters	263
9.4	Stein's Frequentist Approach to a High-dimensional Problem ..	264
9.5	Comparison of High-dimensional and Low-dimensional Problems	268
9.6	High-dimensional Multiple Testing (PEB)	269
9.6.1	Nonparametric Empirical Bayes Multiple Testing	271
9.6.2	False Discovery Rate (FDR)	272
9.7	Testing of a High-dimensional Null as a Model Selection Problem	273
9.8	High-dimensional Estimation and Prediction Based on Model Selection or Model Averaging	276
9.9	Discussion	284
9.10	Exercises	285

10	Some Applications	289
10.1	Disease Mapping	289
10.2	Bayesian Nonparametric Regression Using Wavelets	292
10.2.1	A Brief Overview of Wavelets	293
10.2.2	Hierarchical Prior Structure and Posterior Computations	296
10.3	Estimation of Regression Function Using Dirichlet Multinomial Allocation	299
10.4	Exercises	302
A	Common Statistical Densities	303
A.1	Continuous Models	303
A.2	Discrete Models	306
B	Birnbaum's Theorem on Likelihood Principle	307
C	Coherence	311
D	Microarray	313
E	Bayes Sufficiency	315
	References	317
	Author Index	339
	Subject Index	345