ABSTRACTS

Twelfth Discussion Meeting in Harmonic Analysis

Indian Statistical Institute, Kolkata

December 27--29, 2011

<u>**Thematic lectures**</u>

(1) **Resolution of singularities for analysts: an introduction**
(2) **An analytic resolution of singularities algorithm in two dimensions**
(3) **A multidimensional resolution of singularities algorithm with applications to analysis**

Malabika Pramanik
University of British Columbia

Abstract: The structure of polynomial zero sets is a multi-faceted line of study, with connections to many fields of mathematics, notably analysis, algebra and geometry. In this three-part lecture series I will outline a genre of problems in analysis of both classical and modern vintage where such sets play a central role. A representative problem is the effective computation of the log canonical threshold or critical integrability index, which we will introduce in the first talk. The second lecture will discuss a geometric characterization of the index in the bivariate setting where the problem is well-understood. The concluding presentation will address the challenges inherent in a multivariate generalization of the bivariate result, and discuss recent progress in this direction jointly with Tristan Collins and Allan Greenleaf.

Zeros of polynomials and spectrum of rank one transformations

M. G. Nadkarni
University of Mumbai and Indian Institute of Technology, Indore

Some questions on integral geometry on noncompact symmetric spaces of higher rank

E. K. Narayanan
Indian Institute of Science, Bangalore

Abstract: Let G/K be a noncompact symmetric space of higher rank. We study two types of averages of functions on G/K: one, over the level sets of the heat kernel on G/K, and the other over spheres on G/K. We prove some injectivity results for functions in the L^p class. This is a joint work with A. Sitaram.

Weyl-Heisenberg frame operators

K. Parthasarathy
Ramanujan Institute, University of Madras

Abstract: We seek to characterise, in simple and unsophisticated terms, frame operators of Weyl-Heisenberg frames. We succeed only partially, using the newly introduced concepts of window operators and tile vertices. However, we are able to completely characterise the frame operator in each of two newly introduced classes: window Weyl-Heisenberg frames (a special class of Weyl-Heisenberg frames) and generalised Weyl-Heisenberg frames (a class more general than Weyl-Heisenberg frames).

Wavelet packets and wavelet frame packets on local fields

Qaiser Jahan
Indian Statistical Institute, Kolkata

Abstract: Using a prime element of a local field of positive characteristic, the concepts of multiresolution analysis (MRA) and wavelet can be generalized to such a field. We formulate a version of the splitting lemma for this setup and using this lemma construct the wavelet packets associated with such MRAs. We see that these wavelet packets generate an orthonormal basis by translations only. Similar things can be done for frames and we can construct the wavelet frame packets in this setting.

Riesz transform and multipliers for the Grushin operator

Jotsaroop Kaur
Indian Institute of Science, Bangalore

Abstract: This is a joint work with Sanjay P.K. and Prof. S. Thangavelu. In this talk I will show that Riesz transforms associated to the Grushin operator $G = -\triangle - |x|^2\partial t^2$ are bounded on $L^p(R^{n+1})$. We also establish an analogue of Hormander-Mihlin multiplier theorem for the Grushin operator.

Non-random and random Toeplitz matrices

Arup Bose
Indian Statistical Institute, Kolkata

Abstract: Symmetric and non-symmetric Toeplitz matrices and Toeplitz operators have been around for almost a century now in mathematics. Recently, this matrix has gained prominence in the study of probabilistic aspects of random matrices and in statistical time series analysis.

It is known that these matrices are related to circulant matrices. The spectrum of these infinite dimensional operators may be approached via the discrete Fourier transform of the entries of an appropriate approximating circulant matrix of increasing dimension.

The random symmetric Toeplitz matrices cannot however be approximated by circulant matrices. Nevertheless, when the entries are i.i.d., their limiting spectrum turns out to be non-random, unbounded, and have sub-Gaussian tails. They are also universal, not depending on the exact distribution of the entries but only on their variance. It is noteworthy that the corresponding result for non-symmetric matrices is not yet proven, although simulations do indicate it to be true.

The sample autocovariance matrix in time series is a random Toeplitz matrix but with entries which have heavy dependence. Its limiting spectral distribution also exists but does not bear a one to one relation with the spectrum of the theoretical autocovariance matrix/operator. This leads to interesting statistical issues.

We shall give a brief introduction to this historically famous and currently important and interesting matrix and operator, touching on some of the results mentioned above.

Ramanujan's Master Theorem for Riemannian symmetric spaces

Angela Pasquale
Universit Paul Verlaine-Metz

Abstract: Ramanujan's Master Theorem states that, under suitable conditions, the Mellin transform of a power series provides an interpolation formula for the coefficients of this series:

$$\int_0^\infty x^{-s-1}\left(\sum_{k=0}^{\infty}(-1)^k a(k)x^k\right)dx = -\frac{\pi}{\sin \pi s}\,a(s)\,.$$

By selecting particular values for the function a, Ramanujan applied this theorem to compute several definite integrals and power series. This explains why it is referred to as the "Master Theorem".

Based on the duality of Riemannian symmetric spaces of compact and noncompact type inside a common complexification, we prove an analogue of Ramanujan's Master Theorem for the spherical Fourier transform of a spherical Fourier series.

This is a joint work with Gestur Ólafsson.

Involutions on the second duals of group algebras versus subamenable groups

Ajit Iqbal Singh
Indian Statistical Institute, Delhi

Abstract: Let $L^1(G)^{**}$ be the second dual of the group algebra $L^1(G)$ of a locally compact group G. We study the question of involutions on $L^1(G)^{**}$. A new class of subamenable groups is introduced which is universal for all groups. There is no involution on $L^1(G)^{**}$ for a subamenable group G.

Riesz transforms on Heisenberg groups*

P. K. Sanjay
Indian Institute of Science, Bangalore

Abstract: We characterise higher order Riesz transforms on the Heisenberg group and also show that they satisfy dimension-free bounds under some assumptions on the multipliers. Using transference theorems, we deduce boundedness theorems for Riesz transforms on the reduced Heisenberg group and hence also for the Riesz transforms associated to Hermite expansions.

*Based on a joint work with S. Thangavelu.

Solutions of Schrodinger and wave equations of the Grushin operator

D. Venku Naidu
Indian Institute of Science, Bangalore

Abstract: In this talk we will consider partial differential equations, namely, Schrodinger and wave equations associated to Grushin operator on $\mathbb{R}^{n+1}$: First, we will discuss uniqueness solution of Schrodinger equation under certain decay conditions on initial data f and solution $u(x;t;s_0)$ at some point of time s_0. Secondly, we will concentrate on solutions of the wave equation and we obtain L^2-norm estimates for solutions of the wave equation associated to Grushin operator.

On the rationality of the spectrum in 1-dimension

Debashish Bose
Chennai Mathematical Institute, Chennai

Abstract: A set Ω, of Lebesgue measure 1, in the real line is called spectral if there is a set Λ of real numbers such that the exponential functions $e_\lambda(x) = \exp(2\pi i \lambda x), \lambda \in \Lambda$, form a complete orthonormal system on $L^2(\Omega)$. Such a set Λ is called a spectrum of Ω, and the pair (Ω, Λ) is called a spectral pair.

It is now known that a spectrum Λ of a set Ω which is bounded must be periodic and thus we obtain a structure theorem for such spectral pairs. In particular we get that Λ is contained in the set of Algebraic Integers. The question remains however if $\Lambda \subset \mathbb{Q}$. I will explain the relation of this question with certain questions in number theory and recurrence sequence.

Non linear Schrödinger equation for the special Hermite operator

Vijay Kumar Sohani
Harish-Chandra Research Institute, Allahabad

Abstract: We establish the local well posedness of solution to the non-linear Schrödinger equation associated to the special Hermite operator on $\mathbb{C}^n$ in certain first order Sobolev space. Our approach is based on Strichartz type estimates, and is valid for a general class of non linearities including power type. The case $n = 1$ represents the magnetic Schrödinger equation in the plane with magnetic potential $A(z) = iz, \; z \in \mathbb{C}$.

A characterisation of the Weyl transform

R. Lakshmi Lavanya
Ramanujan Institute, University of Madras

Abstract: A theorem of Alesker et al. says that the Fourier transform on $\mathbb{R}^n$ is essentially the only transform on the space of tempered distributions which interchanges convolutions and products. We obtain a similar characterisation for the Weyl transform.

This is a joint work with Prof. S. Thangavelu.

Some representations of the discrete Heisenberg group

Gerald Folland
University of Washington, Seattle

Abstract: The operators $f(t) \to f(t-a)$ and $f(t) \to \exp(2\pi ibt)f(t)$ on $L^2(\mathbb{R})$ generate a unitary representation of the discrete Heisenberg group H with central character $\exp(2\pi iabz)$. What are the irreducible representations of H with this central character, and how can one synthesize the representation just described from them? When ab is rational, the answers are quite straightforward, but when ab is irrational, things are much more complicated. We shall sketch the results in both cases.

Characterization of weak L^2 eigenfunction of the Laplace-Beltrami operator on rank one symmetric space of non-compact type

Pratyoosh Kumar
Indian Statistical Institute, Kolkata

Abstract: It is known from the work of Harish-Chandra and a subsequent refinement by Anker that $\phi_0 \notin L^{2,\infty}(G)$. However it follows from the Harish-Chandra series for elementary spherical functions that ϕ_λ does belong to $L^{2,\infty}(G)$ if $\lambda \in \mathbb{R}\setminus\{0\}$. In view of this it becomes important to know what are the other weak L^2 eigenfunctions of the Laplace-Beltrami operator on X with eigenvalue $-(\lambda^2+\rho^2)$ where $\lambda \in \mathbb{R}\setminus\{0\}$. The aim of this talk is to discuss the problem of characterizing these weak L^2 eigenfunctions.

Uncertainty Principles for the Schrodinger equation on Riemannian symmetric spaces of the non-compact type

Maddala Sundari
Chennai Mathematical Institute, Chennai

Abstract: Let X be a Riemannian symmetric space of the noncompact type. We prove that the solution of the time-dependent Schrodinger equation on X with square integrable initial condition f is identically zero at all times t whenever f and the solution at a time $t_0 > 0$ are simultaneously very rapidly decreasing. The stated condition of rapid decrease is of Beurling type. Conditions respectively of Gelfand-Shilov, Cowling-Price and Hardy type are deduced.

Some remarks on Beurling's theorem in Euclidean space*

Rahul Garg
Indian Institute of Science, Bangalore

Abstract: We consider the subcritical case in Beurling's theorem due to Hörmander. More precisely, we look at the functions $f \in L^1(\mathbb{R}^n)$ satisfying

$$\int_{\mathbb{R}^n} \int_{\mathbb{R}^n} |f(x)||\hat{f}(y)|e^{a|x \cdot y|} dx dy < \infty$$

for some $0 < a < 1$. Using the Bargmann transform, the decay on the Fourier-Hermite coefficients of f is studied.

* Joint work with Sundaram Thangavelu.

The Brylinski beta function

M. K. Vemuri
West Virginia University and Chennai Mathematical Institute, Chennai

Coxeter system of lines are sets of injectivity for the twisted spherical means on $\mathbb{C}$

Rajesh K. Srivastava
Harish-Chandra Research Institute, Allahabad

Abstract: In an interesting result, Courant and Hilbert ([2], p. 699) had proved that if average of an even function about a line L vanishes over all circles centered at L, then $f \equiv 0$. As a consequence of this result, any line L in $\mathbb{R}^2$ is a set of non-injectivity for the spherical means for the odd functions about L. In 1996, Agranovsky and Quinto have extended the result of Courant and Hilbert and shown that any Coxeter system of lines is a set of non-injectivity for the Euclidean spherical means in $\mathbb{R}^2$.

However, this result does not continue to hold for injectivity of the twisted spherical means (TSM) on complex plane $\mathbb{C}$ (which arise in the study of spherical means on Heisenberg group). The question, in general that any real analytic curve can be a set of injectivity for the TSM for $L^2(\mathbb{C})$, is still an open problem. However, we are able to prove the following partial results for the TSM.

We prove that any line L passing through the origin is a set of injectivity for the TSM for functions $f \in L^2(\mathbb{C})$, whose each of spectral

projection $e^{\frac{1}{4}|z|^2}f \times \varphi_k$ is a polynomial. With the same exponential condition, we observe that any curve $\gamma(t) = (\gamma_1(t), \gamma_2(t))$, which passes through the origin, where $\gamma_j,\ j = 1, 2$ are polynomials is also a set of injectivity for the TSM. It is an interesting question that $\gamma := (\gamma_1, \gamma_2)$ with γ_j's are real analytic is a set of injectivity for the TSM. We believe that this will help in characterizing non-injectivity sets for the TSM.

Further, to complete the arguments of our idea, we prove that any two perpendicular lines is a set of injectivity for the TSM on $L^q(\mathbb{C}),\ 1 \leq q \leq 2$. Moreover, this result implies that any Coxeter system of even number of lines is a set of injectivity for the TSM on $L^q(\mathbb{C})$. However, question of any Coxeter system of odd number of lines can be a set of injectivity for the TSM for $L^q(\mathbb{C})$ is still open.

These results are quite explicit and adverse to the known result for the spherical means on $\mathbb{R}^2$, due to Agranovsky and Quinto [1]. Our result reveals that the Agranovsky-Quinto conjecture, "the sets of non-injectivity for the spherical means on $\mathbb{R}^n$ $(n \geq 2)$ are contained in a certain algebraic variety" does not continue to hold for the spherical means on the Heisenberg group $\mathbb{H}^1 = \mathbb{C} \times \mathbb{R}$.

This work is available at: arXiv:1103.4571.

References

[1] M. L. Agranovsky and E. T. Quinto, *Injectivity sets for the Radon transform over circles and complete systems of radial functions,* J. Funct. Anal., 139 (1996), no. 2, 383–414.

[2] R. Courant and D. Hilbert, *Methods of Mathematical Physics,* Vol. 2.