Morphologic Regularization for inverse problems

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Classical inverse problems:

In the inverse problems, the relation between the observed image **Y** and the desired sharp image **X** can be represented as

$$\mathbf{Y} = H\mathbf{X} + \eta \tag{1}$$

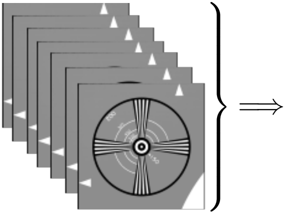
where **Y**, **X** and η represent lexicographically ordered column vectors of the observed image, desired sharp image and the additive noise respectively.

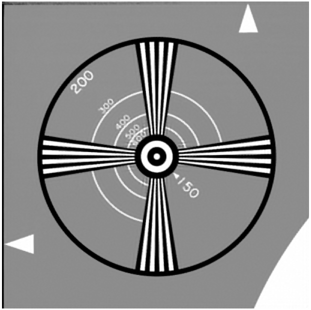
Classical inverse problems:

- Inverse problem mainly used for image restoration.
- Super resolution and deblurring are the most common classical inverse problems in image processing.

What is Super Resolution?

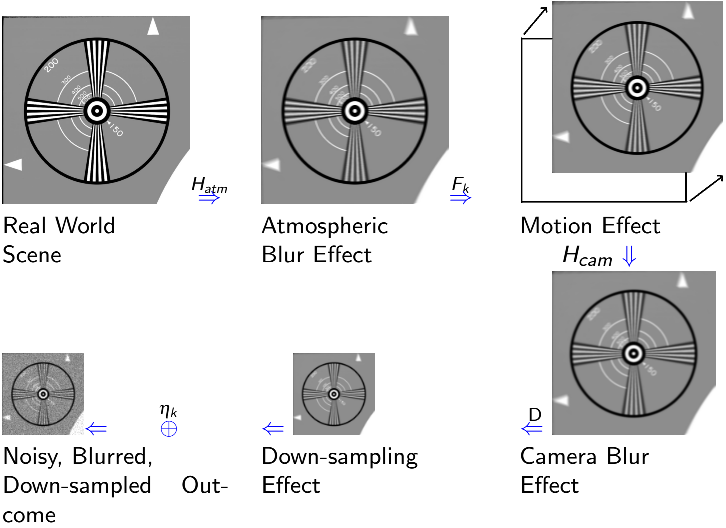
Definition: Super-resolution is largely known as a technique whereby multi-frame motion is used to overcome the inherent resolution limitations of a low resolution camera system.





Estimated High Resolution Image [Purkait and Chanda, 2012].

Block diagram representation



Model Construction

$$\mathbf{Y}_k = DF_k H\mathbf{X} + \eta_k, \qquad \forall k = 1, 2, ..., K$$

Known:	Symbols:	Represents:
✔	$\mathbf{Y}_k$	k^{th} LR Image
✘	$\mathbf{X}$	HR Image ⇐ Predict
✔	D	Decimation Matrix
✔	F_k	Geometric Transformation Matrix [a]
✔	H	Blurring Kernel
✘	η_k	Random Noise
✔	K	Number of LR images

[a] Vandewalle et al., A frequency domain approach to registration of aliased images with application to super-resolution, EURASIP, 2006.

Compact Representation:

- SR reconstruction Model:

$$\mathbf{Y}_k = A_k\mathbf{X} + \eta_k, \qquad \forall k = 1, 2, ..., K$$

  This can be organized and rewritten as follows [Purkait and Chanda, 2012]

$$\mathbf{Y} = RA\mathbf{X} + \eta$$

  where R is an index matrix that incorporates the sub-pixel shifts of all LR images.

- Deblurring problem can be written as

$$\mathbf{Y} = A\mathbf{X} + \eta$$

  where A is the blurring Kernel and η is the atmospheric noise.

- Therefore those classical inverse problem can be written in generalized form

$$\mathbf{Y} = H\mathbf{X} + \eta$$

Solution of classical inverse problems:

Frequency domain

Wiener filter: most popular one that minimizes the mean square error.

$$\hat{X}(k,\ l) = \frac{H(k,\ l)}{|H(k,\ l)|^2 + S_\eta(k,\ l)/S_x(k,\ l)} \quad (2)$$

Spatial domain

Regularized LSE: solves an unconstrained optimization problem, such as

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \{\frac{1}{2}\|H\mathbf{X} - \mathbf{Y}\|_2^2 + \lambda \Upsilon(\mathbf{X})\} \quad (3)$$

where λ is the regularization parameter that controls the emphasis between the data error and the regularization term $\Upsilon(\mathbf{X})$.

♯ $\Upsilon(\mathbf{X})$ mostly represent the smoothness criterion by means of energy in first or second order image derivative.

Solution of classical inverse problems:

- Frequency domain methods are fast however it is impossible to incorporate complicated nonlinear prior knowledge.
- In this investigation we concentrate only on regularization based spatial domain iterative algorithms which solve the minimization problem

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \{ \| H\mathbf{X} - \mathbf{Y} \|_p^p + \lambda \Upsilon(\mathbf{X}) \} \qquad (4)$$

  where $1 \leq p \leq 2$, $\Upsilon(\mathbf{X}) = \|\Gamma(\mathbf{X})\|$ and Γ is a high frequency operator.
- The first term (data error term) takes care of generating high frequency components whereas the second term (smoothness term) would prevent from the inclusion of any ringing artifacts.
- We will mainly concentrate on the smoothness term.

Regularization Techniques

General inverse image reconstruction Model:

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \left[\|H\mathbf{X} - \mathbf{Y}\|_p^p + \lambda \Upsilon(\mathbf{X}) \right]$$

where $1 \leq p \leq 2$, $\Upsilon(\mathbf{X}) = \|\Gamma(\mathbf{X})\|$ and Γ is a high frequency operator.

Different types of regularization:

- $\Upsilon(\mathbf{X}) = |\nabla \mathbf{X}|_2^2$ known as bounded variation (BV) regularization.
- $\Upsilon(\mathbf{X}) = |\nabla \mathbf{X}|_1$ known as total variation (TV) regularization.
- $\Upsilon(\mathbf{X}) = \sum_{l=-w}^{w} \sum_{m=-w}^{w} \alpha^{|l|+|m|} |\mathbf{X} - S_x^l S_y^m \mathbf{X}|_l$ known as bilateral total variation (BTV) regularization, where $l + m \geq 0$ and S_x^l, S_y^m are shift-operators along x and y directions with l and m pixel respectively.

Explicit Forms of Γ

- BV / TV : $\Gamma = \nabla_x + \nabla_y$

$$= \begin{array}{|c|c|}\hline -1 & 1 \\ \hline \end{array} + \begin{array}{|c|}\hline -1 \\ \hline 1 \\ \hline \end{array}$$

- BTV : $\Gamma = \alpha \left(\begin{array}{|c|c|}\hline -1 & 1 \\ \hline \end{array} + \begin{array}{|c|}\hline -1 \\ \hline 1 \\ \hline \end{array} \right)$

$$+\alpha^2 \left(\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline \end{array} + \begin{array}{|c|}\hline -1 \\ \hline 0 \\ \hline 1 \\ \hline \end{array} \right)$$

$$\dots$$

Other existing Regularization Techniques

- Non-local means (NLM) [Zhang et al., 2010]:
  - based on the idea that if a patch occurs inside an image, it is more probable that the same patch would occur at other location within that image.
  - It smoothen the patch according to photometric similarity with the available patches.
- Steering kernel regression (SKR) [Takeda et al., 2008]:
  - based on structure tensor and smoothen the image along the edges and preserve the image details across the edges during the image reconstruction.

Morphological operators as Regularization term

- Most of the existing regularization techniques lead to a stable solution.
- However, developing an edge-preserving regularization method that can suppress noise in degraded images and the ringing artifacts evolved during reconstruction of image without sacrificing edges is yet to achieve.
- It is always interesting to explore different kind of regularization methods for betterment.

Our Investigation

Inverse reconstruction Model:

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \left[\|H\mathbf{X} - \mathbf{Y}\|_p^p + \lambda \Upsilon(\mathbf{X}) \right]$$

We investigate on following:

- We use multi-scale morphological gradient operator to develop the regularization term $\Upsilon(\mathbf{X})$ and then solve the convex problem by formulating efficient sub-gradients of morphological operators.
- We analyze the different regularization methods and show that they all emerge from the same concept involving low-pass filtering which is basically smoothness prior.
- Finally, combining all these, we devise two more adaptive regularization methods as a prior for classical inverse problems.

Morphological operators as Regularization term

Morphological operators as Regularization term

Motivation:

- Existing regularization techniques are only based on direct image gradients.
- Exploring different regularization technique is always interesting.
- Multi-scale morphology is used for structure preserving restoration techniques for image processing successfully, suggesting that it could be a great choice.

Difficulty:

- Highly non-linearity makes the problem difficult to handle.
- Needs to calculate the subgradients for morphological operators.
- Slower than calculating direct derivatives.

Morphological operators as Regularization term

Difficulty: Possible Solution

- Highly non-linearity makes the problem difficult to handle.
  # Non-linear is good because it's locally adaptive property can preserve edges more efficiently.
- Needs to calculate the subgradients for morphological operators.
  # We calculate Morphological subgradients efficiently
- Slower than calculating direct derivatives.
  # We develop advanced Bregman iteration for classical inverse problems for proposed regularization instead of gradient descent.

Proposed Method

Here we solve the following inverse problem

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \{ \frac{1}{2} \| H\mathbf{X} - \mathbf{Y} \|_2^2 + \lambda \Upsilon(\mathbf{X}) \},$$

with the proposed Morphological Regularization as follows:

$$\Upsilon(\mathbf{X}) = \sum_{s=1}^{S} \alpha^s \mathbf{1}^t [C_s(\mathbf{X}) - O_s(\mathbf{X})]$$

where $O_s(\mathbf{X}) = D_s(E_s(\mathbf{X})),\ C_s(\mathbf{X}) = E_s(D_s(\mathbf{X}))$.

$$D_s(\mathbf{X}) = \begin{pmatrix} \max_{r \in (sB)_{(1)}} \{x_r\} \\ \max_{r \in (sB)_{(2)}} \{x_r\} \\ \ddots \\ \max_{r \in (sB)_{(mn)}} \{x_r\} \end{pmatrix}, \quad E_s(\mathbf{X}) = \begin{pmatrix} \min_{r \in (sB)_{(1)}} \{x_r\} \\ \min_{r \in (sB)_{(2)}} \{x_r\} \\ \ddots \\ \min_{r \in (sB)_{(mn)}} \{x_r\} \end{pmatrix}$$

Recently developed solver

Bregman Iteration:

Consider the following minimization problem:

$$\min_{\mathbf{X}}\{\Upsilon(\mathbf{X}) \ : \ T(\mathbf{X})=0\}$$

Now the Bregman iterations [Marquina and Osher, 2008] that solve the above constrained minimization problem are as follows:

Initialize $\mathbf{X}^0 = p^0 = \mathbf{0}$

$$\begin{cases} \mathbf{X}^{(n+1)} = \arg\min_{\mathbf{X}}\{\lambda B_{\Upsilon}^{p^{(n)}}(\mathbf{X},\mathbf{X}^{(n)}) + T(\mathbf{X})\} \\ p^{(n+1)} = p^{(n)} - \nabla T(\mathbf{X}^{(n+1)}) \end{cases}$$

where $B_{\Upsilon}^{p^{(n)}}$ is the Bregman distance corresponding to convex functional $\Upsilon(.)$ and is defined from point $\mathbf{X}$ to point $\mathbf{V}$ as

$$B_{\Upsilon}^{p}(\mathbf{X},\mathbf{V}) = \Upsilon(\mathbf{X}) - \Upsilon(\mathbf{V}) - \langle p, \mathbf{X}-\mathbf{V}\rangle$$

Proximal Map:

Consider the following unconstrained minimization problem:

$$\min_{\mathbf{X}} (\lambda \Upsilon(\mathbf{X}) + T(\mathbf{X}))$$

The solution satisfies the condition:
$\lambda \partial \Upsilon(\mathbf{X}) + \partial T(\mathbf{X}) = 0 \Rightarrow (\mathbf{X} + \gamma\lambda \partial \Upsilon(\mathbf{X})) - (\mathbf{X} - \gamma \partial T(\mathbf{X})) = 0$
This leads to a forward and backward splitting algorithm:

$$\mathbf{X}^{(k+1)} = Prox_{\Upsilon}(\mathbf{X}^{(k)} - \gamma \partial T(\mathbf{X}^{(k)})),$$

where the proximal operator $Prox_{\Upsilon}(\mathbf{V})$ is defined as:

$$Prox_{\Upsilon}(\mathbf{V}) = \arg\min_{\mathbf{X}} \{\lambda \Upsilon(\mathbf{X}) + \frac{1}{2\gamma} \|\mathbf{X} - \mathbf{V}\|_2^2\}$$

Combining Proximal map and Bregman iterations

Solution of the inverse problem with the proposed regularization:
We solve the following inverse problem

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \{ \frac{1}{2} \| H\mathbf{X} - \mathbf{Y} \|_2^2 + \lambda \Upsilon(\mathbf{X}) \}$$

The proposed image reconstruction algorithm combining Bregman iteration and operator splitting as follows:

Proposed Iterative Algorithm:

Initialize $\mathbf{B}^{(0)} = n = 0, \mathbf{X}^{(0)} = \mathbf{Y}$;

$$\begin{aligned}
&\textbf{While}(\| H\mathbf{X}^{(n)} - \mathbf{Y} \|_2^2 > \eta) \\
&\quad \begin{cases} \mathbf{U}^{(n+1)} = \mathbf{X}^{(n)} - \gamma H^T (H\mathbf{X}^{(n)} - \mathbf{B}^n) \\ \mathbf{X}^{(n+1)} = \mathbf{U}^{(n+1)} - \lambda' | \frac{\delta \Upsilon(\mathbf{X})}{\delta \mathbf{X}} |_{\mathbf{X}^{(n)}} \\ \mathbf{B}^{(n+1)} = \mathbf{B}^{(n)} + (\mathbf{Y} - H\mathbf{X}^{(n+1)}) \end{cases} \\
&\quad n = n + 1 \\
&\textbf{end}
\end{aligned} \quad (5)$$

Efficient computation of $\left|\frac{\delta\Upsilon(\mathbf{X})}{\delta\mathbf{X}}\right|$*: Using Chain Rule*

$\Upsilon(\mathbf{X}) = \sum_{s=1}^{S} \alpha^s \mathbf{1}^t [C_s(\mathbf{X}) - O_s(\mathbf{X})]$,
where $O_s(\mathbf{X}) = D_s(E_s(\mathbf{X})),\ C_s(\mathbf{X}) = E_s(D_s(\mathbf{X}))$

$$D_s(\mathbf{X}) = (\mathbf{X} \oplus B) \oplus \cdots \oplus B = D(D(\ldots \text{ s times } \ldots D(\mathbf{X}) \ldots))$$

$$\begin{aligned} \frac{\delta}{\delta\mathbf{X}} D_s(\mathbf{X}) &= \frac{\delta}{\delta\mathbf{X}}[D(D_{s-1}(\mathbf{X}))] \\ &= \frac{\delta D_s}{\delta D_{s-1}} \frac{\delta D_{s-1}}{\delta D_{s-2}} \ldots \frac{\delta D_1(\mathbf{X})}{\delta\mathbf{X}} \end{aligned}$$

Let $\mathbf{Z}^{d_1} = \frac{\delta D}{\delta\mathbf{X}}\mathbf{1}$ and $\mathbf{Z}^{e_1} = \frac{\delta E}{\delta\mathbf{X}}\mathbf{1}$

$$\begin{aligned} \mathbf{Z}^{d_s} := \frac{\delta}{\delta\mathbf{X}}[D_s(\mathbf{X})]\mathbf{1} &= \frac{\delta D_s}{\delta D_{s-1}} \mathbf{Z}^{d_{s-1}} \\ &= \frac{\delta D(D_{s-1})}{\delta D_{s-1}} \mathbf{Z}^{d_{s-1}} \end{aligned}$$

$\frac{\delta D_1(\mathbf{X})}{\delta\mathbf{X}}$ is a subgradients of max operator computed easily and then propagate in higher scale.

Experiment I: Deblurring

Figure: Illustrates results of deblurring technique with various regularization methods. Top row: noisy and blurred input image and TV regularization (PSNR = 29.93dB, SSIM = 0.9224). Bottom row: using BTV (PSNR = 29.95dB, SSIM = 0.9224) and Morphologic regularization (PSNR = 30.05dB, SSIM = 0.9248).

Experiment II: SR image reconstruction

Figure: Illustrates results of SR technique with various regularization methods. Top row: SR image using Bicubic interpolation and TV regularization (PSNR = 29.43dB, SSIM = 0.9077). Bottom row: using BTV (PSNR = 29.47dB, SSIM = 0.9080) and proposed Morphologic regularization (PSNR = 29.55dB, SSIM = 0.9106).

More study on morphologic regularization: Adaptive Regularization

More study on morphologic regularization

- Ideal regularizer is expected to capture only noise present in the image so that minimizing $\Upsilon(\mathbf{X})$ leads to suppression of noise while keeping sharp edges unaltered.
- It would be invariant to low pass filtering that can remove noise and preserve all the image details.(smoothness prior)
- Among existing regularizers, the morphologic regularizer is the closest to the ideal, as the open and close filters remove noise smaller than SE and preserve edges upto the curvature of SE.

Regularizer and smoothing kernel

Let $K = (\alpha_{lm} | l, m = -w : w)$ be the low pass filtering kernel. Then,

$$\begin{aligned} \alpha_{lm} &= 1/S, \text{ for average filter} \\ &= \frac{1}{S} \exp^{-\frac{l^2+m^2}{2\sigma^2}}, \text{ for gaussian filter} \end{aligned}$$

An ideal image prior would be invariant to low pass filtering that can remove noise and preserve all the image details.

Then the regularization term may be defined as

$$\begin{aligned} \Upsilon(\mathbf{X}) &= \|\mathbf{X} - K\mathbf{X}\|_p \\ &= \|\mathbf{X} - \textstyle\sum_{l,m \in W} \alpha_{lm} S_x^l S_y^m \mathbf{X}\|_p \\ &= \|\textstyle\sum_{l,m \in W} \alpha_{lm}(\mathbf{X} - S_x^l S_y^m \mathbf{X})\|_p \text{ (since } \textstyle\sum_{l,m \in W} \alpha_{lm} = 1) \\ &\leq \textstyle\sum_{l,m \in W} \alpha_{lm} \|\mathbf{X} - S_x^l S_y^m \mathbf{X}\|_p \text{ (Jensen's Inequality), for } p > 1 \end{aligned}$$

(Minimizing this results output invariant to low-pass filtering K)

Regularizer and smoothing kernel

For 4-N neighbourhood:

$$\begin{aligned}\Upsilon(\mathbf{X}) &= \sum_{l,m\in N_g} \alpha_{lm} \|\mathbf{X} - S_x^l S_y^m \mathbf{X}\|_p \\ &= 2\|(\nabla_x + \nabla_y)\mathbf{X}\|_p \\ &= \|\nabla \mathbf{X}\|_p\end{aligned}$$

- Advantage of considering $\|\nabla \mathbf{X}\|_p$ or $\sum_{l,m\in W} \alpha_{lm} \|\mathbf{X} - S_x^l S_y^m \mathbf{X}\|_p$ as regularization term is that it essentially minimizes $\|\mathbf{X} - K\mathbf{X}\|_p$ which is our smoothness prior.
- Minimization of TV and BTV regularization try to generate an image which is invariant to some smoothing operators and the data term of is responsible to generate high frequency components of the image. If the data-term doesn't present then ideally it would produce a smooth flat image.

Hence, an attractive alternate regularizer could be develop using an edge preserving efficient spatial filter.

Spatial image filtering techniques:

- Most of the traditional filters use a symmetric non-adaptive kernel. Those filters are able to remove noise and artifacts present in the image, but also blur the edges and wipe out textures.
- Tomasi and Manduchi [1998] used adaptive kernel for edge preserving image smoothing, known as bilateral filtering, given by

$$\alpha_{ij,\,lm} = \exp^{-\left\{\frac{l^2+m^2}{2\sigma_e^2}+\frac{(X(i+l,j+m)-X(i,j))^2}{2\sigma_s^2}\right\}}$$
$$= \exp^{-\left\{\frac{l^2+m^2+\lambda(X(i+l,j+m)-X(i,j))^2}{2\sigma_e^2}\right\}} = \exp^{-\left\{\frac{D_{ij,\,lm}^2}{2\sigma_e^2}\right\}}$$

where σ_e is the scale parameter.

  - λ controls the emphasis between spatial and spectral difference.
  - This kernel may also be viewed as the Gaussian of Euclidean distance $D_{ij,\,lm}$ between value of the pixels.

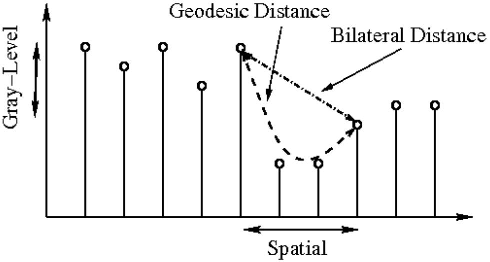
Edge-preserving morphological operators

- Various nonlinear edge preserving morphological smoothing algorithms are also available in the literature [Lerallut et al., 2007, Soille, 1994].
- Lerallut et al. [2007] define adaptive morphological operators, called amoeba filter, where shape of the structuring element varies from pixel to pixel, determined by thresholding the geodesic distance $D_{ij,\,lm}$ of the pixel $(i+l, j+m)$ from the center pixel (i,j).
- Morphologically viewed, an image may be considered as a topographic surface with (i,j) as spatial location and $X(i,j)$ as altitude. The geodesic distance in an image between pixels (i,j) and $(i+l, j+m)$ is defined as the length of the shortest path between them along the topographic surface.

Geodesic Kernel Smoother

Existing approximate Kernels:

- LARK Kernel [Takeda, *et al.* '07]
- Beltrami Kernel [Sochen, *et al.* '98]
- Structure Tensor [Brox, *et al.* '04]



The values of the kernel are computed as:

$$K(x,y) = e^{-\frac{distance(x,y)^2}{\sigma^2}}$$

Figure: Illustrates different kernel representations.

Geodesic Regularization

Motivation/Idea:

- Optimal Regularization is always desired.
- The relationship between the non-linear filtering and regularization technique suggest that an edge preserving filter could be good regularizer.
- The edge preserving nature of the most efficient nonlinear Geodesic kernel based filtering suggests that it can preserve better edge structure during regularization.

Difficulty:

- Non-linear nature results subgradients difficult to compute.
- Usual Geodesic distance computation is slow.

Challenge for geodesic kernel regularization

Difficulty: Possible Solution

- Non-linear nature results subgradients difficult to compute.
  # We use the same idea to compute subgradients as we did for Morphologic Regularization.
- Usual Geodesic distance computation is slow.
  # We develop an efficient algorithm to compute Geodesic distance. We also derive split-Bregman iterations for classical inverse problems using geodesic kernel regularization.

Proposed Morphologic Regularization

- We define more general adaptive regularization as

$$\Upsilon(\mathbf{X}) = \sum_{l,m \in W} \alpha_{lm} |\nabla^{lm}| \quad (6)$$

where α_{lm} is an adaptive kernel that varies from pixel to pixel.
$|\nabla^{lm}| = \|\mathbf{X} - S_x^l S_y^m \mathbf{X}\|_p$ or $|\nabla^{lm}| = [C_s(\mathbf{X}) - O_s(\mathbf{X})]$.

- In essence, a regularization term has two components: a derivative operator ∇^{lm} and a smoothing kernel α_{lm}. The elements of smoothing kernel α_{lm} decreases with geodesic distance from the candidate pixel (i, j).

Topographic morphology based regularization

Based on the above study on the kernels, we can choose an efficient adaptive kernel in the following form:

$$\alpha_{ij,\,lm} = \frac{1}{S_{ij}} \exp\{-\frac{D^p_{ij,\,lm}}{2\sigma^p}\} \tag{7}$$

where $D^p_{ij,\,lm}$ is the geodesic distance of the pixels (i,j) and $(i+l, j+m)$.

We propose a geodesic regularizer as follows:

$$\Upsilon(\mathbf{X}) = \sum_{l,m\in W} \alpha_{lm} |\mathbf{X} - S^l_x S^m_y \mathbf{X}| \tag{8}$$

Morphologic filter based regularization

We define an adaptive structuring element

$$a_{ij,\,lm} = \kappa(\alpha_{ij,\,lm} - 1),$$

where κ is a constant and is fixed over all the pixels.
Accordingly the adaptive morphological dilation is defined as:

$$(X \oplus A)(i,j) = \sup_{lm \in W} \{X(i-l, j-m) + a_{ij,\,lm}\}.$$

And adaptive morphological erosion is defined as:

$$(X \ominus A)(i,j) = \inf_{lm \in W} \{X(i+l, j+m) - a_{ij,\,lm}\}$$

Morphologic filter based regularization

Based on the above adaptive morphological operators, we propose adaptive morphologic regularizer as:

$$\Upsilon(\mathbf{X}) = \sum_{l,m \in W} \alpha_{lm} [C_a(\mathbf{X}) - O_a(\mathbf{X})] \tag{9}$$

where $C_a(\mathbf{X})$ and $O_a(\mathbf{X})$ are adaptive closing and opening operators.

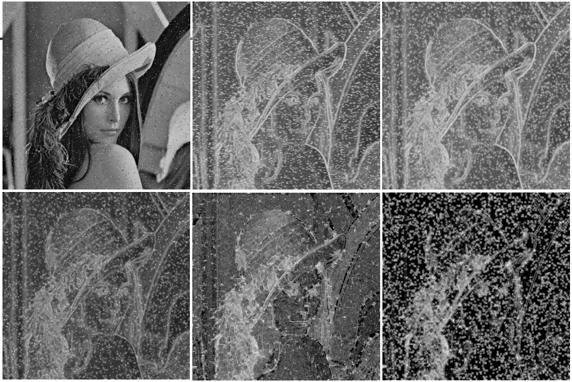


Figure: Illustrates different regularization methods. Top row: noisy input image with salt and paper noise (2%), absolute value of the high frequency operators used for TV and BTV regularization. Bottom row: same used for geodesic, morphologic, and adaptive morphologic regularization method.

Geodesic distance computation

There are two main kinds of algorithms for computing Geodesic distance: raster-scan and wave-front propagation.

- Raster-scan algorithms: based on kernel operations applied sequentially over the image in multiple passes [Borgefors, 1986]. Those methods are the extensions of Dijkstra algorithm for two dimensional grids.
- Wave-front algorithms: based on the iterative propagation of a pixel front with certain velocity. e.g., Fast Marching Method (FMM) [Sethian, 1999].

Geodesic versions of both kinds of algorithms may be found in Toivanen [1996] and Yatziv et al. [2006].

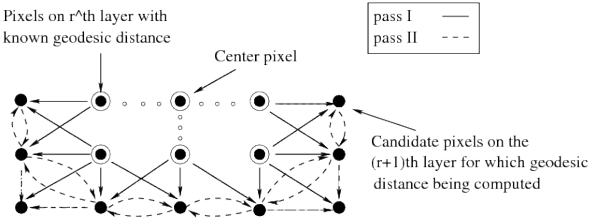
Geodesic distance computation

- Most of the existing geodesic distance computation techniques are developed for generating a distance transformation map for a gray-level image over a binary image.
- Whereas, in the present work, we need to compute the geodesic distance of every pixel within W from the central pixel.
- So here, we develop an efficient two-pass iterative raster-scan algorithm which is an extension of Dijkstra algorithm.

The geodesic distance between two pixels (i,j) and $(i+l,j+m)$ is defined as:

$$D_{ij,\,lm} = \min_{\Pi}\left[\sum_{k=1}^{n-1}\{1+\gamma|X(x_i)-X(x_{i+1})|\}\right]$$

where $\Pi(ij,lm) = \{(i,j)=x_1,x_2,\ldots,(i+l,j+m)=x_n\}$ is a path in a connected domain between the pixels x_k and x_{k+1}.



Algorithm

For central pixel (i, j), initialize $r = 0$, $Geo_{ij} = 0$
While r $<$ w iterate Pass I and Pass II.

- **Pass I** : Compute interim geodesic distances of the pixel (l, m) on the $(r+1)^{th}$ layer from the central pixel along the possible shortest paths through the inner r^{th} layer.
  - Let $N_{lm}^{(r)}$ be the neighborhood of (l, m) consisting of adjacent pixels on the r^{th} layer and consider it as a flat adaptive structuring element at (l, m).
  - Interim geodesic distance of pixel (l, m) from the center of the window by eroding with $N_{lm}^{(r)}$ as

$$Geo_{lm}^{tmp} = \min_{(u,v) \in N_{lm}^{r}} \{Geo_{uv} + \gamma \nabla_{lm}^{uv} + s\}$$

Algorithm

- **Pass II** : Compute actual geodesic distance of the pixel (l, m) on the $(r+1)^{th}$ layer using interim geodesic distances from Pass I.
  - Let $N_{lm}^{(r+1)}$ be the neighborhood consisting of adjacent pixels in the $(r+1)^{th}$ layer and consider it as a flat adaptive SE at (l, m).
  - Actual geodesic distance of pixel (l, m) from the center of the window by eroding with N_{lm}^{r+1} as

$$Geo_{lm} = \min \left[Geo_{lm}^{tmp}, \min_{(u,v) \in N_{lm}^{(r+1)}} \{ Geo_{uv}^{tmp} + \gamma \nabla_{lm}^{uv} + s \} \right]$$

Now, we can compute kernel matrix $K = [\alpha_{ij,\, lm}]$ as in (7) with $D_{ij,\, lm} = Geo_{lm}$.

Experiment I: Deblurring

Figure: Illustrates results of deblurring technique with various regularization methods. Top row: noisy and blurred input image, TV (PSNR = 29.93dB, SSIM = 0.9224) and BTV (PSNR = 29.95dB, SSIM = 0.9224) regularization. Bottom row: using geodesic (PSNR = 29.97dB, SSIM = 0.9228), Morphologic (PSNR = 30.05dB, SSIM = 0.9248), and Adaptive morphologic method (PSNR = 30.10dB, SSIM = 0.9251).

Experiment II: Super Resolution

Figure: Illustrates results of SR technique with various regularization methods. Top row: SR image using Bicubic interpolation, TV (PSNR = 29.43dB, SSIM = 0.9077) and BTV (PSNR = 29.47dB, SSIM = 0.9080) regularization. Bottom row: using geodesic (PSNR = 29.48dB, SSIM = 0.9082), Morphologic (PSNR = 29.55dB, SSIM = 0.9106), and Adaptive morphologic method (PSNR = 29.59dB, SSIM = 0.9107).

Conclusion

- We are the first to explore mathematical morphology in the field of regularization to solve classical inverse problems in image processing.
- We analyze different regularization methods used in this domain and found that they come from the same concept of edge preserving smoother.
- We have proposed two geodesic kernel based regularization for a general inverse problem of image reconstruction.

Conclusion

- We propose a new robust adaptive geodesic regularization and adaptive morphologic regularization methods that can suppress the noise more efficiently while preserving the edges.
- Our experimental section shows that it works quite well, in fact better than existing techniques.
- The adaptive regularization method proposed here are tested for two classic inverse problems, viz, deblurring and SR reconstruction problem, but one can easily extend this work to other applications as well.

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Thank You