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1. DETAILED COURSE STRUCTURE

1.1 M. Math. Curriculum

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1.2 List of Compulsory Courses

1. Analysis of Several Variables
2. Topology I
3. Linear Algebra
4. Algebra I
5. Measure Theoretic Probability
6. Complex Analysis
7. Functional Analysis
8. Algebra II
9. Topology II
10. Differential Geometry I
11. Basic Probability Theory (for non-BMath non-BStat students)
12. Fourier Analysis
13. Differential Topology
1.3 List of Elective Courses

**Group A (At least one elective course is to be chosen from this group.)**

1.3.1 Number Theory  
1.3.2 Advanced Number Theory  
1.3.3 Algebraic Number Theory

**Group B**

1. Differential Equations  
2. Graph Theory and Combinatorics  
3. Advanced Functional Analysis  
4. Operator Theory  
5. Partial Differential Equations  
6. Advanced Linear Algebra  
7. Advanced Probability  
8. Markov Chains  
9. Ergodic Theory  
10. Stochastic Processes  
11. Topology III  
12. Topology IV  
13. Differential Geometry II  
14. Algebra III  
15. Commutative Algebra I  
16. Commutative Algebra II  
17. Algebraic Geometry (PRQ: Op 1)  
18. Elliptic Curves  
19. Representations of Locally Compact Groups  
20. Lie Groups & Lie Algebra  
21. Linear Algebraic Groups  
22. Mathematical Logic  
23. Set Theory  
24. Game Theory  
25. Automata, Languages and Computation  
26. Advanced Fluid Dynamic  
27. Quantum Mechanics I  
28. Quantum Mechanics II  
29. Analytical Mechanics  
30. Special Topics (to be suggested by the faculty)  
31. Projects I and II
2. BRIEF SYLLABI

2.1 Compulsory Courses

C1. Analysis of Several Variables

Differentiability of maps from $\mathbb{R}^m$ to $\mathbb{R}^n$ and the derivative as a linear map. Higher derivatives, Chain Rule, Taylor expansions in several variables, Local maxima and minima, Lagrange multiplier. Multiple integrals, Existence of the Riemann integral for sufficiently well-behaved functions on rectangles, i.e., product of intervals. Multiple integrals expressed as iterated simple integrals. Brief treatment of multiple integrals on more general domains. Change of variables and the Jacobian formula, illustrated with plenty of examples. Inverse and implicit functions theorems. Picard’s Theorem.

Curves in $\mathbb{R}^2$ and $\mathbb{R}^3$ Line integrals, Surfaces in $\mathbb{R}^3$, Surface integrals, Integration of forms, Divergence, Gradient and Curl operations, Green’s, Stokes’ and Gauss’ (Divergence) theorems.

References:
- M. Spivak: *Calculus on manifolds*, Benjamin (1965).
- T. Apostol: *Mathematical Analysis*.
- R. Courant and F. John: *Introduction to Calculus and Analysis*.

C2. Topology I


Covering spaces, Path Lifting and Homotopy Lifting Theorems, Fundamental Group.

References:
C3. Linear Algebra


2. Review of basic concepts from rings and ideals required for module theory (if necessary). Modules over commutative rings: examples. Basic concepts: submodules, quotients modules, homomorphisms, isomorphism theorems, generators, annihilators, torsion, direct product and sum, direct summand, free modules, finitely generated modules, exact and split exact sequences.

3. Properties of \( K[\lambda] \) over a field \( K \). Structure theorem for finitely generated modules over a PID; applications to Abelian groups, rational and Jordan canonical forms.

Time permitting, snake’s lemma, complexes and homology sequences may be introduced.

References:
- S. Lang, *Algebra*, GTM (211), Springer (Indian reprint 2002).

C4. Algebra I


Note: It is desirable that Item No. 1 of Algebra-I is covered before Item No. 2 of Linear Algebra begins.

References:
• S. Lang, *Algebra*, GTM (211), Springer (Indian reprint 2004).

C5. Measure Theoretic Probability


Probability: (If needed a quick review of concepts and results (without proof) from basic Discrete and Continuous Probability.) Distribution Functions of Probability Measures on R, Borel-Cantelli Lemma, Weak and Strong Laws of Large Numbers in i.i.d. case, various Modes of Convergence, Characteristic Functions, Uniqueness/Inversion/Levy Continuity Theorems, Proof of the Central Limit Theorem for i.i.d. case with Finite Variance.

References:

C6. Functional Analysis


$L^p$ spaces, Riesz representation theorem for the space $C[0, 1]$.

Hilbert spaces, adjoint operators, self-adjoint and normal operators, spectrum, spectral radius, analysis of the spectrum of a compact operator on a Banach space, spectral theorem for bounded self-adjoint operators.

Time permitting: Goldsteins Theorem, reflexivity; spectral theorem for normal and unitary operators.

References:
C7. Complex Analysis

A review of basic Complex Analysis: Cauchy-Riemann equations, Cauchy’s theorem and estimates, power series expansions, maximum modulus principle, Classification of singularities and calculus of residues. Normal families, Arzela’s theorem. Product developments, functions with prescribed zeroes and poles, Hadamard’s theorem. Conformal mappings, the Riemann mapping theorem, the linear fractional transformations.

Depending on time available, some of the following topics may be done:
(i) Subharmonic functions, the Dirichlet problem and Green’s functions.
(ii) An introduction to elliptic functions.
(iii) Introduction to functions of several complex variables.

References:
- J. B. Conway, Functions of one complex variable II. GTM (159), Springer-Verlag, 1995.

C8. Algebra II

Results on finite groups: permutation groups, simple groups, solvable groups, simplicity of An. Topics like semi-direct product (if not covered in Algebra-I).

Algebraic and transcendental extensions; algebraic closure; splitting fields and normal extensions; separable, inseparable and purely inseparable extensions; finite fields.

Galois extensions and Galois groups, Fundamental theorem of Galois theory, cyclic extensions, solvability by radicals, constructibility of regular n-gons, cyclotomic extensions.

Time permitting, additional topics may be selected from:
(i) Traces and norms, Hilbert theorem 90, Artin-Schrier theorem, Galois cohomology, Kummer extension.
(ii) Transcendental extensions.
(iii) Real fields: ordered fields, real closed fields, Sturm’s theorem, real zeros and homomorphisms.
(iv) Integral extensions and the Nullstellensatz.

References:
- S. Lang, Algebra, GTM (211), Springer (Indian reprint 2004).
- N.S. Gopalakrishnan, University Algebra, Wiley Eastern (1986).
- TIFR pamphlet on Galois theory.
C9. Topology II

Review of fundamental groups, necessary introduction to free product of groups, Van Kampen’s theorem. Covering spaces, lifting properties, Universal cover, classification of covering spaces, Deck transformations, properly discontinuous action, covering manifolds, examples.

Categories and functors. Singular homology groups, axiomatic properties, Mayer-Vietoris sequence, homology with coefficients, statement of universal coefficient theorem for homology, simple computation of homolgy groups.

CW-complexes and Cellular homology, Simplicial complex and simplicial homology as a special case of Cellular homology, Relationship between fundamental group and first homology group.

References :

C10. Differential Geometry I

Parametrized curves in $\mathbb{R}^3$, length of curves, integral formula for smooth curves, regular curves, parametrization by arc length. Osculating plane of a space curve, Frenet frame, Frenet formula, curvatures, invariance under isometry and reparametrization. Discussion of the cases for plane curves, rotation number of a closed curve, osculating circle, ‘Umlaufsatz’.

Smooth vector fields on an open subset of $\mathbb{R}^3$, gradient vector field of a smooth function, vector field along a smooth curve, integral curve of a vector field. Existence theorem of an integral curve of a vector field through a point, maximal integral curve through a point.


References :
C11. Basic Probability Theory

(Compulsory for non BStat/BMath Students. B.Stat./B.Math. students can’t opt for this.)


Joint and conditional distributions, Independence of random variables, Transformation of variables.

Laws of Large Numbers (proofs optional).

References:

C12. Differential Topology

Manifolds in RN, submanifolds, smooth maps of manifolds, derivatives and tangents, Inverse function theorem and immersions, submersions, Transversality, Homotopy and stability, Sard’s theorem and Morse functions, embedding manifolds in Euclidean space.

Differential Forms and Integration of forms, Stokes’ Theorem, Definition of DeRham Cohomology.

References:

C13. Fourier Analysis


References:
2.2 Elective Courses

Group A
(A student must choose at least one elective from this group.)

A1. Number Theory

Review of unique factorization; properties of the rings $\mathbb{Z}[i]$ and $\mathbb{Z}[\omega]$ (chapter 1 of IR).

Review of congruences, Euler’s $\phi$-function, results of Fermat, Euler and Wilson; linear congruences, Chinese remainder theorem. Primitive roots and the group structure of $U(\mathbb{Z}/n\mathbb{Z})$; applications to congruences of higher degree; Hensel’s Lemma (chapter 4 of IR and sections 2.1 to 2.7 of NZM).

Quadratic Reciprocity: Quadratic Residues, Gaussian reciprocity law, the Jacobi symbol (chapter 5 of IR).

Arithmetic Functions, Moebius inversion formula and combinatorial methods like principle of inclusion-exclusion and pigeonhole etc (sections 4.2,4.3,4.5 of NZM).

Diophantine equations. Linear equations, the equation $x^2+y^2=z^2$. Method of Descent; the equation $x^4+y^4=z^2$ (section 5.1 to 5.4 of NZM).

Binary Quadratic forms. Sum of two squares. Legendre’s Theorem (section 3.4 to 3.7 of NZM).

Simple continued fractions. Infinite continued fractions and irrational numbers. Periodic continued fractions, algorithms for solving Brahmagupta-Pell equation, numerical computations. Dirichlet’s box principle and solution of Pell’s equation (chapter 7 of NZM).

Elementary results on the function $\pi(x)$, Bertrand’s postulate (sections 8.1, 8.2 of NZM).

Time permitting, additional topics may be chosen from:

(i) Partitions. Euler’s identity and Euler’s formula (sections 10.1 to 10.4 of NZM).
(ii) Gauss and Jacobi Sums, Cubic and Biquadratic Reciprocity (from chapters 8,9 of IR).
(iii) Irrational numbers: Hurwitz’s theorem on rational approximations; irrationality of certain values of trigonometric functions; irrationality of $\pi$ (chapter 6 of NZM).
(iv) Diophantine equations over finite fields (chapter 10 of IR).
(v) Introduction to Zeta function, Dirichlet’s $L$-functions and Elliptic Curves (from IR).

References:


A2. Advanced Number Theory

Review of finite fields; Polynomial equations over finite fields: theorems of Chevalley and Warning;
Quadratic Forms over prime fields. Review of the law of quadratic reciprocity.

The ring of $p$-adic integers; the field of $p$-adic numbers; completion; $p$-adic equations and Hensel’s lemma; Quadratic Forms with $p$-adic coefficients. Hilbert’s symbol.


References:

A3. Algebraic Number Theory

Review of norm and trace, Number fields and their rings of integers, Prime decomposition in number rings, Kummer-Dedekind discriminant criterion for ramification, The Ideal Class Group, ray class group, their finiteness and Dirichlet’s Unit theorem, Valuations and completions of number fields, Decomposition and inertia groups, Frobenius automorphism, Artin symbol, Dedekind zeta function and the Distribution of ideals in a number ring, Kronecker limit formula, Frobenius density theorem. Time permitting, introduction to class field theory.

References:
- D.A. Marcus: *Number Fields*, Springer.
- TIFR pamphlet on *Algebraic Number Theory*.

Note: For students opting for “Algebraic Number Theory”, a prior knowledge of “Commutative Algebra” is desirable.

Group B

B1. Differential Equations
*Only for non BStat/BMath students*

Ordinary differential equations – first order equations, Picard’s theorem (existence and uniqueness of solution to first order ordinary differential equation), Second order linear equations – second order linear differential equations with constant co-efficients, Systems of first order differential equations, Equations with regular singular points, Introduction to power series and power series solutions, Special ordinary differential equations arising in physics and some special functions (eg. Bessel’s functions,
Legendre polynomials, Gamma functions).


*Note: The course may be combined with the B.Math./B.Stat. course on “Differential Equations”.*

References:

**B2. Graph Theory and Combinatorics**

Graphs and digraphs, connectedness, trees, degree sequences, connectivity, Eulerian and hamiltonian graphs, matchings and SDR’s, chromatic numbers and chromatic index, planarity, covering numbers, flows in networks, enumeration, Inclusion-exclusion, Ramsey’s theorem, recurrence relations and generating functions.

References:

*Note: The course may be combined with the “Graph Theory and Combinatorics” course of M.Stat. 2nd year.*

**B3. Advanced Functional Analysis**

Brief introduction to topological vector spaces (TVS) and locally convex TVS. Linear Operators. Uniform Boundedness Principle. Geometric Hahn-Banach theorem and applications (Markov-Kakutani fixed point theorem, Haar Measure on locally compact abelian groups, Liapounovs theorem). Extreme points and Krein-Milman theorem. In addition, one of the following topics:
(b) Banach algebras, spectral radius, maximal ideal space, Gelfand transform.
(c) Unbounded operators, Domains, Graphs, Adjoints, spectral theorem.

References:
B4. Operator theory

2. \( C^* \)-algebras—noncommutative states and representations, Gelfand-Neumark representation theorem.
3. Von-Neumann Algebras; Projections, Double Commutant theorem, \( L^\infty \) functional Calculus.
4. Toeplitz operators.

References :

B5. Partial Differential Equations

Theory of Schwartz distributions and Sobolev spaces; local solvability and Lewys example; existence of fundamental solutions for constant coefficient differential operators; Laplace, heat and wave equations, hypoelliptic and analytic hypoelliptic operators, elliptic boundary value problems interior regularity, local existence.

References :

B6. Advanced Linear Algebra

Topics from:

References :
B7. Advanced Probability


References:

Note: The syllabus for the “Advanced Probability” course of M.Stat. 2nd year may also be used.

B8. Markov Chains


Suggested Texts:

B9. Ergodic Theory

Measure preserving systems; examples: Hamiltonian dynamics and Liouvilles theorem, Bernoulli shifts, Markov shifts, Rotations of the circle, Rotations of the torus, Automorphisms of the Torus, Gauss transformations, Skew-product.


The Isomorphism problem; conjugacy, spectral equivalence.

Transformations with discrete spectrum, Halmosvon Neumann theorem.


References:

**B10. Stochastic Processes**

Weak Convergence of probability measures on Polish spaces including $C[0, 1]$, Brownian motion; construction, simple properties of paths, Connections between Brownian Motion / Diffusion and PDEs. Time permitting: Stationary processes, Markov processes and generators.

References:
- D. Revuz and M. Yor, *Continuous martingales and Brownian motion*, Grundlehren der Mathematischen Wissenschaften (293), Springer-Verlag (1999).

Note: (1) For students opting for this elective, a prior knowledge of the contents of the courses Probability I, II, III of the B.Stat. programme (ISI) is essential. (2) The course may be combined with the course “Stochastic Processes-I” of M.Stat. 2nd year.

**B11. Topology III**

CW–complexes, cellular homology, comparison with singular theory, computation of homology of projective spaces.


Definition of higher homotopy groups, homotopy exact sequence of a pair. Definition of fibration, examples of fibrations, homotopy exact sequence of a fibration, its application to computation of homotopy groups. Hurewicz homomorphism, The Hurewicz theorem. The Whitehead Theorem.

References:
B12. Topology-IV

Smooth manifolds, Differential forms on manifolds, Integration on manifolds, Stoke’s theorem, computation of cohomology rings of projective spaces, Borsuk-Ulam theorem.
Degree, linking number and index of vector fields, The Poincare-Hopf theorem.
Definition and examples of principal bundles and fibre bundles, clutching construction, description of classification theorem (without proof).

References :
- R. Bott and L. W. Tu, Differential forms in algebraic topology, GTM (82), Springer-Verlag (1982).
- M. J. Greenberg, Lectures on algebraic topology, Benjamin (1967).

B13. Differential Geometry II

A quick review of tensors, alternating forms, manifolds, immersion, submersion and submanifolds.

References :
- S. Helgason, Differential geometry, Lie groups, and symmetric space, Graduate Studies in Mathematics (34), AMS (2001).

B14. Algebra III

Balanced maps, Tensor product of modules and algebras: definitions, basic properties, right exactness, change of base.
Semisimple rings and modules; Wedderburn’s structure theorem.
Nilradical and Jacobson radical; NAK lemma; Jacobson radical of an Artinian ring is nilpotent; Ring semi-simple if and only if Artinian with trivial radical; Artinian ring is Noetherian.
Central Simple Algebras and the Brauer Group; Examples.
Representation of finite groups: group algebra, Maschke’s Theorem, Simple Modules over
Group Algebras; Characters and Orthogonality relations; Burnside’s two-prime theorem; Induced representation; Frobenius reciprocity; Brauer’s theorem on induced characters.

References:
- P.M. Cohn, *Further Algebra and Applications*, Springer.
- TIFR notes on *Semisimple rings and modules*.

B15. Commutative Algebra I

Rings and ideals: review of ideals in quotient rings; prime and maximal ideals, prime ideals under quotient, existence of maximal ideals; operations on ideals (sum, product, quotient and radical); Chinese Remainder theorem; nilradical and Jacobson radical; extension and contraction of ideals under ring homomorphisms; prime avoidance.

Free modules; Projective Modules; Tensor Product of Modules and Algebras; Flat, Faithfully Flat and Finitely Presented Modules; Shanan’s Lemma.

Localisation and local rings, universal property of localisation, extended and contracted ideals and prime ideals under localisation, localisation and quotients, exacteness property. Results on prime ideals like theorems of Cohen and Isaac. Nagata’s criterion for UFD and applications; equivalence of PID and one-dimensional UFD.


Valuations, Discrete Valuation Rings, Dedekind domains.

References:
B16. Commutative Algebra-II

Pre-requisite: Commutative Algebra I


Valuations, Discrete Valuation Rings, Dedekind Domains, Fractional and Invertible Ideals, Ramification Formula.

Results on Normal and Regular Rings: Local property of Normal Domains, Normality and DVR at height one primes, Intersection of DVRs; Jacobian criterion for regular local rings (of affine algebras).


Properties of Regular Local Rings. Homological Characterisation of Regular Local Rings. Regular Local Ring is a UFD.

Derivations and Modules of Differentials.

References:
- TIFR pamphlet on *Homological Methods in Commutative Algebra*.

B17. Algebraic Geometry

References:


*Note:* For students opting for “Algebraic Geometry”, a prior knowledge of “Commutative Algebra” is desirable.

**B18. Elliptic Curves**

*Pre-requisites:* A course in complex analysis, a course in number theory, a course in algebraic number theory (could be simultaneous), a course in algebraic geometry (could be simultaneous).

1. Algebraic curves, divisors, Riemann-Roch theorem.
2. Definition of elliptic curves, Weierstrass form, isogeny, Tate module, Weil pairing, Endomorphism ring.
3. Elliptic functions and integrals, Elliptic curves over complex numbers, Uniformization.
5. Elliptic curves over local fields, Minimal Weierstrass equation, Torsion, Good and bad reduction and Neron-Ogg-Shafarevich criterion for good reduction.
6. Elliptic curves over global fields, weak Mordell-Weil, Kummer pairing, Mordell-Weil theorem over $\mathbb{Q}$.

If time permits: Heights on projective spaces and elliptic curves and Mordell-Weil theorem; Nagell-Lutz theorem.

**References:**


**B19. Representations of Locally Compact Groups**

Topological Groups, basic properties like subgroups, quotients and products, fundamental systems of neighbourhoods, open subgroups, connectedness and compactness. Existence of Haar measure on locally compact groups, properties of Haar measures.

Group actions on topological spaces, the space $X/G$ in the topological as also in the analytical case assuming regularity conditions of the group action.

Representation of a locally compact group on a Hilbert space, the associated representation of group algebra, invariant subspaces and irreducibility, Schurs lemma.

Compact groups: Unitarity of finite dimensional representations, Peter-Weyl theorem, Representations of $SU(2,\mathbb{C})$, Representation of a finite group.
Induced representation and Frobenius reciprocity theorem, Representations of Heisenberg groups and of Euclidean motion group, Principal series representations of $SL(2,\mathbb{R})$.

**B20. Lie Groups and Lie Algebras**

1. Linear Lie groups: the exponential map and the Lie algebra of linear Lie group, some calculus on a linear Lie group, invariant differential operators, finite dimensional representations of a linear Lie group and its Lie algebra. Examples of linear Lie group and their Lie algebras, e.g., Complex groups: $GL(n,\mathbb{C}), SL(n,\mathbb{C}), SO(n,\mathbb{C})$, Groups of real matrices in those complex groups: $GL(n,\mathbb{R}), SL(n,\mathbb{R}), SO(n,\mathbb{R})$, Isometry groups of Hermitian forms $SO(m, n), U(m, n), SU(m, n)$. Finite dimensional representations of $su(2)$ and $SU(2)$ and their connection. Exhaustion using the lie algebra $su(2)$. [2 weeks]


3. Semisimple Lie algebra and structure theory: Definition of Linear reductive and linear semisimple groups. Examples of Linear connected semisimple/reductive Lie groups along with their Lie algebras (look back at 2 above and find out which are reductive/semisimple). Cartan involution and its differential at identity; Cartan decomposition $g = k + p$, examples of $k$ and $p$ for the groups discussed above. Definition of simple and semisimple Lie algebras and their relation, Cartans criterion for semisimplicity. Statements and examples of Global Cartan decomposition, Root space decomposition; Iwasawa decomposition; Bruhat decomposition. [6 weeks]

If time permits, one of the following topics:

(i) A brief introduction to Harmonic Analysis on $SL(2,\mathbb{R})$.
(ii) Representations of Compact Lie Groups and Weyl Character Formula.
(iii) Representations of Nilpotent Lie Groups.

**References:**

- J.E. Humphreys: *Introduction to Lie algebras and representation theory*, GTM (9), Springer.
- S.C. Bagchi, S. Madan, A. Sitaram and U.B. Tiwari: *A first course on representation theory and linear Lie groups*, University Press.
- Serge Lang: *SL(2,\mathbb{R})*. GTM (105), Springer.

**B21. Linear Algebraic Groups**

**Pre-requisites:** A course in commutative algebra, a course in Lie algebras (could be simultaneous), a course in algebraic geometry (could be simultaneous).

Review of background commutative algebra and algebraic geometry (as in chapter 1 of Humphreys’s book or chapter 1 of Springer’s book). Definition of affine algebraic groups and homomorphisms over algebraically closed fields, examples. Orbit-closures under actions, linearity of affine groups. Lie

References:
- J. E. Humphreys, *Linear algebraic groups*, (chapters 1 to 10), GTM, Springer-Verlag (1975).
- R. Steinberg, *Conjugacy classes in algebraic groups*, (Chapters 1 and 2 only as reference, some proofs are not given), Lecture Notes in Mathematics (366), Springer-Verlag (1974).

**B22. Mathematical Logic**

Syntax of First-Order Logic: First Order Languages, Terms and Formulas of a First Order language, First Order Theories.

Semantics of First-Order Languages: Structures of First-Order Languages, Truth in a Structure, Model of a Theory.

Propositional Logic: Tautologies and Theorems of propositional Logic, Tautology Theorem.

Proof in First Order Logic, Metatheorems of a first order theory, e.g. , theorems on constants, equivalence theorem, deduction and variant theorems etc., Consistency and Completeness, Lindenbaum Theorem.

Henkin Extension, Completeness theorem, Extensions by definition of first order theories, Interpretation theorem.

Model Theory: Embeddings and Isomorphisms, L¨owenheim-Skolem Theorem, Compactness theorem, Categoricity, Complete Theories.

Recursive functions, Arithmatization of first order theories, Decidable Theory, Representability, Godel’s first Incompleteness theorem.

References:

**B23. Set Theory**

Either (A) or (B)
(A) Descriptive Set Theory.
1. A quick review of elementary cardinal and ordinal numbers, transfinite induction, induction on trees, Idempotence of Souslin operation.
3. Analytic and coanalytic sets and their regularity properties, separation and reduction theorems, Von Neumann and Kuratowski-Ryll Nardzewskis selection theorems, Uniformization of Borel sets with large and small sections. Kondos uniformization theorem.

References:

(B) Axiomatic Set Theory
A naive review of cardinal and ordinal numbers including regular and singular cardinals, some large cardinals like inaccessible and measurable cardinals. Martins Axiom and its consequences. Axiomatic development of set theory upto foundation axiom, Class and Class as models, relative consistency, absoluteness, Reflection principle, Mostowski collapse lemma etc., non-decidability of large cardinal axioms, Godel’s second incompleteness theorem, Godel’s constructible universe, Forcing lemma and independence of CH.

References:
- S. M. Srivastava, *A Course on Axiomatic Set Theory*

B24. Game Theory

B. Introduction to Cooperative Games (TU Games).

Note: The course may be combined with the course “Game Theory-I” of M.Stat. 2nd year or the course “Game Theory-I” of the MSQE programme at ISI.

B25. Automata, Languages and Computation

1. Automata and Languages: Finite automata, regular languages, regular expressions, equivalence of deterministic and non-deterministic finite automata, minimisation of finite automata, closure properties, Kleene’s theorem, pumping lemma and its applications, Myhill-Nerode theorem and
its uses. Context-free grammar, context-free languages, Chomsky normal form, closure properties, pumping lemma for CFL, pushdown automata.

2. **Computability:** Computable functions, primitive and partial recursive functions, universality and halting problem, recursive and recursively enumerable sets, parameter theorem, diagonalisation and reducibility, Rice’s theorem and its applications, Turing machines and its variants, equivalence of different models of computation and Church-Turing thesis.

3. **Complexity:** Time complexity of deterministic and non-deterministic Turing machines, P and NP, NP-completeness, Cook’s theorem: other NP-complete problems.

**References:**


*Note:* The course may be combined with the corresponding course of M. Tech.(CS) 1st Year.

**B26. Advanced Fluid Dynamics**


Viscous incompressible fluid: Equations of motion of a viscous fluid, Reynold’s number, circulation in a viscous liquid, Flow between parallel plates, flow through pipes of circular, elliptic and annular section under constant pressure gradient. Prandtl’s concept of boundary layer.

**References:**

B27. Quantum Mechanics I

2. (i) Schrodinger wave equation (ii) Perturbation theory.
3. Problem of two or more degrees of freedom without spherical symmetry; Stark effect.
4. Angular momentum, SU(2) algebra.
7. WKB approximation.

References:
- L.I. Schiff, Quantum Mechanics.
- J.J. Sakurai, Modern Quantum Mechanics.

B28. Quantum Mechanics II


References:
- L.I. Schiff, Quantum Mechanics.
- H. Weyl, *The theory of groups and Quantum Mechanics*.

**B29. Analytical Mechanics**


**References:**