Nonparametric Analysis of Shapes with Applications to Morphometrics and Medical Diagnostics

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1. Introduction

The goal of this project is to develop methods for nonparametric analysis of shapes and images based on landmark data. Among significant applications are those in the discrimination and classification of biological shapes and medical images, and in machine vision.

Each observation under study consists of a set of k points, or landmarks, chosen with expert help on a two- or three-dimensional image. Such an observation is called a k-ad or a configuration of k points. In order to compare samples of such data from different groups, collected with possibly different magnifications, different camera angles and orientations, one needs to remove the effects of scale, location and rotation from each k-ad, yielding its "**shape**". The spaces of such shapes are geometric spaces known as differential manifolds.

A number of one- and two-sample tests have been carried out in the past using parametric models. The present project focuses on nonparametric tests, since the parametric models often do not represent this type of data very well. It turns out that the new procedures are not only more robust, but they also yield much smaller p-values, often by orders of magnitude, than those of the parametric tests, in cases where the groups are different. This is an indication that the new nonparametric procedures are more powerful than the ones currently in use.

Examples of two-sample tests are presented, including (1) shapes based on thirteen landmarks on a two-dimensional slice from a Magnetic Resonance brain scan of each of 14 schizophrenic children and 14 normal children, (2) shapes of male and female gorilla skulls based on eight landmarks on the mid-line plane of the skull.

2. Inference on the shape space

Consider a random sample of k-ads from some unknown distribution Q on the shape space. For inference about Q, we define a concept of center or "**mean shape**" and a measure of spread or "**variation in shape**". Most often, comparing these two parameters for different populations is sufficient to distinguish between them. For that we need sample estimates of mean shape and variation from a random sample of shapes, and two sample tests to compare them.

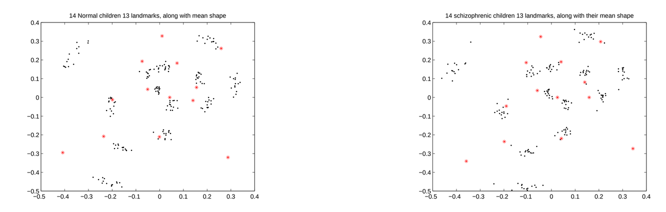
There are two common notions of mean shape, namely **extrinsic mean** and **intrinsic mean**, based on the distance chosen on the shape space. In many examples, the two means are very close, and tests carried out with either of them give similar results. The two means give rise to two concepts of variation, namely extrinsic and intrinsic variation.

For more details on the concept of mean shapes and variations, and the two sample nonparametric tests carried out to compare them, we refer to Bhattacharya and Patrangenaru [3], [4] and Bhattacharya and Bhattacharya [1], [2].

In the subsequent examples, we present the computations of mean shapes and variations, and the results of nonparametric tests for comparing means and variations for two populations. Also mentioned are the results of two sample tests carried out in the past using parametric models.

3. Example 1 (Schizophrenic Children)

In this example from Bookstein [5], 13 landmarks are recorded on a midsagittal two-dimensional slice from a Magnetic Resonance brain scan of each of 14 schizophrenic children and 14 normal children. Figures 1a,b show the k-ads for the patient and normal samples along with the respective sample extrinsic means. The individual observations have been translated and scaled so that each k-ad has mean 0 and norm 1. Then they are rotated appropriately so as to minimize their distance from the mean.



(a) (b)

Figure 1: *(a) 13 landmarks for 14 normal children along with the mean shape, (b) Corresponding landmarks for 14 schizophrenic patients along with the mean shape. The * refers to the mean landmarks.*

Figure 2 shows the shapes of the normal and the patient sample extrinsic means along with the pooled sample mean. Again the two mean shapes have been rotated to bring them closest to the pooled sample mean.

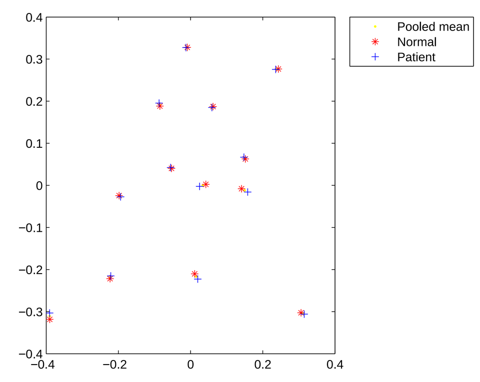


Figure 2: *Sample extrinsic means for the 2 groups along with the pooled sample mean.*

The values of the nonparametric two-sample test statistics for testing equality of the mean shapes for the two populations, along with the p-values are as follows:

- Extrinsic: 95.5476, p-value = 3.8×10^{-11}.
- Intrinsic: 95.4587, p-value = 3.97×10^{-11}.

The extrinsic sample variations for the patient and normal samples are 0.0107 and 0.0093 respectively. The value of the two-sample test statistic for testing equality of extrinsic variations for the two populations is 0.9461, and the p-value is 0.3441.

The analyses suggest that the two groups have different mean shapes but same variation in shape.

The value of the likelihood ratio test statistic, using the so-called **offset normal shape distribution** (Dryden and Mardia [6], pp. 145-146) is $-2\log\Lambda = 43.124$, p-value = 0.005.
The corresponding values of Goodall's F-statistic and Bookstein's Monte Carlo test (Dryden and Mardia [6], pp. 145-146) are $F_{22,572} = 1.89$, p-value = 0.01.
P-value for Bookstein's test = 0.04.

Thus our conclusions are consistent with those of parametric tests, however, the nonparametric p-values are smaller by orders of magnitude. Hence the nonparametric tests can distinguish between the two classes much more efficiently, which in turn suggests that the paramteric models used do not fit the data set well.

4. Example 2 (Gorilla Skulls)

To test the difference in the shapes of skulls of male and female gorillas, eight landmarks are chosen on the mid-line plane of the skull of 29 male and 30 female gorillas. We use the data of O'Higgins and Dryden reproduced in Dryden and Mardia [6], pp. 317-318. The nonparametric two sample tests carried out to compare the mean skull shapes for males and females yield the following values:

- Extrinsic:392.6, p-value $< 10^{-16}$.
- Intrinsic: 391.63, p-value $< 10^{-16}$.

The extrinsic sample variations for male and female samples are 0.005 and 0.0038 respectively. The value of the two-sample test statistic for testing equality of extrinsic variations is 0.923, and the p-value is 0.356.
A parametric F-test (Dryden and Mardia [6], pp. 154) yields $F = 26.47$, p-value = 0.0001.
A parametric (Normal) model for Bookstein coordinates leads to the Hotelling's T^2 test (Dryden and Mardia [6], pp. 170-172) yields the p-value 0.0001.

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