Data and models for problem 18 of Chapter 9

Burnham and Anderson (1998) (to be abbreviated below as B-A) generated data to mimic the experiment presented by Stromborg et al. (1988). The experiment is designed to study the survival effect of a pesticide administered to nestling European starlings in an island. All birds under study are leg-banded with uniquely numbered coloured bands. Half of these birds are randomly assigned to a treatment group and receive a dose of pesticide and the remaining birds are assigned to a control group. Birds in these two groups are believed to be under otherwise very similar conditions. After a 4-day period following the dosage, all birds are released. Surviving starlings are potentially resighted and resighting efforts are made on a day (say, Friday) in each of the following few weeks. The birds captured in a week are released again. For more details of the experiment and the data see B-A and the references therein.

B-A generated data with 300 birds in both the treatment and control groups with 8 resighting occasions – weeks $2, \cdots, 9$. Thus the birds are released on 8 occasions – weeks $1, \cdots, 8$. Table 1 below is reproduced from B-A and presents the data as a matrix $(n(i,j))$ for both the groups where $n(i,j)$ denotes the number of birds released at week i and captured in week j for the first time after week i ($i = 1, \cdots, 8,\ j = i+1, \cdots, 9$). It also gives the total number $(N(i))$ of birds released at week i ($i = 1, \cdots, 8$).

Table 1: The starling data presented as a matrix $(n(i,j))$ for both the treatment and control groups where $(n(i,j))$ denotes the number of birds first captured in week j after last being released at time i $(i = 1,\cdots,8,\ j = i+1,\cdots,9)$ and the total number $(N(i))$ of birds released at week i $(i = 1,\cdots,8)$.

		Recaptures for Treatment Group							
Week	$N(i)$	$n(i,j)$							
		$j=2$	3	4	5	6	7	8	9
1	300	158	43	15	5	0	0	0	0
2	158		82	23	7	1	1	0	0
3	125			69	17	6	1	0	0
4	107				76	8	2	0	0
5	105					67	20	3	0
6	82						57	14	1
7	81							53	12
8	70								46

		Recaptures for Control Group							
Week	$N(i)$	$n(i,j)$							
		$j=2$	3	4	5	6	7	8	9
1	300	210	38	5	1	0	0	0	0
2	210		157	20	8	2	0	0	0
3	195			138	24	2	1	0	0
4	163				112	24	2	0	0
5	145					111	16	6	0
6	139						105	16	4
7	124							93	12
8	115								89

A product multinomial model is assumed for the given data. Each row corresponds to a multinomial distribution. As denoted in B-A, let ϕ_{ti} and ϕ_{ci} be the conditional probabilities of survival from week i to $i+1$ $(i = 1,\cdots,8)$ for the treatment group and the control group respectively and p_{ti} and p_{ci} be the conditional probabilities of resighting at week i, $i = 2,\cdots,9$. The possible models are

$$M_{r,s}: \quad \phi_{t1} \neq \phi_{c1}, \cdots, \phi_{tr} \neq \phi_{cr},\ \phi_{ti} = \phi_{ci},\ i = r+1, \cdots, 8$$
$$p_{t2} \neq p_{c2}, \cdots, p_{t,s+1} \neq p_{c,s+1},\ p_{tj} = p_{cj},\ i = s+2, \cdots, 9$$
$$r = 0, \cdots, 8,\ s = 0, \cdots, 8,\ r = s, s+1.$$

For example, the hypothesis of no treatment effect corresponds to the model $M_{0,0}$. For discussion on these models and other related matters see B-A where some other models employing transformations on the parameters are also considered. We, however, restrict only to the above models for the sake of simplicity. For Problem 18 of Chapter 9, consider only the models $M_{0,0}, M_{1,0}, M_{1,1}, M_{2,1}, \ldots, M_{4,4}$.

It is to be noted that the parameters ϕ_8 and p_9 are not separately estimable, only the product $\phi_8 p_9$ is estimable, so this product is to be treated as a single parameter. Therefore, e.g., for Model $M_{0,0}$, the number of estimable parameters is taken to be 15.