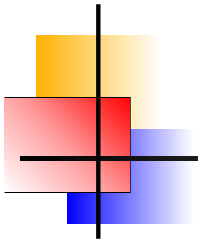

Efficient priors for Multi-frame and Single-frame Super Resolution

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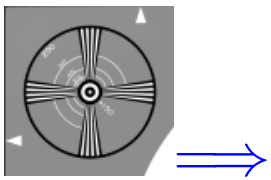
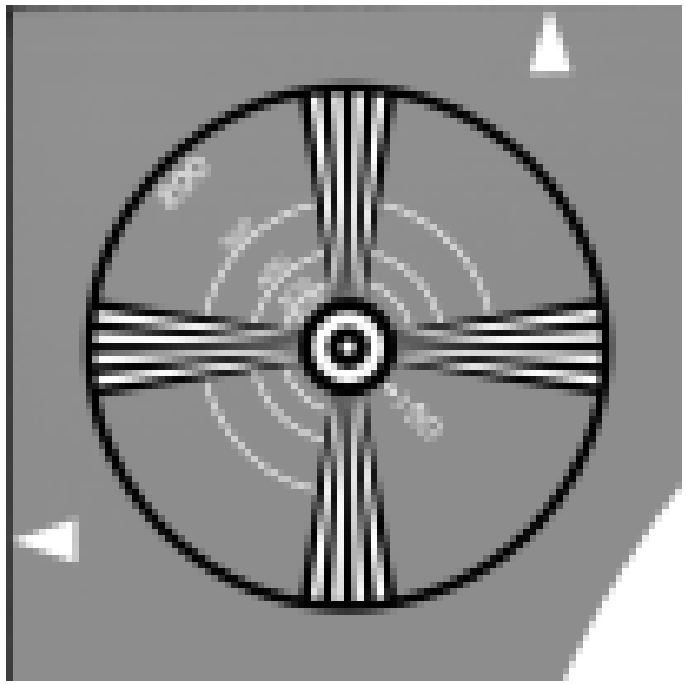
August 11, 2012

What is Super Resolution?

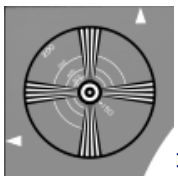


LR Image 1

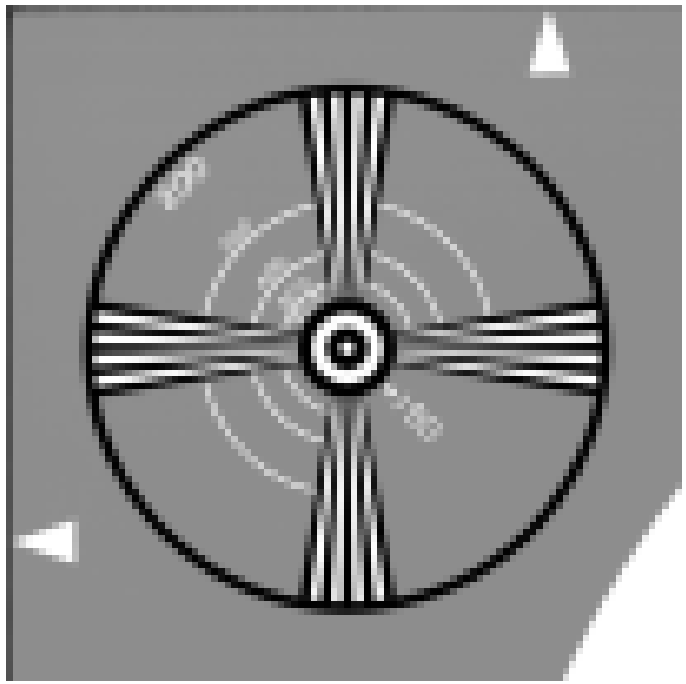


Bicubic Interpolated Image

What is Super Resolution?

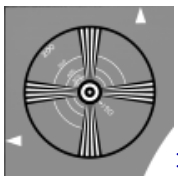


LR Image 2

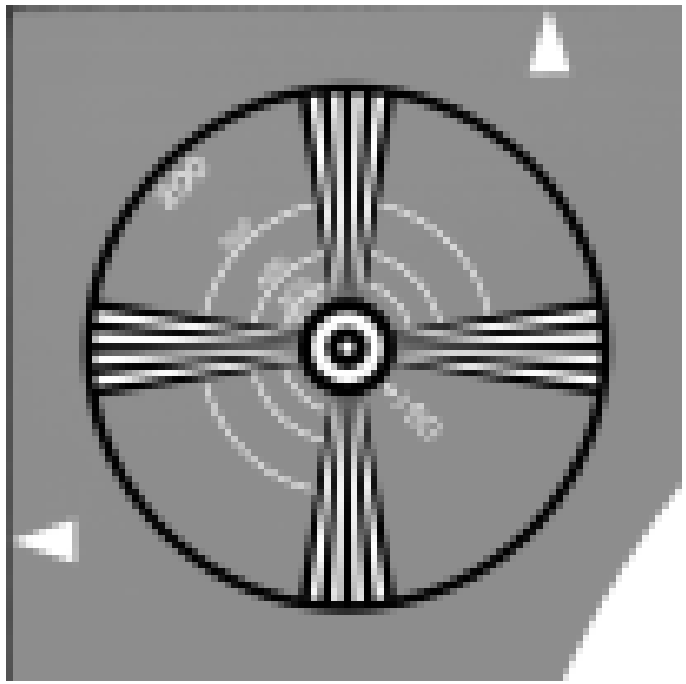


Bicubic Interpolated Image

What is Super Resolution?

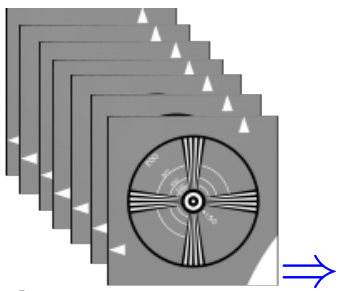


LR Image 3

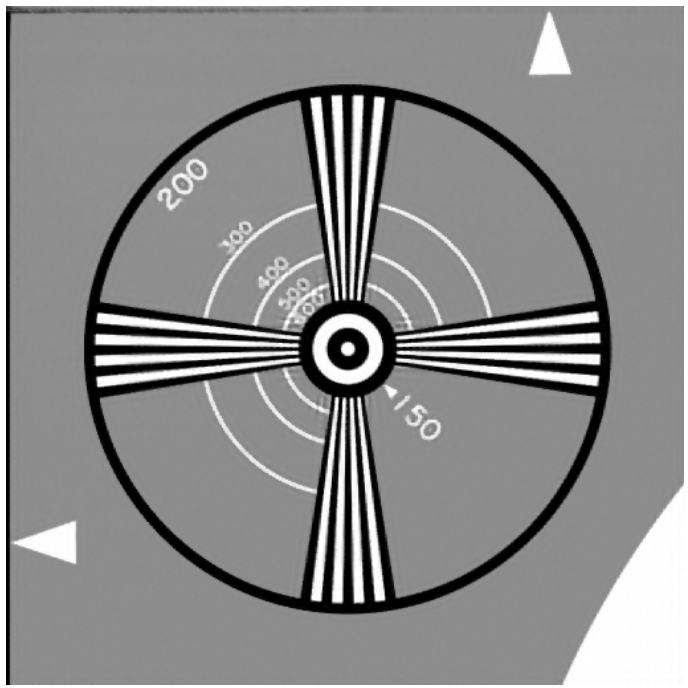


Bicubic Interpolated Image

What is Super Resolution?



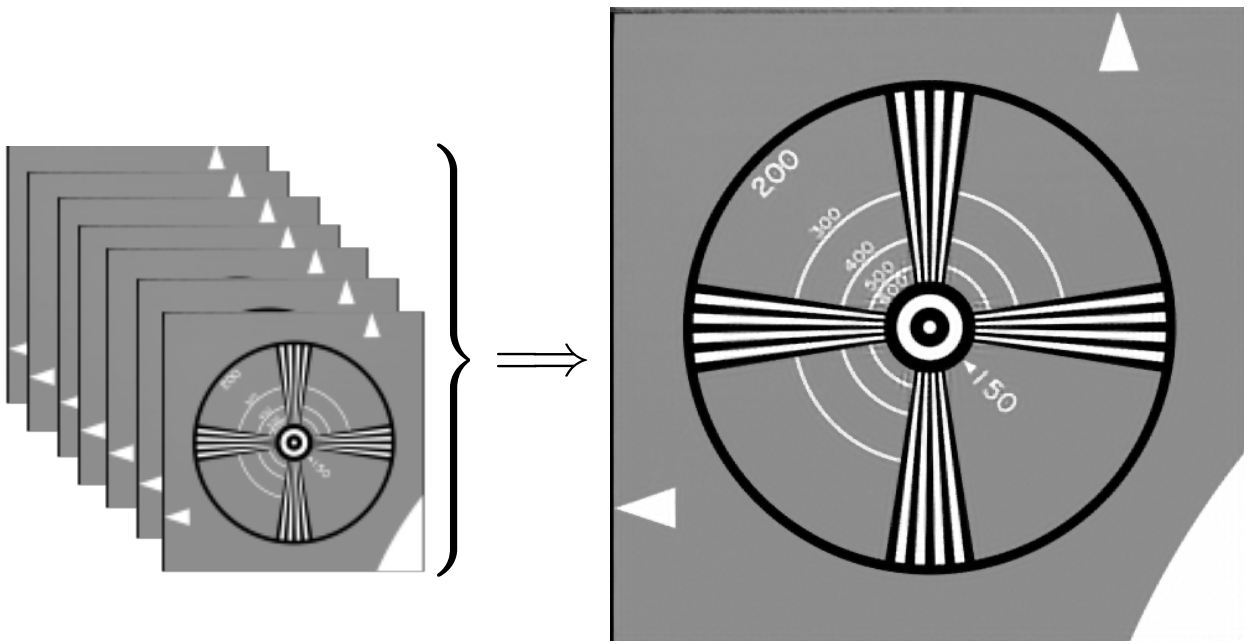
Stack of LR Images



Super Resolution Image.

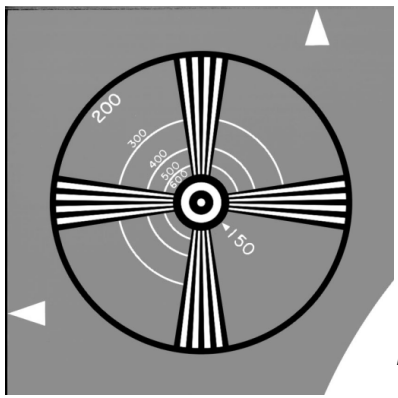
What is Super Resolution?

Definition: Super-resolution is largely known as a technique whereby multi-frame motion is used to overcome the inherent resolution limitations of a low resolution camera system.



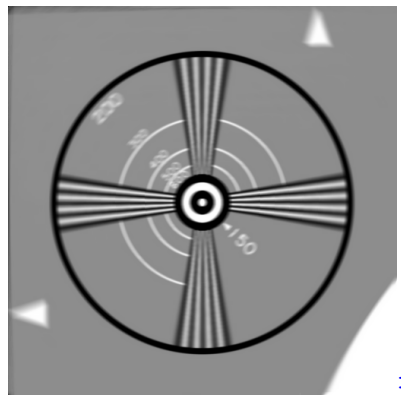
Estimated High Resolution Image [11].

Block diagram representation



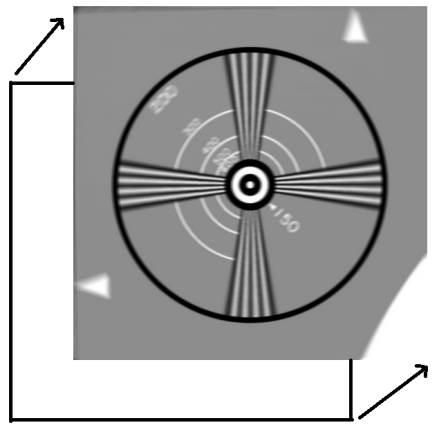
Real World Scene

$\Rightarrow H_{atm}$



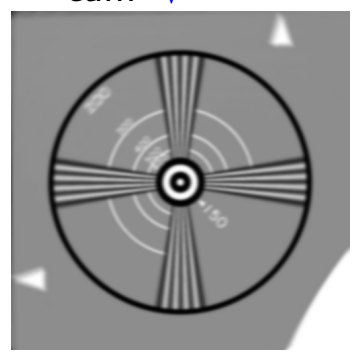
Atmospheric Blur Effect

$\Rightarrow F_k$



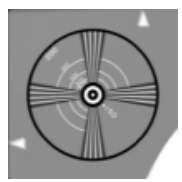
Motion Effect

$\Downarrow H_{cam}$



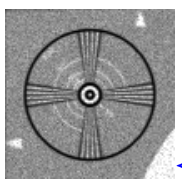
Camera Blur Effect

$\Leftarrow D$

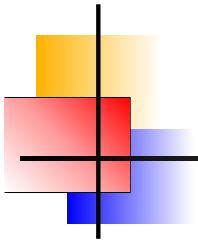


Down-sampling Effect

$\Leftarrow \oplus \mathbf{e}_k$



Noisy, Blurred, Down-sampled Out-come



Model Construction

$$\mathbf{Y}_k = DF_kH\mathbf{X} + \mathbf{e}_k, \qquad \forall k = 1, 2, ..., K$$

Known:	Symbols:	Represents:
✔	$\mathbf{Y}_k$	k^{th} LR Image
✘	$\mathbf{X}$	HR Image
✔	D	Decimation Matrix
✘	F_k	Geometric Transformation Matrix
✘	H	Blurring Kernel
✘	$\mathbf{e}_k$	Random Noise
✔	K	Number of LR images

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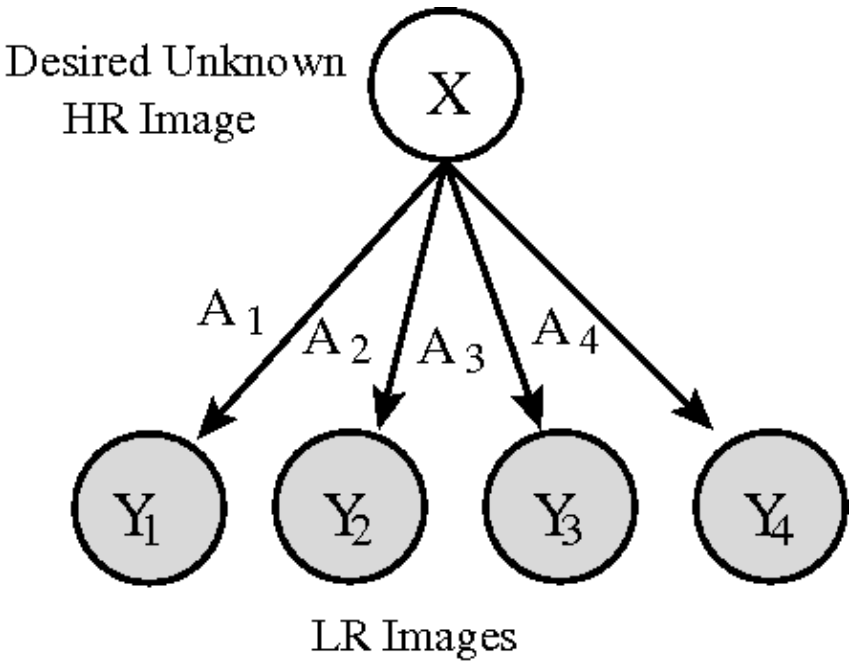
✔	$\mathbf{Y}_k$	k^{th} LR Image
✘	$\mathbf{X}$	HR Image $\Leftarrow$ Predict
✔	D	Decimation Matrix
✔	F_k	Geometric Transformation Matrix [a]
✔	H	Blurring Kernel
✘	$\mathbf{e}_k$	Random Noise
✔	K	Number of LR images

[a] Vandewalle et al., A frequency domain approach to registration of aliased images with application to super-resolution, EURASIP, 2006.

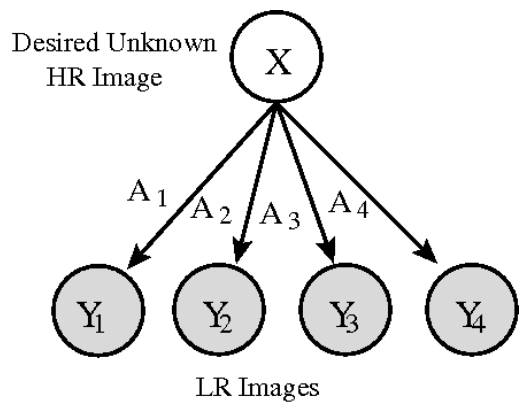
Generative Model Representation



In our case it is not a fully generative model as we assumed some model parameters. Shaded circles are observed variables. $A_k = DF_kH$ are the weights corresponding to LR image Y_k.

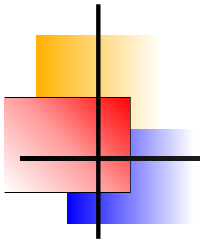
Maximum a Posteriori (MAP) solution

$$F = \underbrace{\frac{1}{2\sigma^2}\sum_{k=1}^{K}\rho(Y_k, A_k X)}_{\text{Likelihood}} - \underbrace{\log(p(X))}_{\text{prior}}$$



Standard method: Assume $\rho(Y_k, A_k X) = \|Y_k - A_k X\|_2^2$, $p(X) = \exp\{-\Upsilon(X)\}$, and $\Upsilon(X) = |\Gamma(X)|$ solve the above using gradient descent.

$$\frac{\partial F}{\partial X} = -\frac{1}{2\sigma^2}\sum_{k=1}^{K} A_k^T(Y_k - A_k X) - \frac{\partial}{\partial X}\Gamma(X)$$



Algebraic Approach

SR reconstruction Model:

$$Y_k = A_k X + e_k, \qquad \forall k = 1, 2, ..., K$$

LSE technique would give the solution of the form :

$$\hat{X} = \arg\min_X \left[\sum_{k=1}^{K} \| Y_k - A_k X \|_2^2 \right]$$

For unique solution, we add a regularization term $\Upsilon(X)$ as:

$$\hat{X} = \arg\min_X \left[\sum_{k=1}^{K} \| Y_k - A_k X \|_2^2 + \lambda \Upsilon(X) \right]$$

where λ is a regularization parameter and $\Upsilon(X) = |\Gamma(X)|_1$. This method is popularly known as LSE with TV regularization.

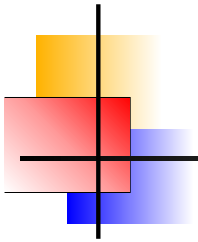
Thesis contribution 'in multi-frame SR'

SR reconstruction Model:

$$\hat{X} = \arg\min_{X} \left[\sum_{k=1}^{K} \| Y_k - A_k X \|_2^2 + \lambda \Upsilon(X) \right]$$

We investigate on following:

- We develop a locally adaptive regularization parameter λ from multi-scale morphological Gain-map [7] .
- We use non-linear multi-scale morphological gradient operator $\Upsilon(X)$ and then solve the convex problem by formulating efficient subgradients of morphological operators [11].
- We show the relationship between a non-linear edge-preserving filtering and a regularization technique that leads to a geodesic kernel smoother for super resolution [10].

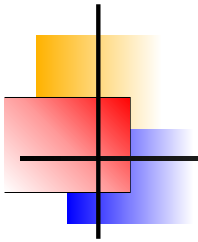


Thesis contribution 'in single-frame SR'

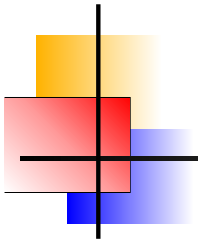
Single-frame Super Resolution: only one LR image is available. (Sometimes it is called as model based zooming)

We also investigate on following:

- Learn image prior from a database of natural images. Learn the correspondence of patch pairs of LR and HR images by multiple sparse dual Dictionary Learning [9].
- Learn the patch prior from the image itself using local self-similarity in multiple scales without help of any external database of natural images [8].



Chapter I: SR reconstruction using locally adaptive regularization with morphologic gain map



Regularization Techniques

General SR reconstruction Model:

$$\hat{X} = \arg\min_{X} \left[\sum_{k=1}^{K} \| Y_k - A_k X \|_p^p + \lambda \Upsilon(X) \right]$$

where $1 \leq p \leq 2$, $\Upsilon(X) = \|\Gamma(X)\|$ and Γ is a high frequency operator.

Different types of regularization:

- $\Upsilon(X) = |\nabla X|_2^2$ known as bounded variation (BV) regularization.
- $\Upsilon(X) = |\nabla X|_1$ known as total variation (TV) regularization.
- $\Upsilon(X) = \sum_{l=-w}^{w} \sum_{m=-w}^{w} \alpha^{|l|+|m|} |X - S_x^l S_y^m X|_l$ known as bilateral total variation (BTV) regularization, where $l + m \geq 0$ and S_x^l, S_y^m are shift-operators along x and y directions with l and m pixel respectively.

Regularization Techniques

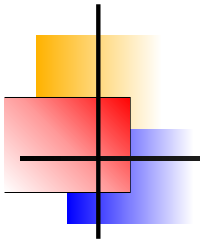
- BV / TV : $\Gamma = \nabla_x + \nabla_y$

$$= \begin{array}{|c|c|}\hline -1 & 1 \\ \hline \end{array} + \begin{array}{|c|}\hline -1 \\ \hline 1 \\ \hline \end{array}$$

- BTV : $\Gamma = \alpha \left(\begin{array}{|c|c|}\hline -1 & 1 \\ \hline \end{array} + \begin{array}{|c|}\hline -1 \\ \hline 1 \\ \hline \end{array} \right)$

$$+\alpha^2 \left(\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline \end{array} + \begin{array}{|c|}\hline -1 \\ \hline 0 \\ \hline 1 \\ \hline \end{array} \right)$$

$$\ldots$$



Reconstruction Methods

The modified ART like algorithm [1] using BTV regularization [1]:

$$X_j^{(n+1)} = X_j^{(n)} + \beta^{(n)} \frac{(A_{k,j})^T}{||A_{k,j}||^2} sign(\mathbf{Y}_k - A_k \mathbf{X}^{(n)}) +$$

$$\lambda \underbrace{\sum_{l=-w}^{w} \sum_{m=-w}^{w}}_{l+m \geq 0} \alpha^{|m|+|l|} [I - S_y^{-m} S_x^{-l}] sign(X^n - S_x^l S_y^m X^n)\}$$

$$\text{for} \quad j = 0, 1, 2, \ldots, N-1 \quad \text{and} \quad k = 0, 1, 2, \ldots, K-1$$

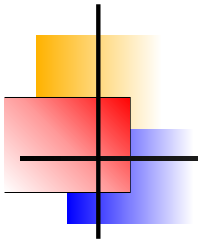
Here both the terms data term and regularization terms taken as l_1 norm.

---

[1] S. Farsiu, M. D. Robinson, M. Elad, and P. Milanfar. Fast and robust multi-frame super-resolution. IEEE Transactions on Image Processing, 13(10):13271344, October 2004.

Reconstruction Result

Output with BTV regularization, with 10 LR images upsampled 5×5 times [noise variation $\sigma = 5$].



Our observation

- Noise and Edges both contribute high-frequency components.
  # Both suppressed or enhanced during reconstruction process.
- A periodic ringing noise appears parallel to the edges in the estimated image.
  # Most distracting in smooth regions.

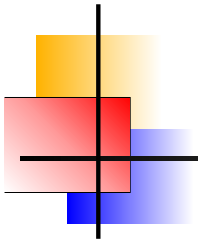
Possible Solution:

Employ some adaptive technique where the relative effect at each iteration is controlled by a gain map at each pixel depending upon its neighbourhood information.

Proposed Solution

$$X_j^{(n+1)} = X_j^{(n)} + \beta^{(n)} \Bigg[\underbrace{[\omega I(j,j) + (1-\omega) I_{gain}^{(p)}(j,j)]}_{\text{Weight for Data term}} \times \frac{(A_{k,j})^T}{||A_{k,j}||^2} sign(\mathbf{Y}_k - A_k \mathbf{X}) + \underbrace{[I(j,j) - I_{gain}^{(p)}]}_{\text{Weight for Regularization term}} \times \underbrace{\sum_{l=-w}^{w} \sum_{m=-w}^{w}}_{l+m \geq 0} \alpha^{|m|+|l|} [I - S_y^{-m} S_x^{-l}] sign(\mathbf{X}^{(n)} - S_x^l S_y^m \mathbf{X}^{(n)}) \Bigg],$$

$$\text{for} \quad j = 0, 1, 2, \ldots, N-1 \quad \text{and} \quad k = 0, 1, 2, \ldots, K-1$$

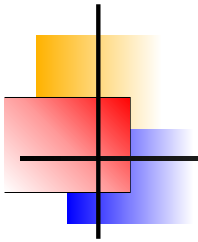


Construction of gain map

Idea behind the gain map I_{gain} formulation:

- It should be small in smooth region (ideally zero) and large along the edge or boundary (ideally one).
- Robust to noise: should be small in the noisy pixels in smooth region even if it contains high-frequency.

We use morphological operators to compute gain map I_{gain}.
Morphological opening and closing remove bright and dark noise pixel respectively, without effecting edges.



Construction of gain map

Algorithm to compute I_{gain}:

Step 1: Initialize image $F = \mathbf{0}$.

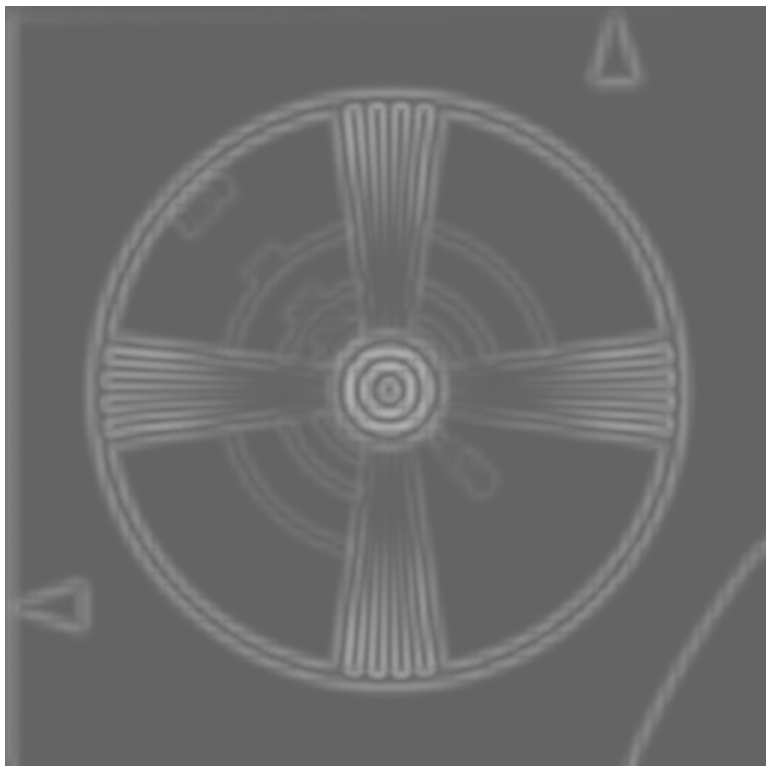
Step 2: Iterate the following steps for $r = 1$ to m

- $F_1 = X^n \circ rS$
  $F_2 = X^n \bullet rS$
- $F_1 = F_1 \ominus rS$
- $F_2 = F_2 \oplus rS$
- Calculate intermediate gradient image
  $F = F + (F_2 - F_1)$

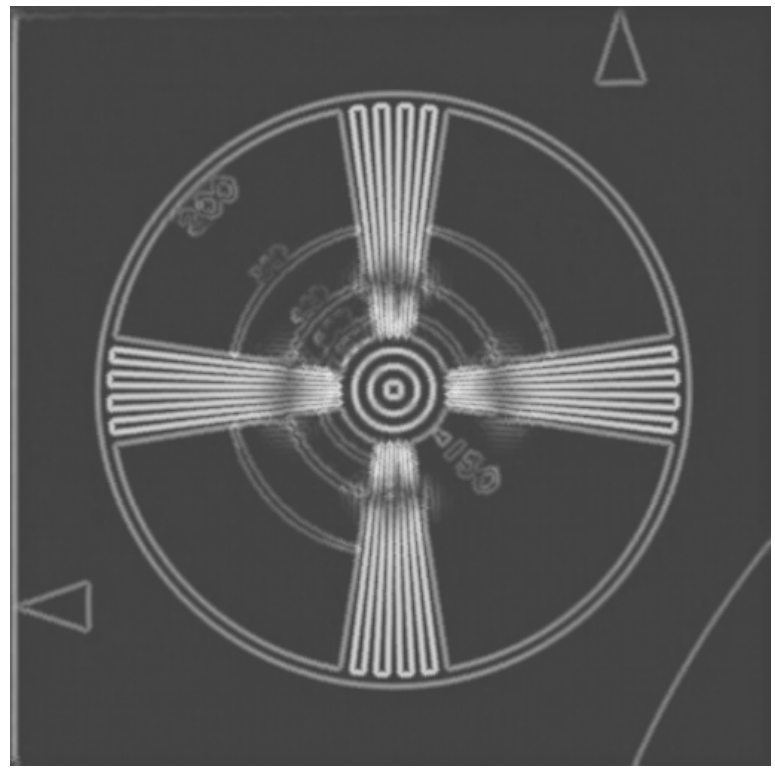
Step 3: $F(i,j) = \text{Sigmoid}\left(F(i,j) - \frac{F_{max}+F_{min}}{2}\right)$, where
$\text{Sigmoid}(x) = \frac{1}{1+e^{-\sigma x}}$

Step 4: $dia(I_g) = F$.

Example of gain map

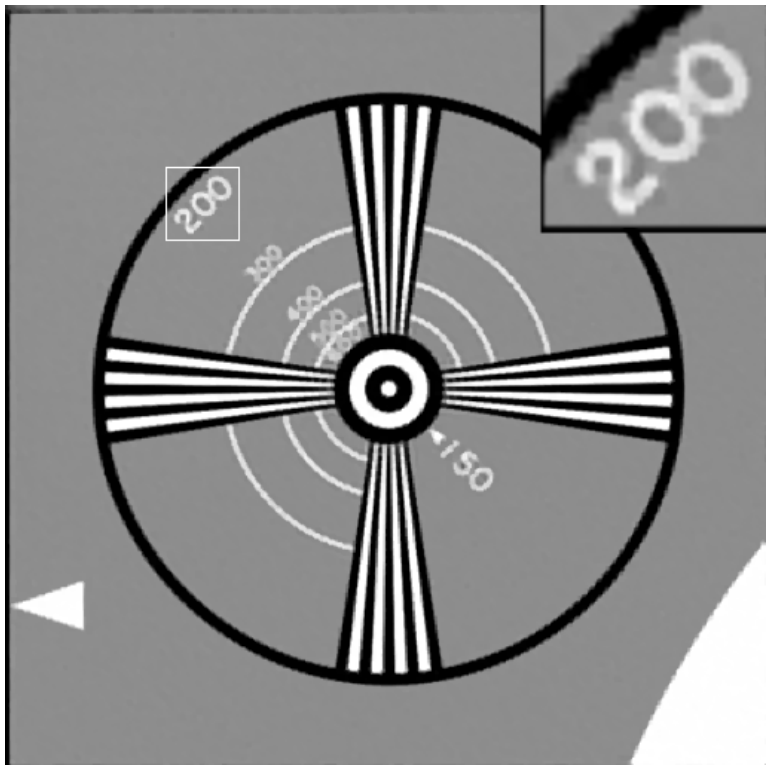


gain map after 1^{st} iteration

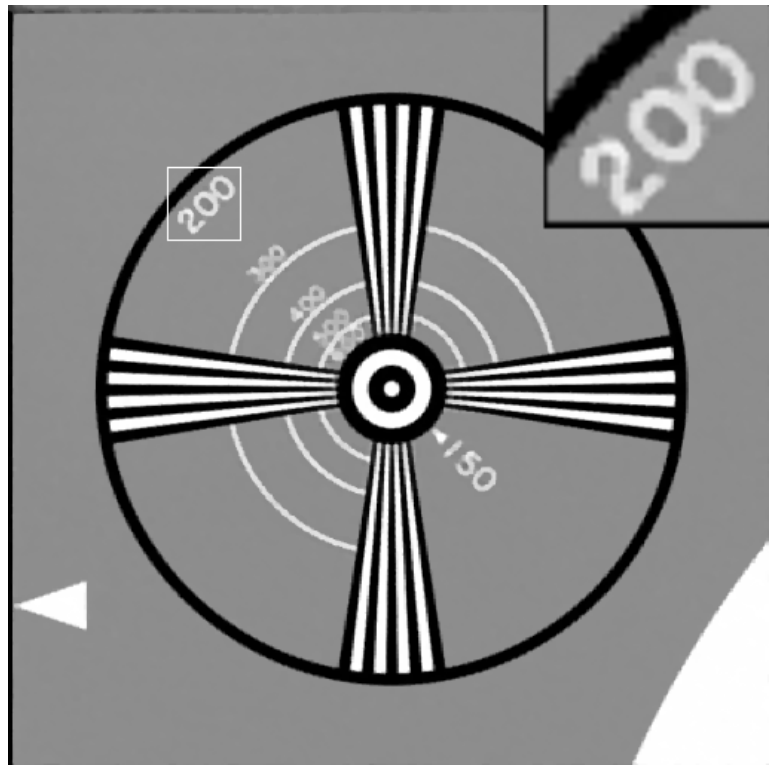


gain map after 10^{th} iteration

SR image reconstruction



BTV regularization(PSNR=29.56)



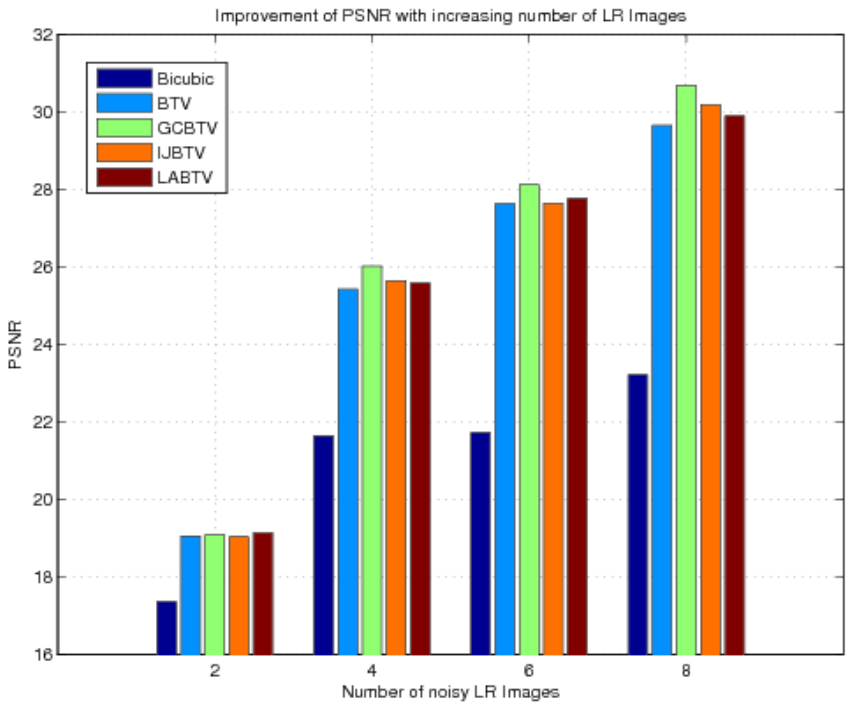
Proposed regularization(PSNR=30.69)

SR image reconstruction

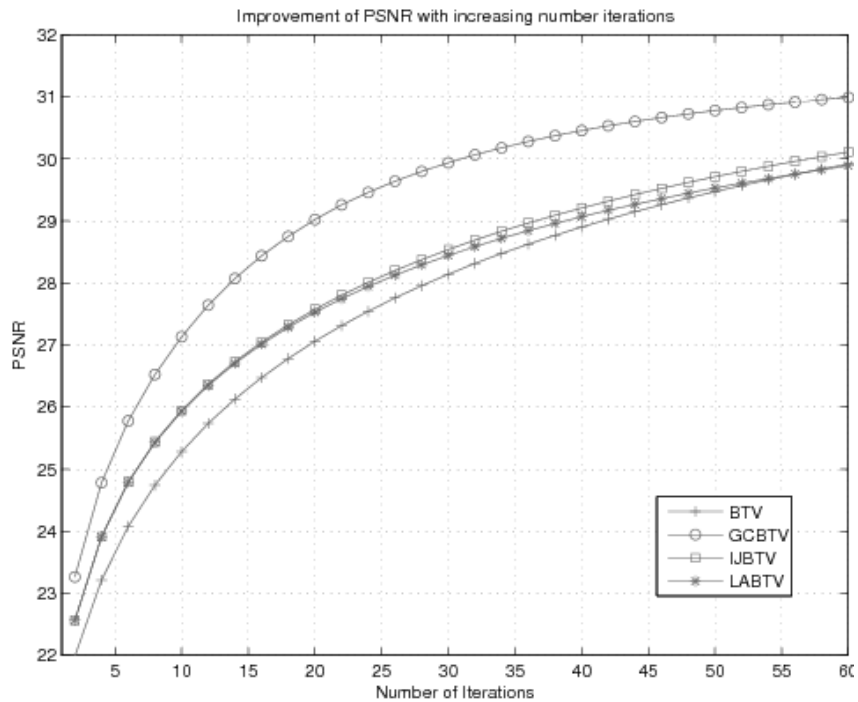
BTV regularization(PSNR=29.65)

Proposed regularization(PSNR=30.01)

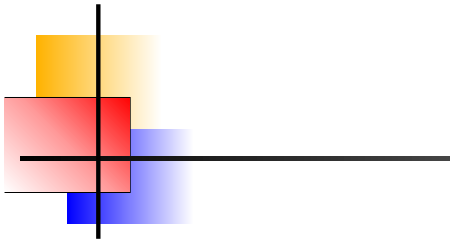
PSNR Plot diagrams



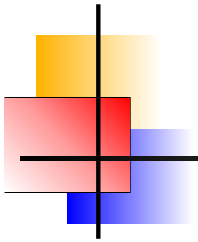
PSNR Vs. # LR images



PSNR Vs. # Iteration



Chapter II: Fast SR reconstruction with Morphological Regularization



Revisit different Regularization Methods:

General SR reconstruction Model:

$$\hat{X} = \arg\min_X \left[\sum_{k=1}^{K} \| Y_k - A_k X \|_p^p + \lambda \Upsilon(X) \right]$$

where $1 \leq p \leq 2$, $\Upsilon(X) = \|\Gamma(X)\|$ and Γ is a high frequency operator.

Different types of regularization:

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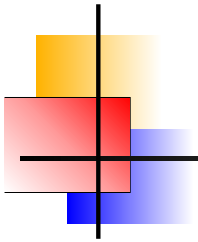
- $\Upsilon(X) = \sum_{l=-w}^{w} \sum_{m=-w}^{w} \alpha^{|l|+|m|} |X - S_x^l S_y^m X|_l$ known as bilateral total variation (BTV) regularization. Where $l + m \geq 0$ and S_x^l, S_y^m are shift-operators along x and y directions with l and m pixel respectively.



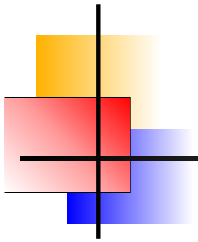
Morphological operators as Regularization term

Motivation:

- Existing regularization techniques are only based on direct image gradients.
- Exploring different feature based regularization technique.
- The last work suggesting that morphological operators could be a great choice.

Difficulty:

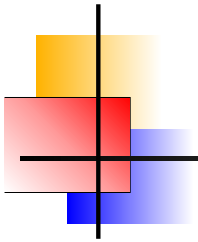
- Highly non-linearity makes the problem difficult to handle.
- Needs to calculate the subgradients for morphological operators.
- Slower than calculating direct derivatives.



Morphological operators as Regularization term

Difficulty: Possible Solution

- Highly non-linearity makes the problem difficult to handle.
  # Non-linear is good because it can be made locally adaptive
- Needs to calculate the subgradients for morphological operators.
  # We calculate Morphological subgradients efficiently
- Slower than calculating direct derivatives.
  # We develop advanced Bregman iteration for super resolution problem instead of gradient descent.



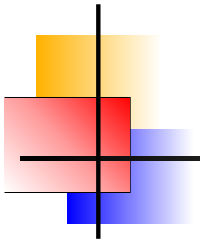
Proposed Method

Proposed Morphological Regularization

$$\Upsilon(\mathbf{X}) = \sum_{s=1}^{S} \alpha^s \mathbf{1}^t [C_s(\mathbf{X}) - O_s(\mathbf{X})]$$

where $O_s(\mathbf{X}) = D_s(E_s(\mathbf{X}))$, $C_s(\mathbf{X}) = E_s(D_s(\mathbf{X}))$

$$D_s(\mathbf{X}) = \begin{pmatrix} \max_{r \in (sB)_{(1)}} \{x_r\} \\ \max_{r \in (sB)_{(2)}} \{x_r\} \\ \ddots \\ \max_{r \in (sB)_{(mn)}} \{x_r\} \end{pmatrix}, \quad E_s(\mathbf{X}) = \begin{pmatrix} \min_{r \in (sB)_{(1)}} \{x_r\} \\ \min_{r \in (sB)_{(2)}} \{x_r\} \\ \ddots \\ \min_{r \in (sB)_{(mn)}} \{x_r\} \end{pmatrix}$$



Proposed Method

Bregman Iteration:

Consider the following minimization problem:

$$\min_{\mathbf{X}}\{\Upsilon(\mathbf{X}) \ : \ T(\mathbf{X}) = 0\}$$

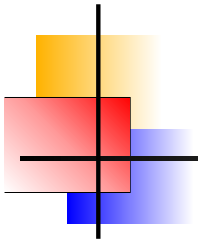
Now the Bregman iterations that solve the above constrained minimization problem are as follows:

Initialize $\mathbf{X}^0 = p^0 = \mathbf{0}$

$$\begin{cases} \mathbf{X}^{(n+1)} = \arg\min_{\mathbf{X}}\{\mu B_{\Upsilon}^{p^{(n)}}(\mathbf{X}, \mathbf{X}^{(n)}) + T(\mathbf{X})\} \\ p^{(n+1)} = p^{(n)} - \nabla T(\mathbf{X}^{(n+1)}) \end{cases}$$

where $B_{\Upsilon}^{p^{(n)}}$ is the Bregman distance corresponding to convex functional $\Upsilon(.)$ and is defined from point $\mathbf{X}$ to point $\mathbf{V}$ as

$$B_{\Upsilon}^{p}(\mathbf{X}, \mathbf{V}) = \Upsilon(\mathbf{X}) - \Upsilon(\mathbf{V}) - \langle p, \mathbf{X} - \mathbf{V} \rangle$$



Proposed Method

Proximal Map:

Consider the following unconstrained minimization problem:

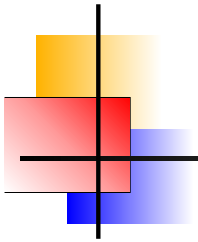
$$\min_{\mathbf{X}}(\mu\Upsilon(\mathbf{X}) + T(\mathbf{X}))$$

The solution satisfies the condition:
$\mu\partial\Upsilon(\mathbf{X}) + \partial T(\mathbf{X}) = 0 \Rightarrow (\mathbf{X} + \gamma\mu\partial\Upsilon(\mathbf{X})) - (\mathbf{X} - \gamma\partial T(\mathbf{X})) = 0$
This leads to a forward and backward splitting algorithm:

$$\mathbf{X}^{(k+1)} = Prox_{\Upsilon}(\mathbf{X}^{(k)} - \gamma\partial T(\mathbf{X}^{(k)})),$$

where the proximal operator $Prox_{\Upsilon}(\mathbf{V})$ is defined as:

$$Prox_{\Upsilon}(\mathbf{V}) = \arg\min_{\mathbf{X}}\{\mu\Upsilon(\mathbf{X}) + \frac{1}{2\gamma}\|\mathbf{X} - \mathbf{V}\|_2^2\}$$



Proposed Method

Super resolution unconstrained minimization Problem:

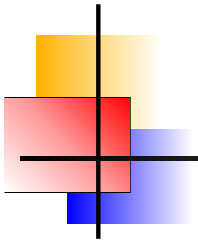
$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \{ \Upsilon(\mathbf{X}) \ : \ \|RH\mathbf{X} - \underline{\mathbf{Y}}\|_2^2 < \eta \}$$

The proposed SR image reconstruction algorithm combining Bregman iteration and operator splitting as follows:

Proposed Iterative Algorithm:

Initialize $\mathbf{Y}^{(0)} = n = 0, \underline{\mathbf{Y}}$, $\mathbf{X}^{(0)} =$ FillUnknown$(\underline{\mathbf{Y}})$;

$$\textbf{While}(\|RH\mathbf{X}^{(n)} - \underline{\mathbf{Y}}\|_2^2 > \eta)$$
$$\begin{cases} \mathbf{U}^{(n+1)} = \mathbf{X}^{(n)} - \gamma H^T R^T (RH\mathbf{X}^{(n)} - \mathbf{Y}^n) \\ \mathbf{X}^{(n+1)} = \mathbf{U}^{(n+1)} - \mu' \mid \frac{\delta \Upsilon(\mathbf{X})}{\delta \mathbf{X}} \mid_{\mathbf{X}^{(n)}} \\ \mathbf{Y}^{(n+1)} = \mathbf{Y}^{(n)} + (\underline{\mathbf{Y}} - RH\mathbf{X}^{(n+1)}) \end{cases} \quad (1)$$
$$n = n + 1$$
$$\textbf{end}$$



Proposed Method

Efficient computation of $|\frac{\delta \Upsilon(\mathbf{X})}{\delta \mathbf{X}}|$: Using Chain Rule

$\Upsilon(\mathbf{X}) = \sum_{s=1}^{S} \alpha^s \mathbf{1}^t [C_s(\mathbf{X}) - O_s(\mathbf{X})]$,
where $O_s(\mathbf{X}) = D_s(E_s(\mathbf{X}))$, $C_s(\mathbf{X}) = E_s(D_s(\mathbf{X}))$

$$D_s(X) = (X \oplus B) \oplus \cdots \oplus B = D(D(\ldots \text{ s times } \ldots D(X)\ldots))$$

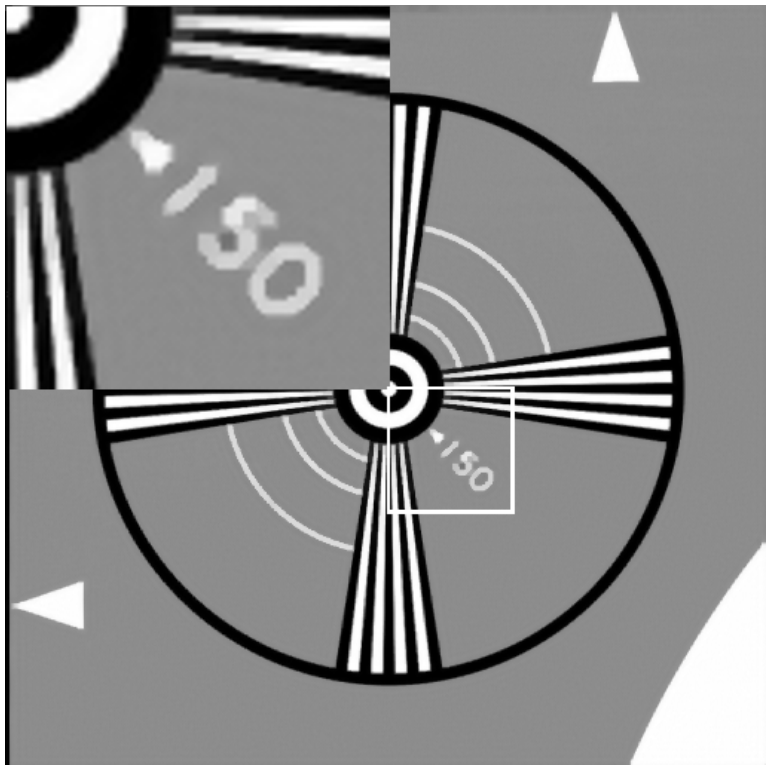
$$\begin{aligned} \frac{\delta}{\delta X} D_s(X) &= \frac{\delta}{\delta X}[D(D_{s-1}(X))] \\ &= \frac{\delta D_s}{\delta D_{s-1}} \frac{\delta D_{s-1}}{\delta D_{s-2}} \cdots \frac{\delta D_1(X)}{\delta X} \end{aligned}$$

Let $Z^{d_1} = \frac{\delta D}{\delta X}\mathbf{1}$ and $Z^{e_1} = \frac{\delta E}{\delta X}\mathbf{1}$

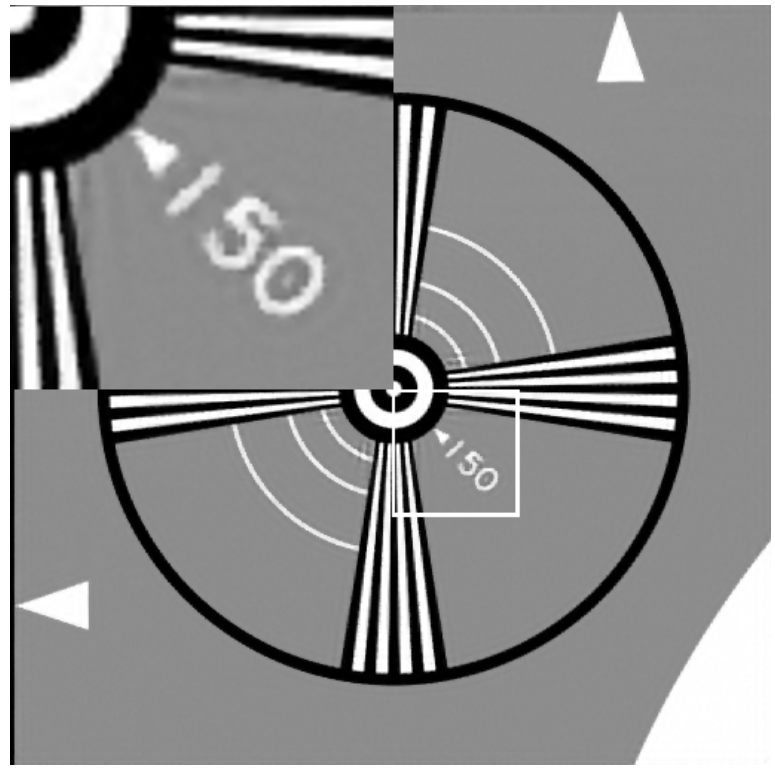
$$\begin{aligned} Z^{d_s} := \frac{\delta}{\delta X}[D_s(X)]\mathbf{1} &= \frac{\delta D_s}{\delta D_{s-1}} Z^{d_{s-1}} \\ &= \frac{\delta D(D_{s-1})}{\delta D_{s-1}} Z^{d_{s-1}} \end{aligned}$$

$\frac{\delta D_1(X)}{\delta X}$ is a subgradients of max operator computed easily and then propagate in higher scale.

SR image reconstruction



BTV regularization(PSNR=31.29)

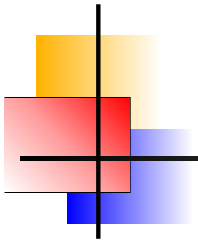


Proposed regularization(PSNR=32.46)

SR image reconstruction

BTV regularization(PSNR=29.80)

Proposed regularization(PSNR=31.45)

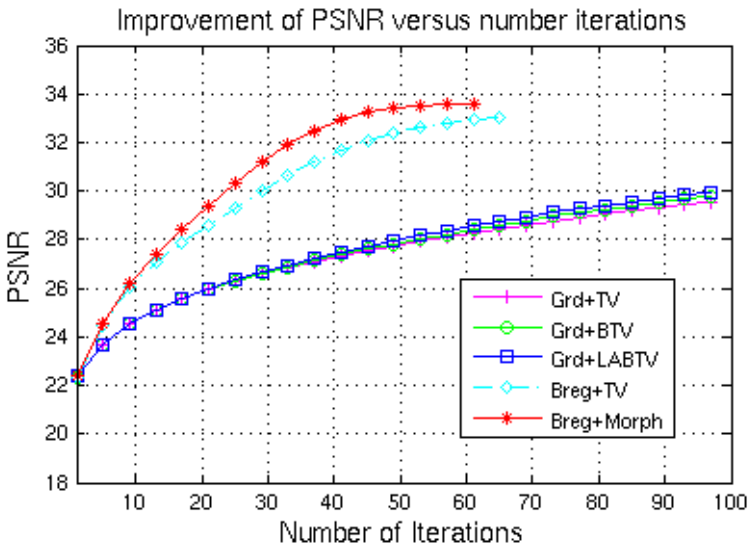


SR image reconstruction : Time Comparison

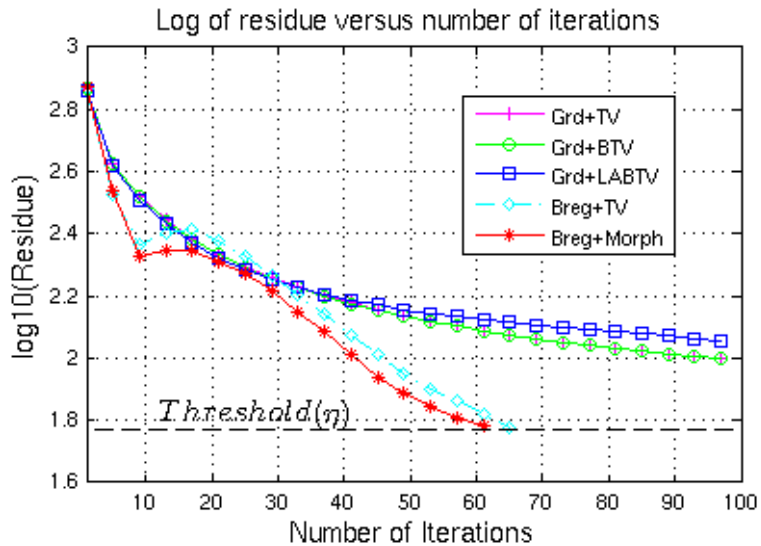
Table: Time comparison of different methods. 10 LR images upsampled with resolution factor 5×5.

Method	Grd+TV	Grd+BTV	Grd+LABTV	Breg+TV	Breg+Morph
Iteration	221	211	308	68	63
Time	19.72s	32.47s	290.48s	6.83s	12.45s

Performance comparison

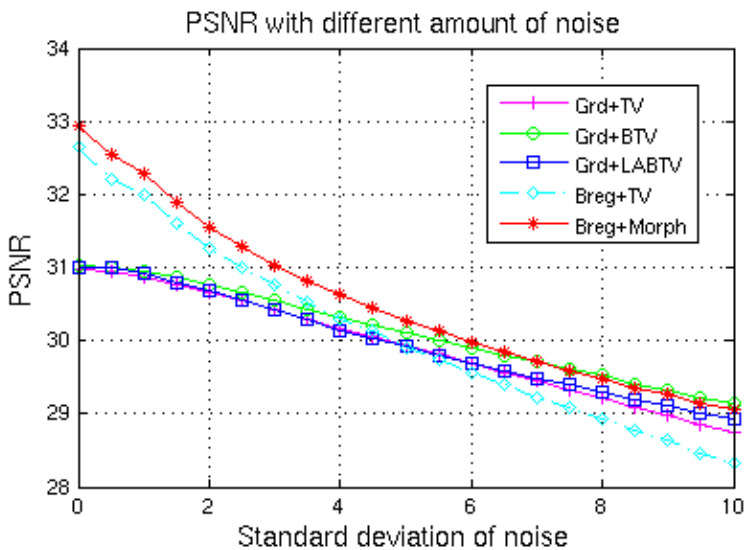


PSNR Vs. # Iteration

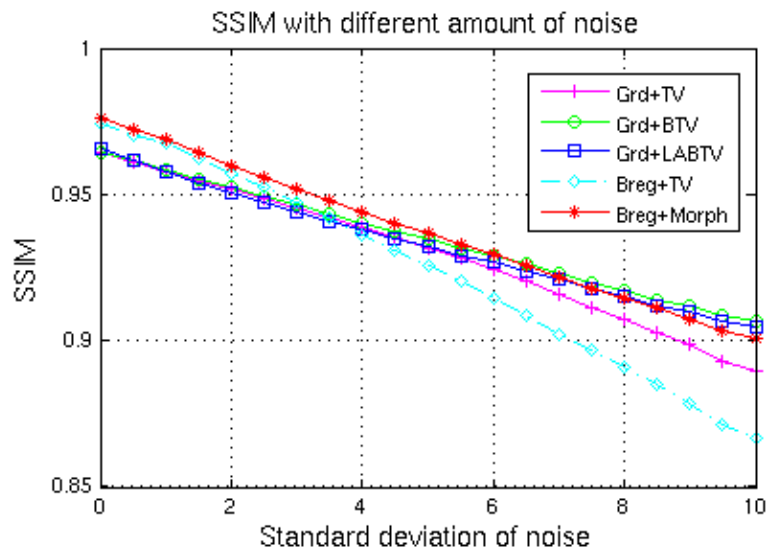


log10(Residue) Vs. # Iteration

Performance comparison

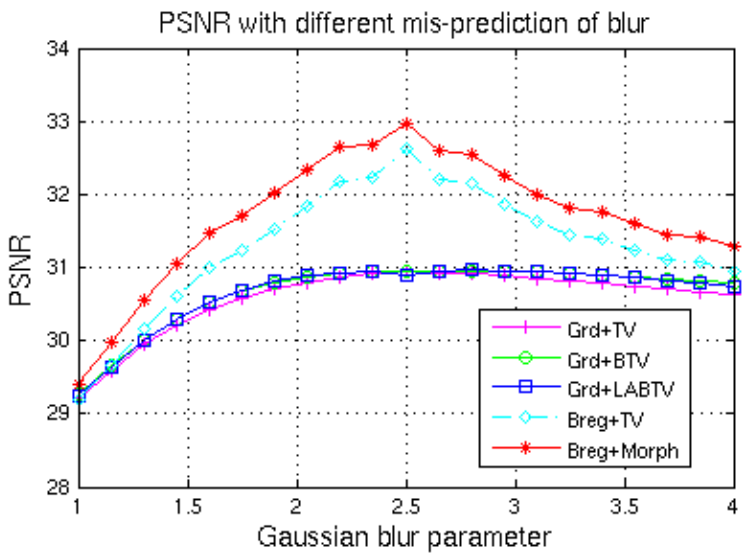


PSNR Vs. noise variation

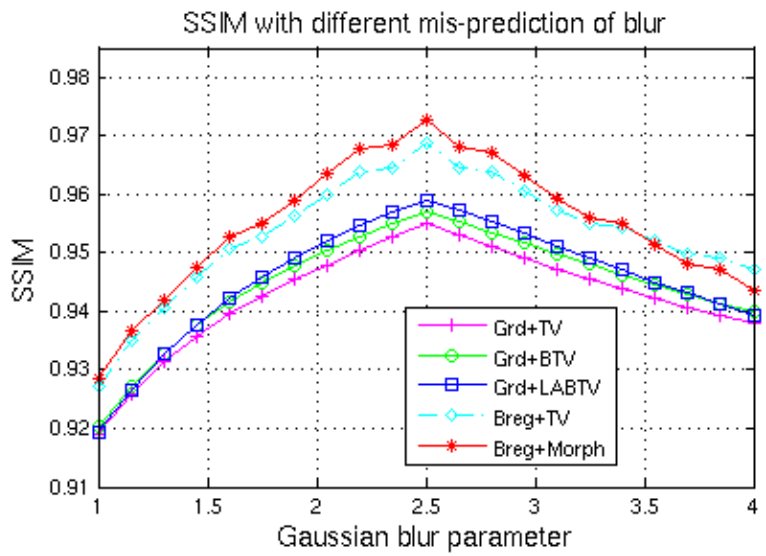


SSIM Vs. noise variation

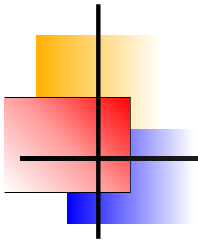
Performance comparison



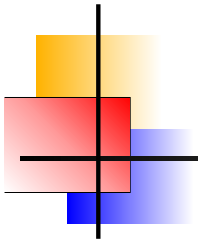
PSNR Vs. blurring kernel



SSIM Vs. blurring kernel



Chapter III: Geodesic Kernel smoother for SR image reconstruction.



The non-linear filtering Vs regularization technique

For different kind of regularizer with pth-norm:

$$\begin{aligned}
\Upsilon(X) &= \sum_{l,m\in N_a} \alpha_{lm} \|X - S_x^l S_y^m X\|_p \\
&\geq \|X - \sum_{l,m\in N_a} \alpha_{lm} S_x^l S_y^m X\|_p \\
&\quad \text{(using Jensen's Inequality), for } p > 1 \\
&= \|X - R \otimes X\|_p \\
&\quad \text{(Invariant to non-linear filtering } R, \\
&\quad \text{removes only noise present in the image)}
\end{aligned}$$

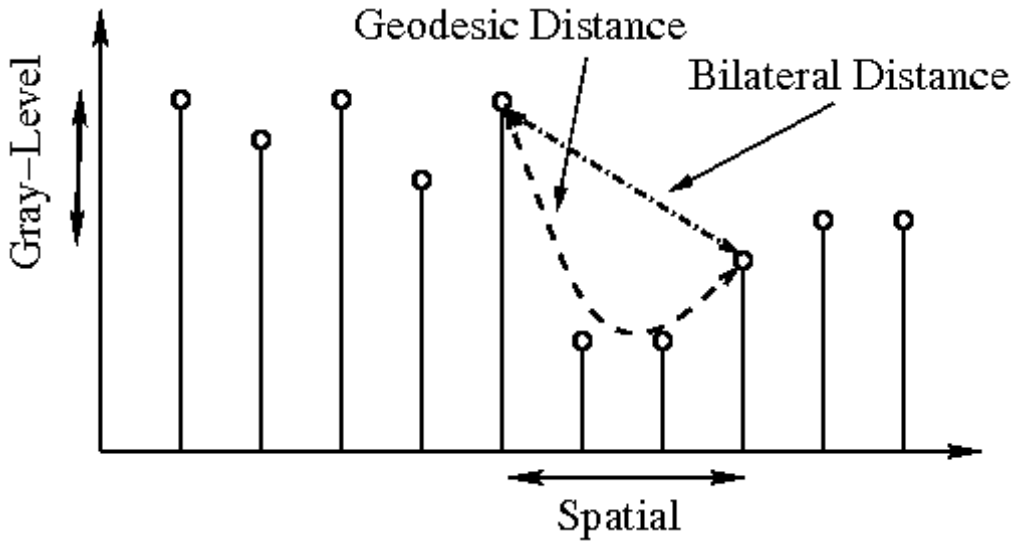
For 4-N neighbourhood:

$$\begin{aligned}
\Upsilon(X) &= \sum_{l,m\in N_g} \alpha_{lm} \|X - S_x^l S_y^m X\|_p \\
&= 2\|(\nabla_x + \nabla_y)X\|_p \\
&= \|\nabla X\|_p
\end{aligned}$$

Geodesic Kernel Smoother

Existing approximate Kernels:

- LARK Kernel [Takeda, *et al.* '07]
- Beltrami Kernel [Sochen, *et al.* '98]
- Structure Tensor [Brox, *et al.* '04]

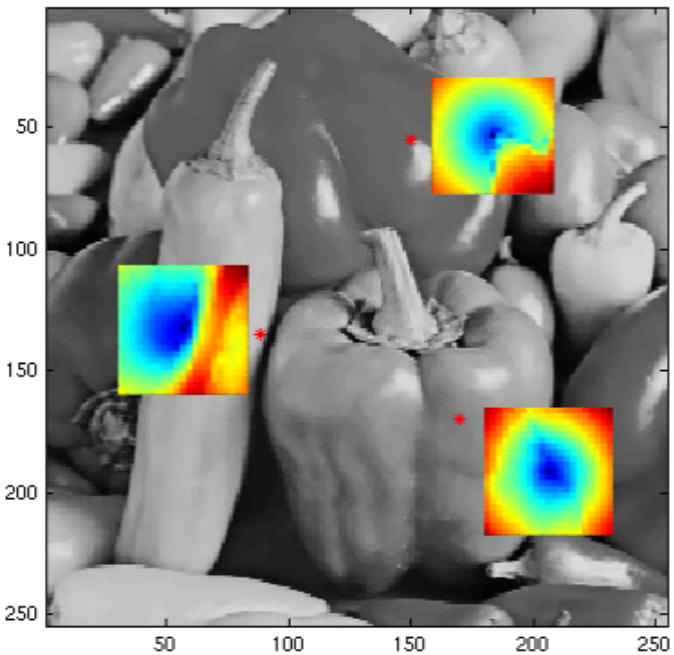


The values of the kernel are computed as:

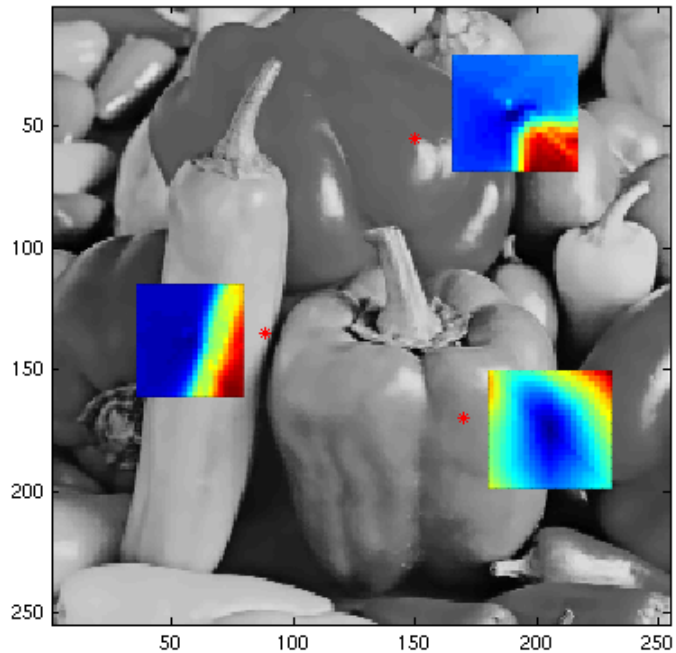
$$K(x,y) = e^{-\frac{distance(x,y)^2}{\sigma^2}}$$

Figure: Illustrates different kernel representations.

Bilateral Kernel Vs. Geodesic Kernel



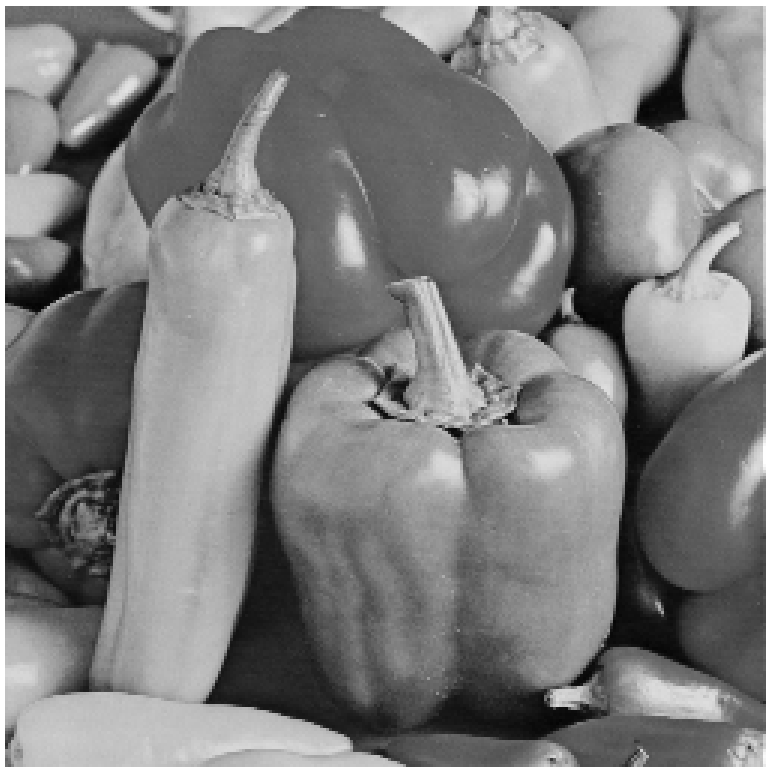
Bilateral Kernel



Geodesic Kernel

Visualization of bilateral and geodesic kernels at different points in the image (blue-red represents higher-smaller values of the kernels).

Image smoothing using Bilateral Kernel

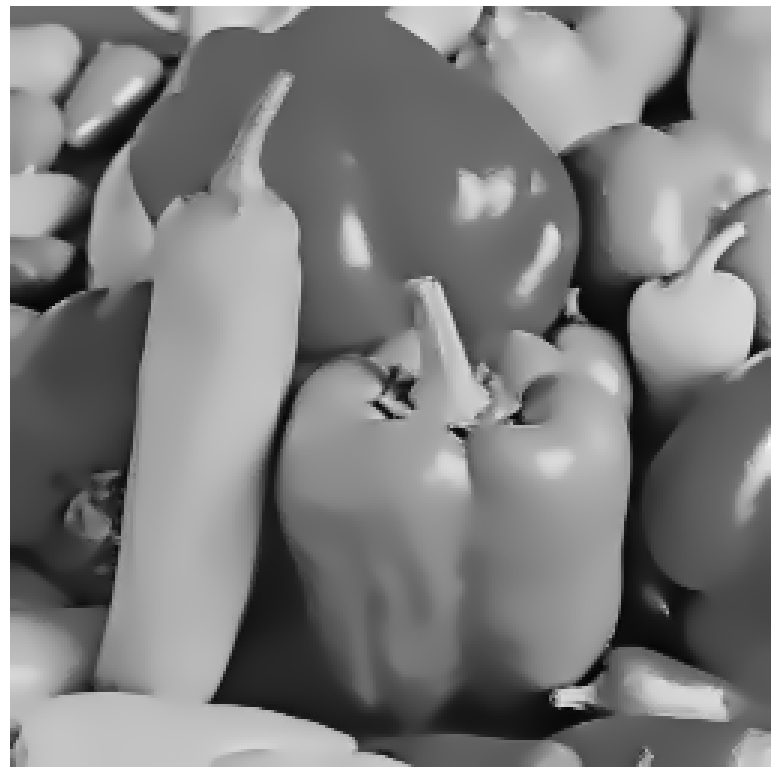


Original Image

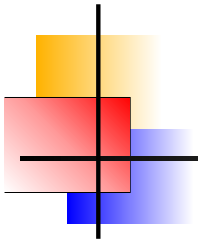
After one iterations

Image smoothing using Bilateral Kernel

After ten iterations



After twenty iterations



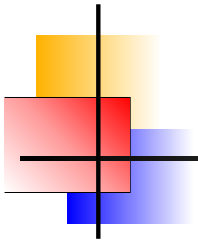
Geodesic Regularization

Motivation:

- Optimal Regularization is always desired.
- The edge preserving nature of the previous example suggests that Geodesic kernel can preserve better edge structure.
- Locally adaptive nature best suits for regularization problem.

Difficulty:

- Non-linear nature results subgradients difficult to compute.
- Usual Geodesic distance computation is slow.



Challenge for geodesic kernel regularization

Difficulty: Possible Solution

- Non-linear nature results subgradients difficult to compute.

  # We use the same idea to compute subgradients as we did for Morphologic Regularization.

- Usual Geodesic distance computation is slow.

  # We develop an efficient algorithm to compute Geodesic distance. We also derive split-Bregman iterations for SR image reconstruction using geodesic kernel regularization.

Algorithm:

Our Iterative Algorithm for SR Reconstruction:

Initialize $\mathbf{Z}^{(0)} = \mathbf{V}^{(0)} = \mathbf{U_k}^{(0)} = \mathbf{0}$, $n = 0$, $\mathbf{X}^{(0)} = \sum_{k=1}^{K} \mathbf{A_k^T Y_k}$;

While $\sum_{k=1}^{K} \|A_k \mathbf{X}^{(n)} - \mathbf{Y_k}\| > \varepsilon$

$$
\begin{cases}
\mathbf{X}^{(n+1)} &= \mathbf{X}^{(n)} + \mu \frac{\delta \Upsilon(\mathbf{X})}{\delta X}|_{\mathbf{X}^{(n)}} (\Upsilon(\mathbf{X}^{(n)}) - \mathbf{Z^{(n)}} - \mathbf{V^{(n)}}) \\
& \quad + \sum_{k=1}^{K} A_k^T \left(A_k \mathbf{X}^{(n)} - \mathbf{Y_k} + \mathbf{U_k}^{(n)} \right) \\
\mathbf{Z}^{(n+1)} &= \delta * Shrink \left(\Upsilon(\mathbf{X^{(n+1)}}) - \mathbf{V^{(n)}}, 1/\mu \right) \\
\mathbf{V}^{(n+1)} &= \mathbf{V}^{(n)} + (\Upsilon(\mathbf{X^{(n+1)}}) - \mathbf{Z}^{(n+1)}) \\
\mathbf{U_k}^{(n+1)} &= \mathbf{U_k}^{(n)} + (A_k \mathbf{X}^{(n+1)} - \mathbf{Y_k}), \ \forall k = 1, 2 \ldots K
\end{cases}
$$

$n = n + 1$

end

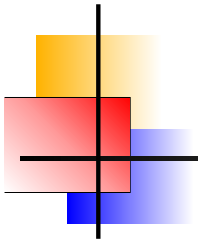
Result of SR method with geodesic kernel regularization

Figure: Top row : SR image using Bicubic interpolation (PSNR = 25.53dB, SSIM = 0.7936), TV+Grd method[12] (PSNR = 28.58dB, SSIM = 0.8614), BTV+Grd method[1] (PSNR = 28.96dB, SSIM = 0.8653). Bottom row : SR reconstructed image using LABTV+Grd method[5] (PSNR = 29.04dB, SSIM = 0.8712), TV+Breg[6] (PSNR = 31.34dB, SSIM = 0.9135), and proposed method Geo+split-Breg (PSNR = 31.54dB, SSIM = 0.9180)

Result of SR method with geodesic kernel regularization

Images	Bicubic	TV + Grd[12]	BTV + Grd[1]	LABTV + Grd[5]	TV + Brg[6]	Geo + split-Breg
Man	25.69 0.7328	29.04 0.8463	29.10 0.8494	28.90 0.8529	31.90 0.9164	**32.16** **0.9213**
Peppers	25.55 0.8592	29.67 0.9177	29.66 0.9185	29.86 0.9199	33.71 0.9566	**34.28** **0.9599**
Barbara	26.48 0.7856	30.07 0.8627	30.17 0.8653	30.20 0.8682	31.86 0.8999	**32.15** **0.9047**
Boat	24.20 0.7490	28.37 0.8589	28.46 0.8626	28.42 0.8631	32.16 0.9386	**32.24** **0.9408**
Lake	21.88 0.7441	25.40 8596	25.44 0.8615	25.45 0.8607	**28.60** **0.9267**	28.58 0.9259
Airplane	23.81 0.8106	28.26 0.8979	28.32 0.8995	28.30 0.8999	32.02 0.9485	**32.16** **0.9503**
Baboon	23.37 0.5396	25.13 0.6884	25.24 0.6963	25.35 0.7096	26.72 0.8091	**26.85** **0.8162**
Couple	26.29 0.7188	30.12 0.8403	30.21 0.8450	30.24 0.8500	33.71 0.9306	**33.91** **0.9346**
Splash	28.37 0.9007	33.86 0.9368	33.90 0.9374	33.74 0.9392	**39.57** 0.9631	39.50 **0.9667**
Tiffany	26.25 0.8640	32.95 0.9095	32.90 0.9113	32.80 0.9124	36.03 0.9440	**36.54** **0.9502**
Zelda	29.57 0.8901	33.53 0.9283	33.56 0.9299	33.58 0.9332	35.88 0.9542	**36.32** **0.9597**
Bird	21.82 0.7787	23.58 0.8445	23.62 0.8466	23.65 0.8472	24.99 0.8943	**25.02** **0.8998**

Table: PSNR (dB) AND SSIM OF RECONSTRUCTED SR IMAGES (NOISE LEVEL $\sigma_n = 0$)



Chapter IV: Single-frame Super Resolution with multiple Dictionary learning

Algebraic Image Zooming

Input Image

Nearest Neighbour Interpolated Image

Algebraic Image Zooming

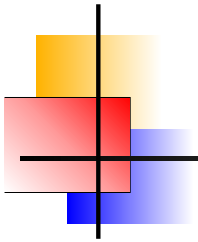
Input Image

Bicubic Interpolated Image

Model-Based Image Zooming

Input Image

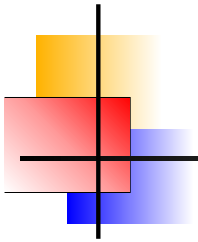
Super Resolution Image



Model-Based Image Zooming

State-of-Art:

- Soft edge smoothness prior [Dai *et al.* 07].
- Sparse Coding Techniques [Yang *et al.* 08, 10].
- Self Similarity Measures [Glasner *et al.* 09].

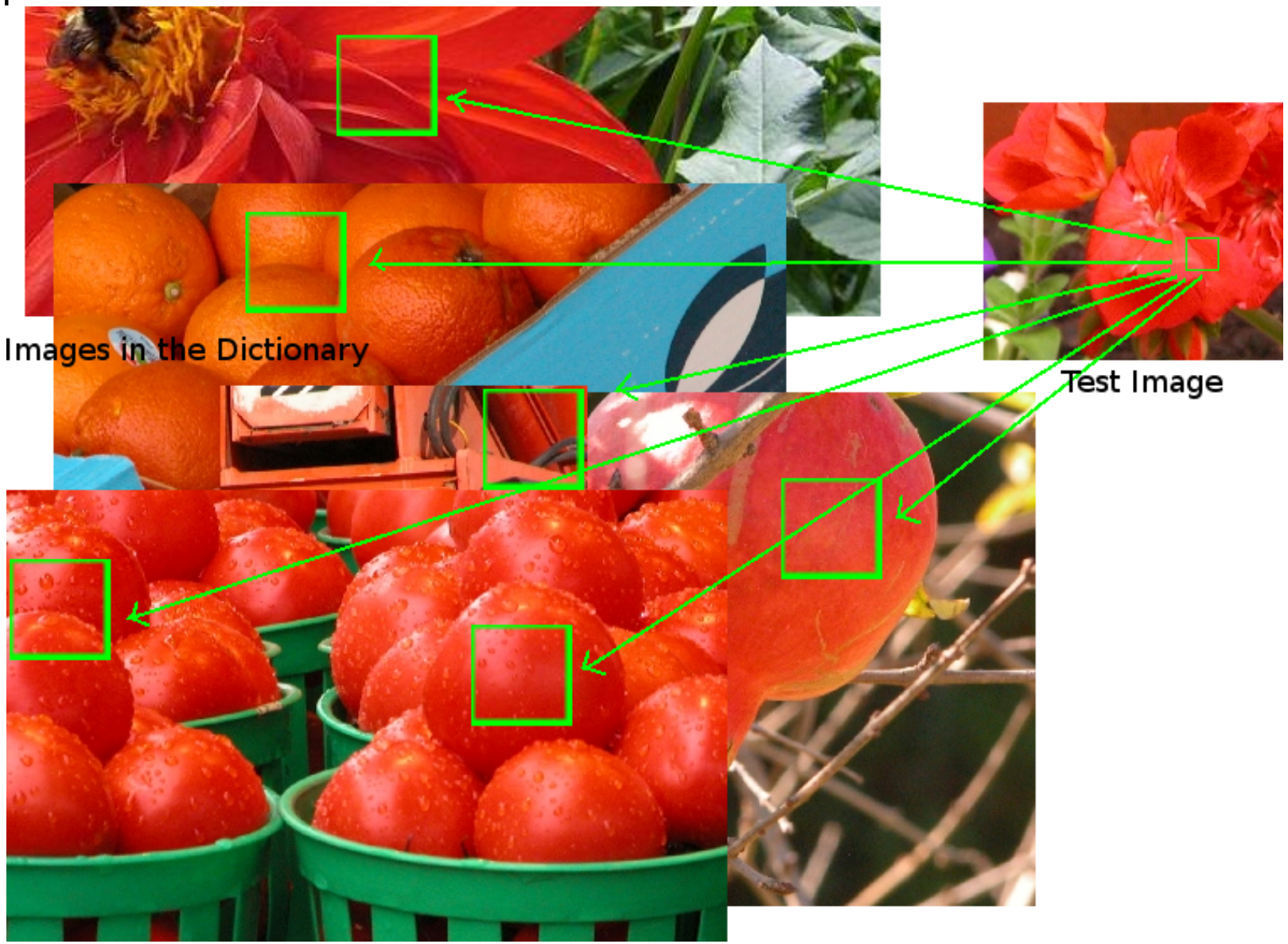


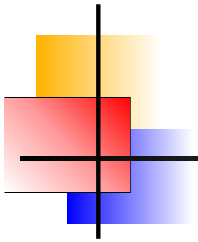
Model-Based Image Zooming

Motivation Behind this work:

- A blur and down-sampled patch can be generated from multiple possible LR patches.
- All the existing methods (mostly patch-based techniques) predict sharp patches corresponding to a blur down-sampled patch.
- Better SR image can be estimated if we predict all the possibilities and choose the best one that suits the neighbourhood patches.

Motivation Behind this work:



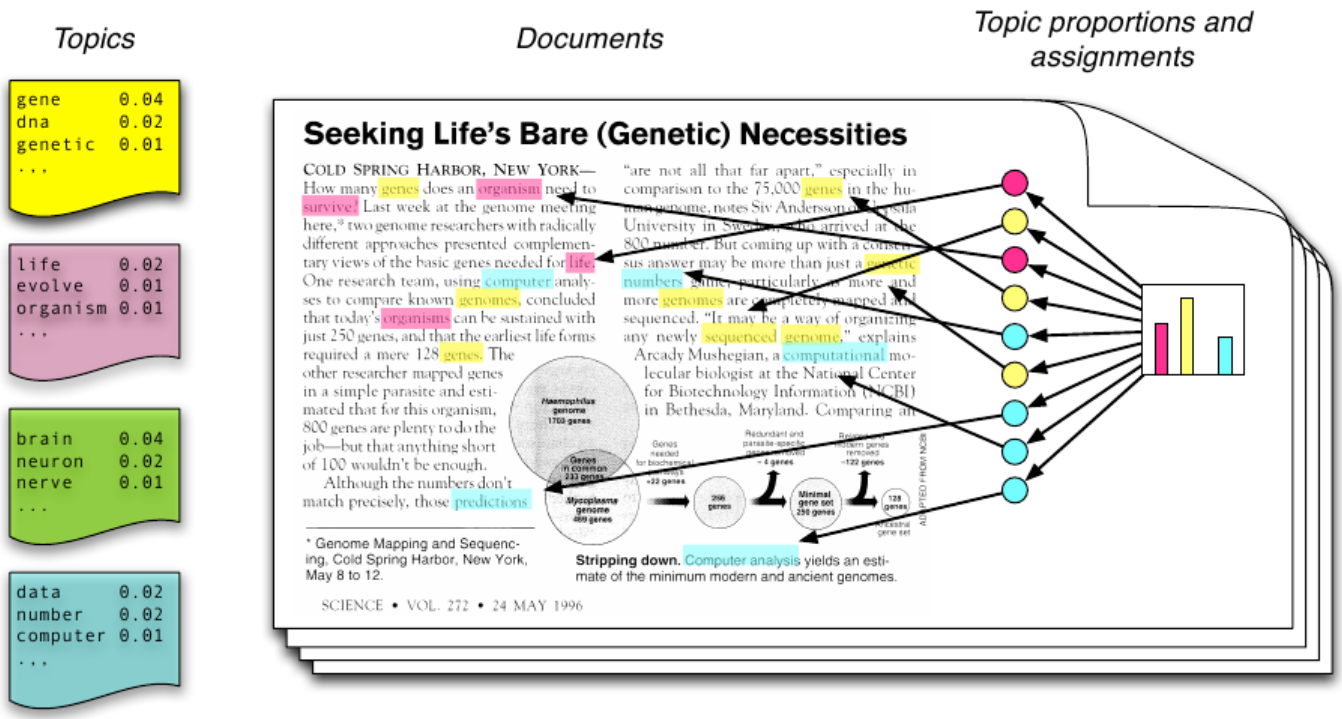


Model-Based Image Zooming

Proposed Model:

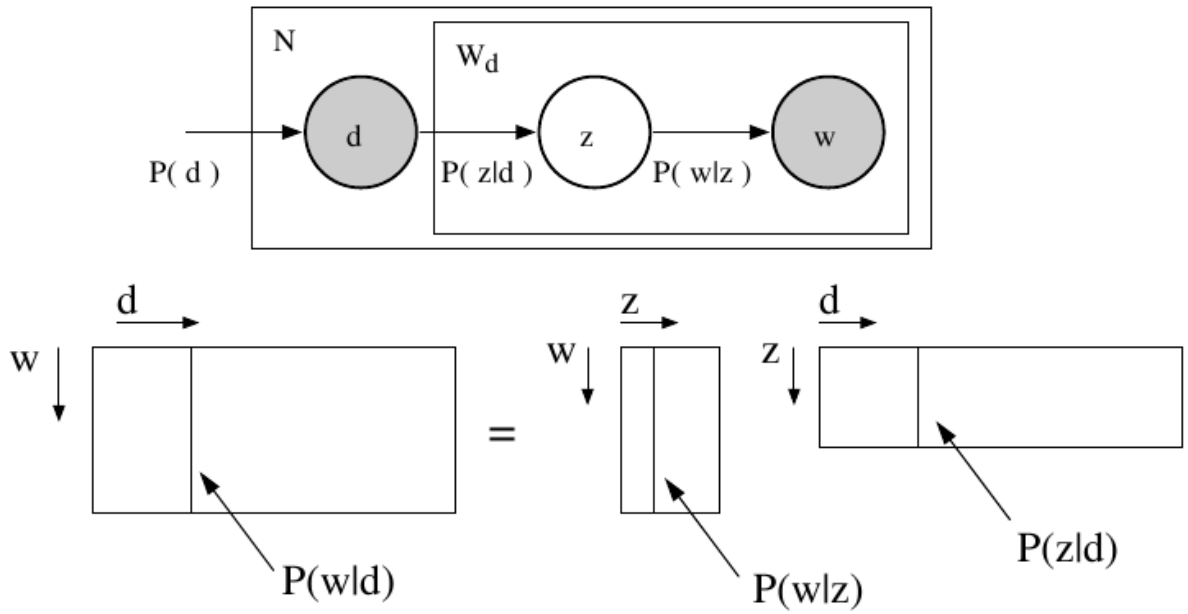
- Cluster /Classify patches using Probabilistic Latent Symantic Analysis (pLSA).
- Use sparse dual dictionary technique to predict HR patch.

Topic Modelling



- Each topic is a distribution of words.
- Each document is a mixture of topics in corpus.
- Each word is drawn from one of those topics.

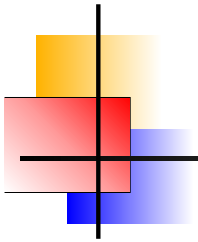
Probabilistic Latent Symantic Analysis (pLSA)



Model Assumption:

$$P(w_i|d_j) = \sum_{k=1}^{K} P(z_k|d_j)P(w_i|z_k),$$

where $P(z_k|d_j)$ is the probability of topic z_k occurring in document d_j, and $P(w_i|z_k)$ is the probability of word w_i occurring in a particular topic z_k.



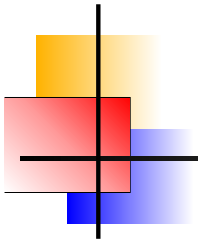
Incorporate Topic Model for image zooming

Why topic model?

- It can compute inherent topics (soft-clustering of documents) in unsupervised way.
- After training it can assign soft-label of topics on test documents.
- An EM algorithm is used to compute all the probability distributions.

Context of SR:

- "Document" as cropped portion of an image consisting of smaller patches as called "Words".
- "Topic" is a higher level concept (Document consist of smooth regions/ edge regions/ texture regions).



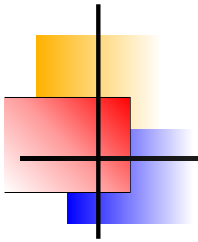
Proposed Methodology:

Offline Phase:

- Collect a set of natural images downloaded from a popular photographic forum.
- Generate HR and LR document pairs and run a topic learning algorithm. Separate out all the documents based on its topic assignment.
- For each cluster run a dual sparse dictionary learning algorithm.

Online Phase:

- For test image, divide it into overlapping documents and infer the topic assignment of each document.
- Predict HR patch for all LR patches inside the document using the corresponding dual dictionary it assigned to.



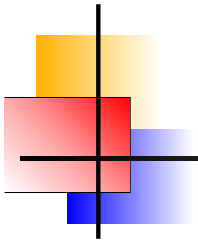
Learning Based Super Resolution : Detail approach

Sparse coding approach :

- Train coupled dictionary D_l and D_h of HR and LR image patches from similar images.
- For each patch y in LR image, solve

$$\hat{\alpha} = arg\min \|\alpha\|_1 \quad \text{subject to} \quad \begin{matrix} \|Fy - FD_l\alpha\|_2^2 \leq \varepsilon_1 \\ \|w - PD_h\alpha\|_2^2 \leq \varepsilon_2 \end{matrix}$$

- Generate the high-resolution patch $x = D\hat{\alpha}$ Put the patch x into a high-resolution image X_0.
- For color images Y channel is upsampled via sparse coding. Other channels are interpolated using bicubic interpolation.



Learning Based Super Resolution : Important issues

Sparse coding approach :

- Patch size : 5×5 to 10×10 gives good quality image.
- Size of dictionary : depends on size of the patch size (usually grater than dimension of the feature vector). We used 100 and 500 size dictionaries for 5×5 and 10×10 patches respectively.
- Dictionary learning : must be learned on similar type of images. If SR image is generated from sparsely reconstructed patches of a global dictionary then using back-projection solve :

$$X^{\star} = arg \min_{X} \|X - X_0\| \text{ subject to } LX = Y$$

$X^{\star}$ is the output SR image.

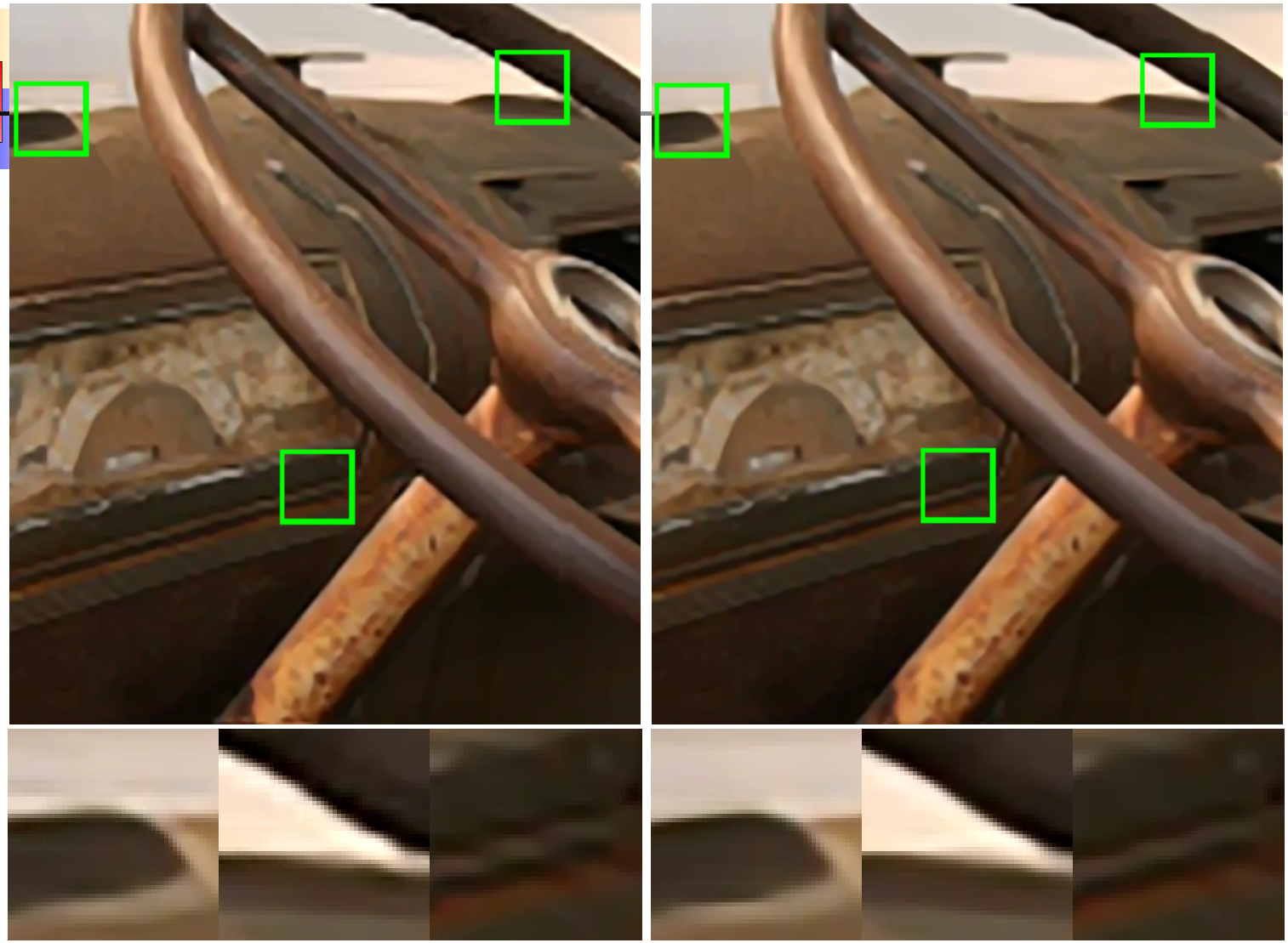
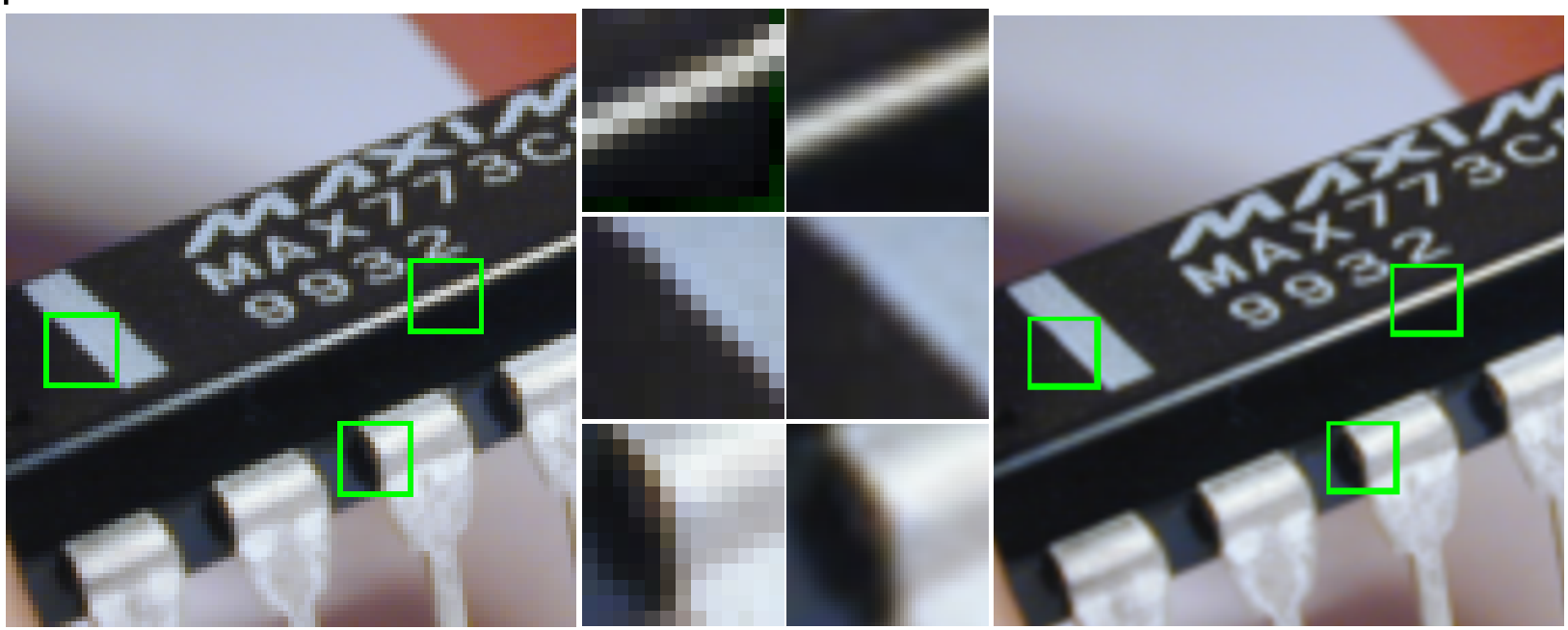
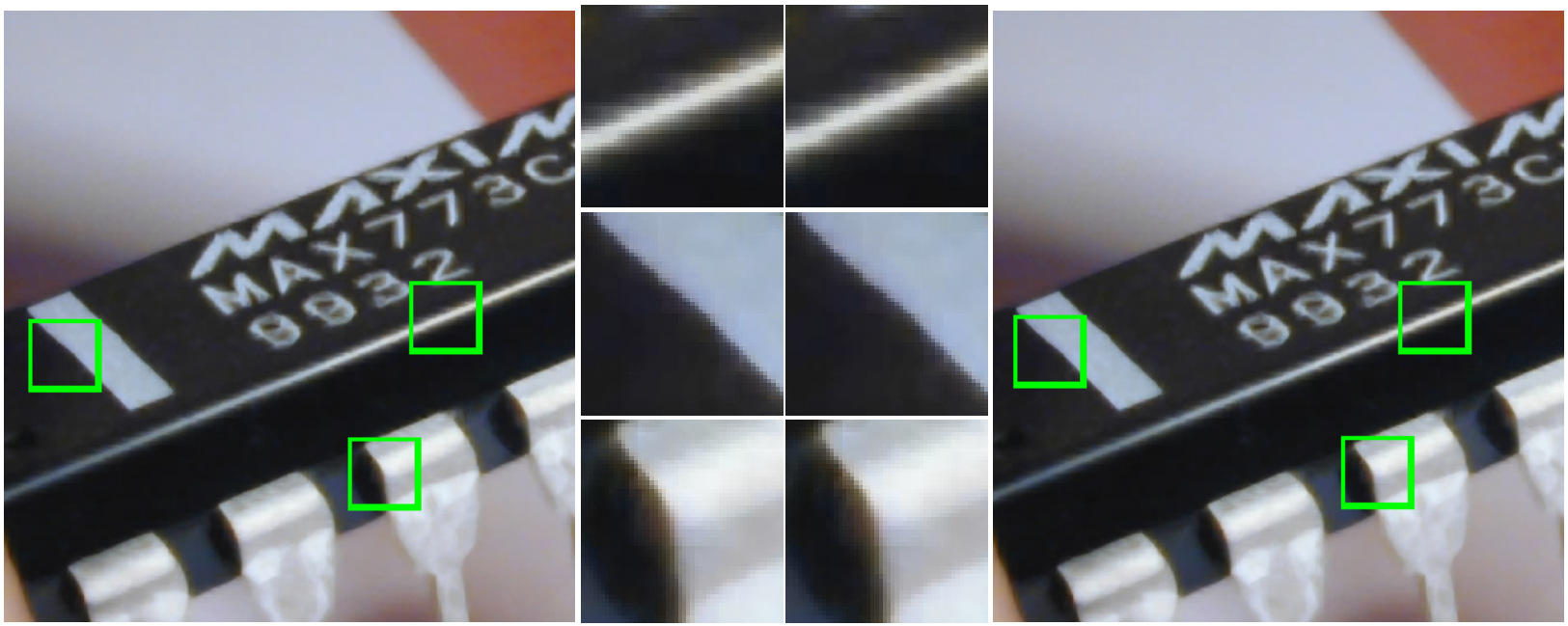


Figure: Using one single global dictionary proposed by Yang *et al.* [14] and proposed method with multiple dictionaries.

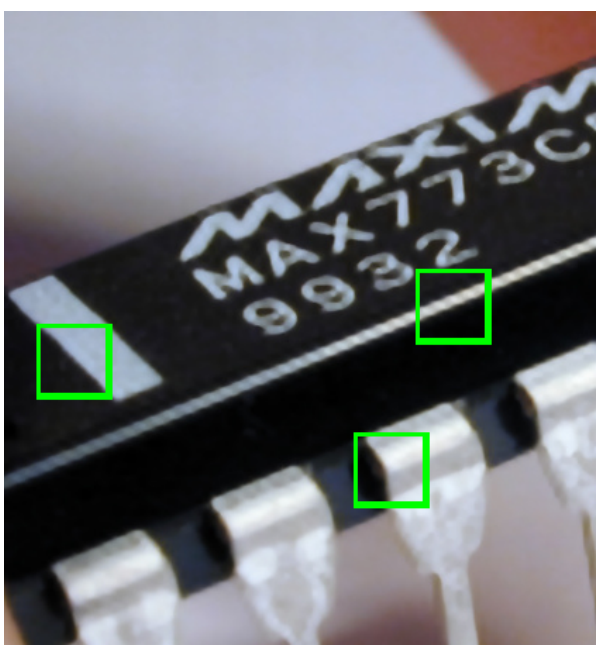


Input image (Scaled for display)

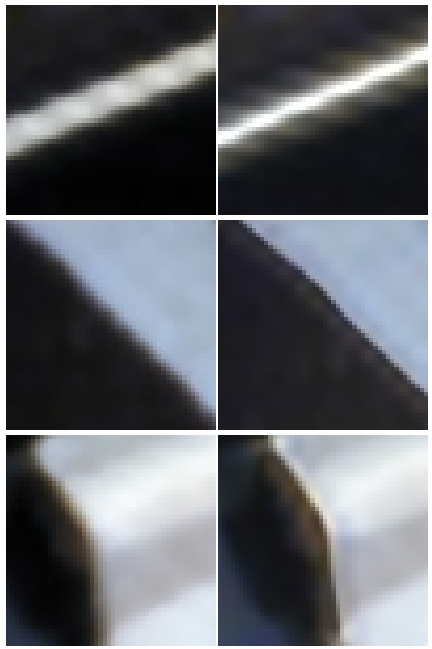
Bicubic interpolation.

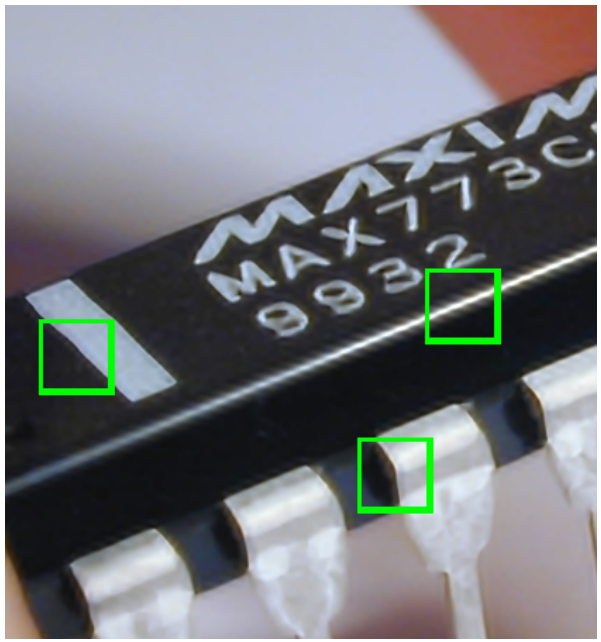


Genuine Fractals^{TM}(commercial product) Sun *et al.* [13]

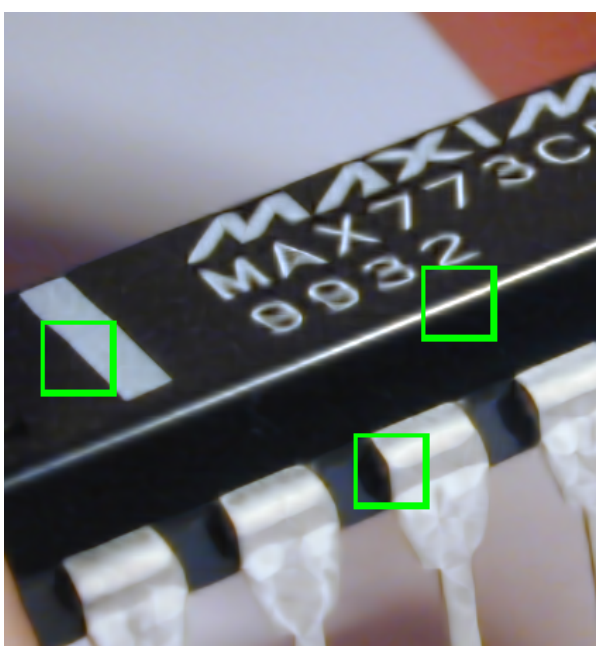


Fattal *et al.* [2],

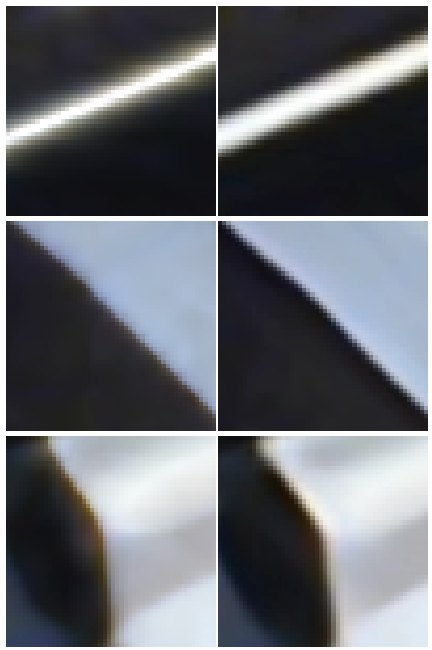


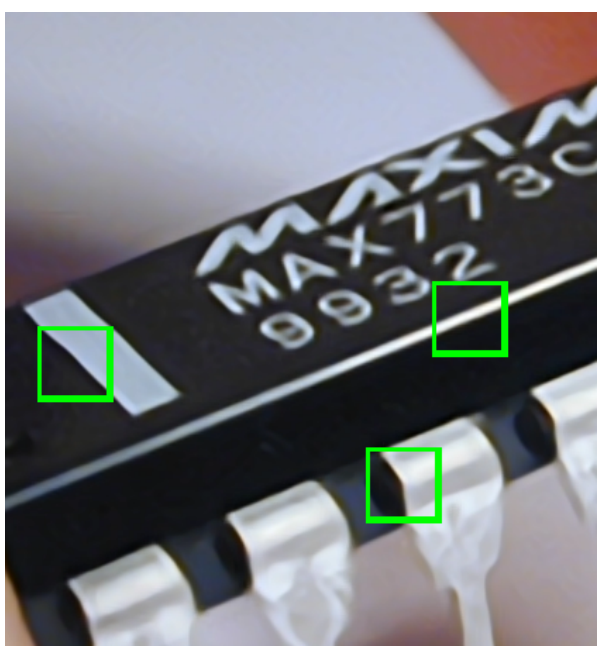


Glasner *et al.* [4]

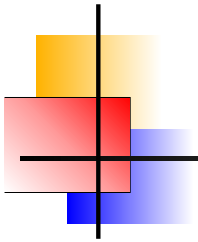


Freeman *et al.* [3]

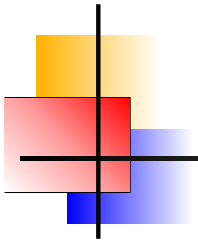




Proposed method.



Chapter V: Single-frame Super Resolution by exploring local self-similarity between patches.



Exploring Local self-similarity for Single-frame super resolution:

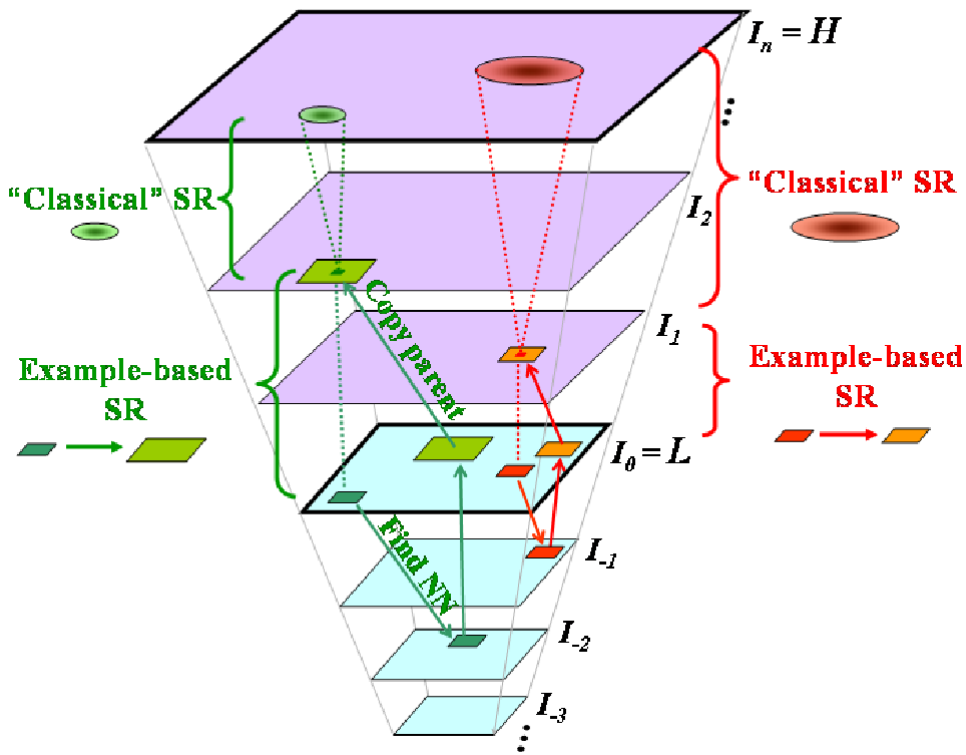
Motivation:

- Recent study suggest that patches are more tend to re-occur in the image itself in same scale or different scales, rather than patches in the other natural images.
- This already established in Glasner *et al.* [4] in naive way.

Difficulty:

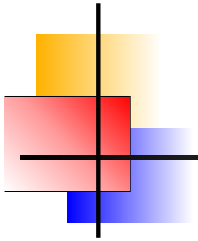
- No learning-testing mechanism for LR and HR patch correspondence. Learn online in the image itself.
- High computation : search nearest neighbour patch for all patches in the down-scaled images.

Glasner et al. technique:

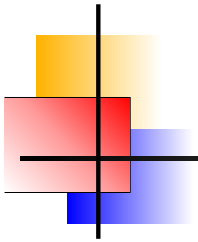


[2]

[2] This image taken from Glasner *et al.* [4]



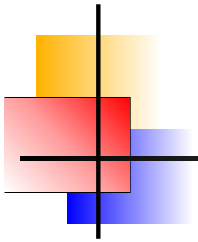
AN EFFICIENT AND FAST METHOD IS REQUIRED
STILL WORKING ON IT



Other Works:

Document Processing:

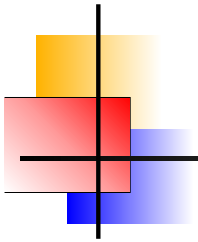
- Off-line Recognition of Hand-Written Bengali Numerals Using Morphological Features, P. Purkait, B. Chanda, *Frontiers in Handwriting Recognition (ICFHR), 2010 International Conference*
- Writer identification for handwritten telugu documents using directional morphological features, P. Purkait, R. Kumar, B. Chanda, *Frontiers in Handwriting Recognition (ICFHR), 2010 International Conference*



Other Works:

Sketch-Photo synthesis and Recognition:

- A novel technique for sketch to photo synthesis, P Purkait, S Kulkarni, B Chanda, *Proceedings of the Seventh Indian Conference on Computer Vision, Graphics and Image Processing (ICVGIP), 2010*
- Face Image Retrieval Based on Probe Sketch Using SIFT Feature Descriptors, K Atal, A Arora, P Purkait, B Chanda, *Perception and Machine Intelligence (PerMin), 2012*



Other Works:

Dance Classification:

- Indian Classical Dance Classification by Learning Dance Pose Bases, S Samanta, P Purkait, B Chanda, *IEEE Workshop on the Applications of Computer Vision (WACV), 2012*

S. Farsiu, M. D. Robinson, M. Elad, and P. Milanfar.
Fast and robust multiframe super-resolution.
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Image and video upscaling from local self-examples.
ACM Trans. Graph., 28(3):1–10, 2010.

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Xuelong Li, Yanting Hu, Xinbo Gao, Dacheng Tao, and Beijia Ning.
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J. of Sci. Comput., 37(3):367–382, Dec. 2008.

P. Purkait and B. Chanda.
Morphologic gain-controlled regularization for edge-preserving super-resolution image reconstruction.
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P. Purkait and B. Chanda.
Digital restoration of damaged mural images.
In *Proceedings of The 8th indian Conference on Vision, Graphics and Image Processing (ICVGIP-12) Submitted*, December 2012.

P. Purkait and B. Chanda.
Image upscaling using multiple dictionaries of natural image patches.

In *Proceedings of The 11th Asian Conference on Computer Vision (ACCV-12) Submitted*, November 2012.

P. Purkait and B. Chanda.
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Unpublished (On preparation), PP:1, 2012.

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IEEE Trans. Image Process., 21(9):–, 2012.

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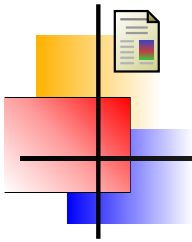
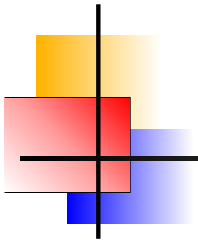
Jian Sun, Zongben Xu, and Heung-Yeung Shum.
Image super-resolution using gradient profile prior.
In *Proceedings of CVPR*, pages 1–8, Jun. 2008.



Jianchao Yang, John Wright, Thomas S. Huang, and Yi Ma.
Image super-resolution via sparse representation.
IEEE Trans. Image Process., 19(11):2861–2873, Nov. 2010.



Thanks !