

Oscillations and Warming Trend in Global Temperature time series

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Introduction

- The goal of this project is to distinguish a warming trend and periodic oscillations from natural variability in global surface air temperatures.
- First I use a **Singular Value decomposition** (SVD) analysis of the three dimensional temperature data over different stations all over the world and over different months. That gives the leading spatial patterns explaining most of the variation. Since those patterns concentrate mostly over the North Atlantic region, I focus on the analysis of that region. I use the leading patterns to reconstruct the annual average temperature over the North Atlantic.

- Then I use **Singular Spectrum Analysis** (SSA) to analyze the time series of annual average surface air temperatures over North Atlantic for the past 136 years, allowing a secular warming trend and a small number of oscillatory modes to be separated from the noise. There is a rising trend since 1910. The oscillations exhibit periods of around 5 or 6 years.

Data Source

- The source of the climatic data is UEA CRU Jones HAD CRUT2v temperature anomaly data.
- The data has the following grid structure:
- Time: In months from 1/1870 to 12/2005 by 1. 1632 grid points
- Longitude: 177.5W to 177.5E by 5. 72 grid points.
- Latitude: 87.5N to 87.5S by 5. 32 grid points.
- Temperature: In degree Celsius. Missing value -99.99.

Figure 1 shows the plot of the temperature over certain months.

Figure 1a: Jan 1870 Global Surface Air temperature

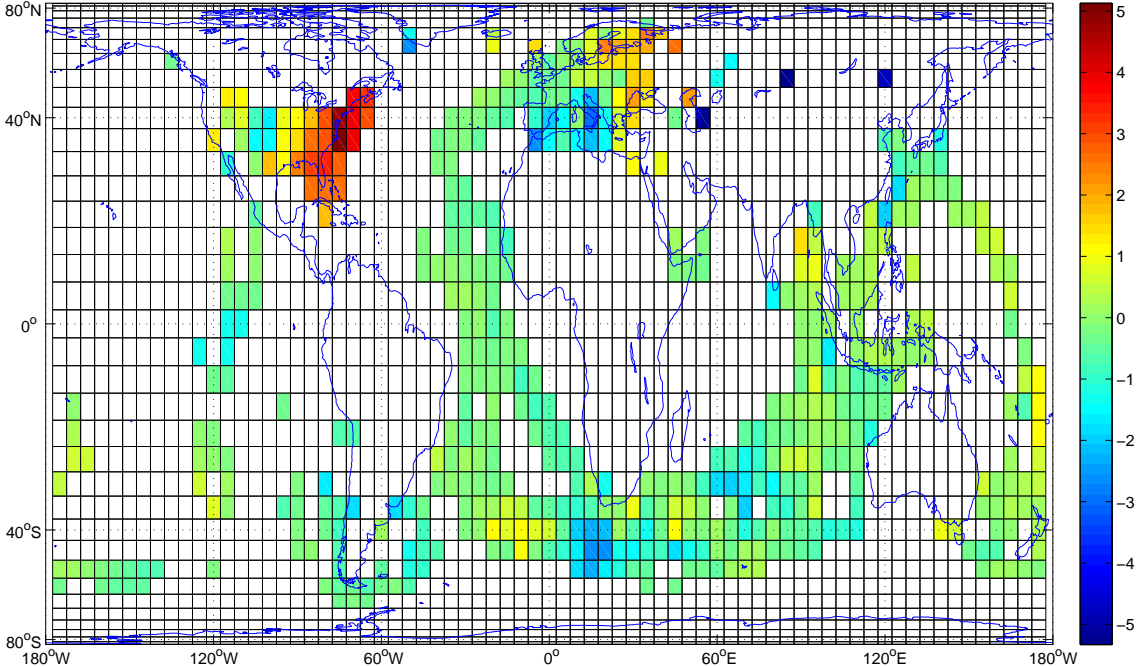
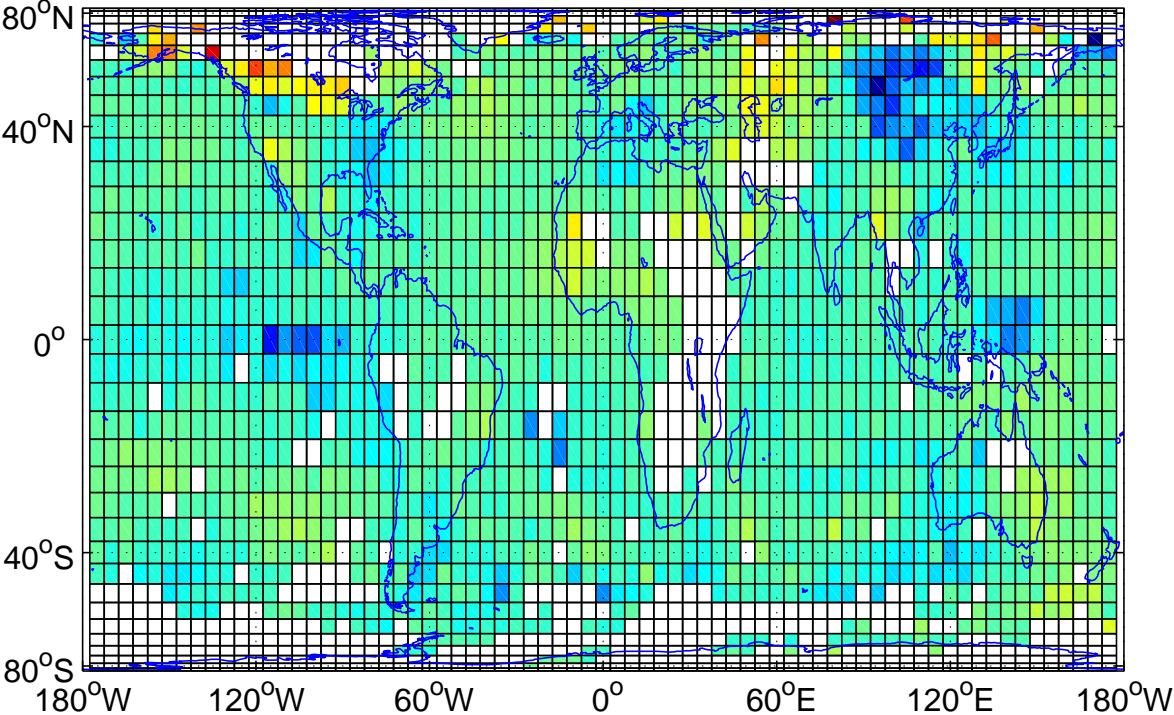


Figure 1b: Dec 2005 Global Surface Air temperature



Empirical Orthogonal Function (EOF) decomposition Analysis

- Prior to the analysis, I weight the data appropriately to take into account the distortion in high latitudes in a Mercator projection. I apply weights to the grid points, the weight being proportional to the cosine of the latitude.
- Then I compute the spatial covariance matrix. To have significant number of observations, I use the last 100 years: 01/1906 to 12/2005 in my covariance calculation. Also I use only those grid points which have at least 50 years of data. That would ensure that the pairs have at least 5 years of overlap.

- Figure 2 shows the grid points used in my analysis. Most of them concentrate around Northern Atlantic (NA).
- Then I compute the eigen vectors and eigen values of the covariance matrix.
- Figure 3 shows the spatial pattern of the first four EOFs. All of them concentrate on NA and have a single pole in that region. EOFs 1 and 2 are positive correlated with space while EOFs 3 and 4 are close to negative. They explain 22, 13, 9 and 7 percent of the variation in the data respectively. Together they explain about 51% of the variance.
- Since the EOF plots are very localized, they explain the spatial directions well.

- I perform a white noise test to select the EOFs that explain a significant amount of variation.
- Figure 4 shows the plot of the first 40 eigen values of the sample covariance matrix vs the corresponding 95th percentile from a white noise time series with same variance structure. The plot suggests that the first 11 eigen values and hence the corresponding EOF patterns are significantly above noise level.
- I use these EOF patterns to reconstruct the time series from 01/1870 to 12/2005. I look at the annual average temperature averaged over the NA for this time period.

- Figure 5 shows the annualized Principal components (PC) of the 4 leading EOFs. These represent the time pattern of those EOFs. As the plot shows, the first 2 PCs are negatively correlated with time, the third is positively correlated while the fourth is close to being uncorrelated.
- Figure 6 shows the average annual temperature over NA timeseries, the true one vs the one reconstructed using the first 11 leading patterns. They are pretty close.
- The time series shows a rising trend towards the later years and some irregular oscillations. The next goal would be to separate the trend and oscillations from noise.

Figure 2: Dec.2005 temperatures over selected grids

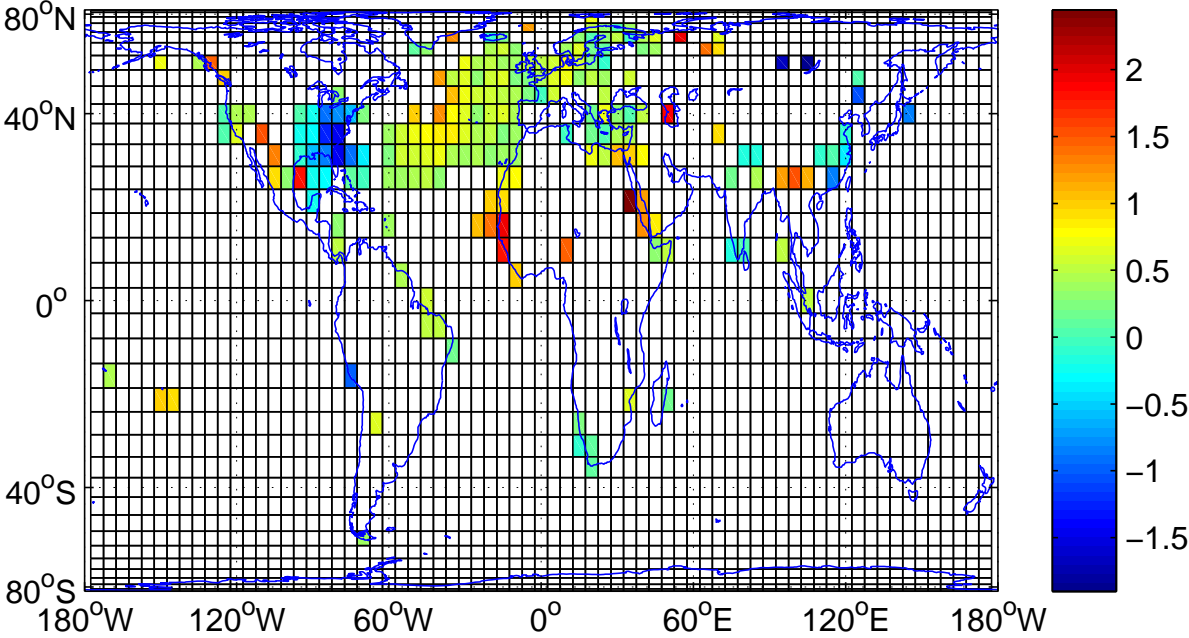


Figure 3: The first 4 EOF patterns

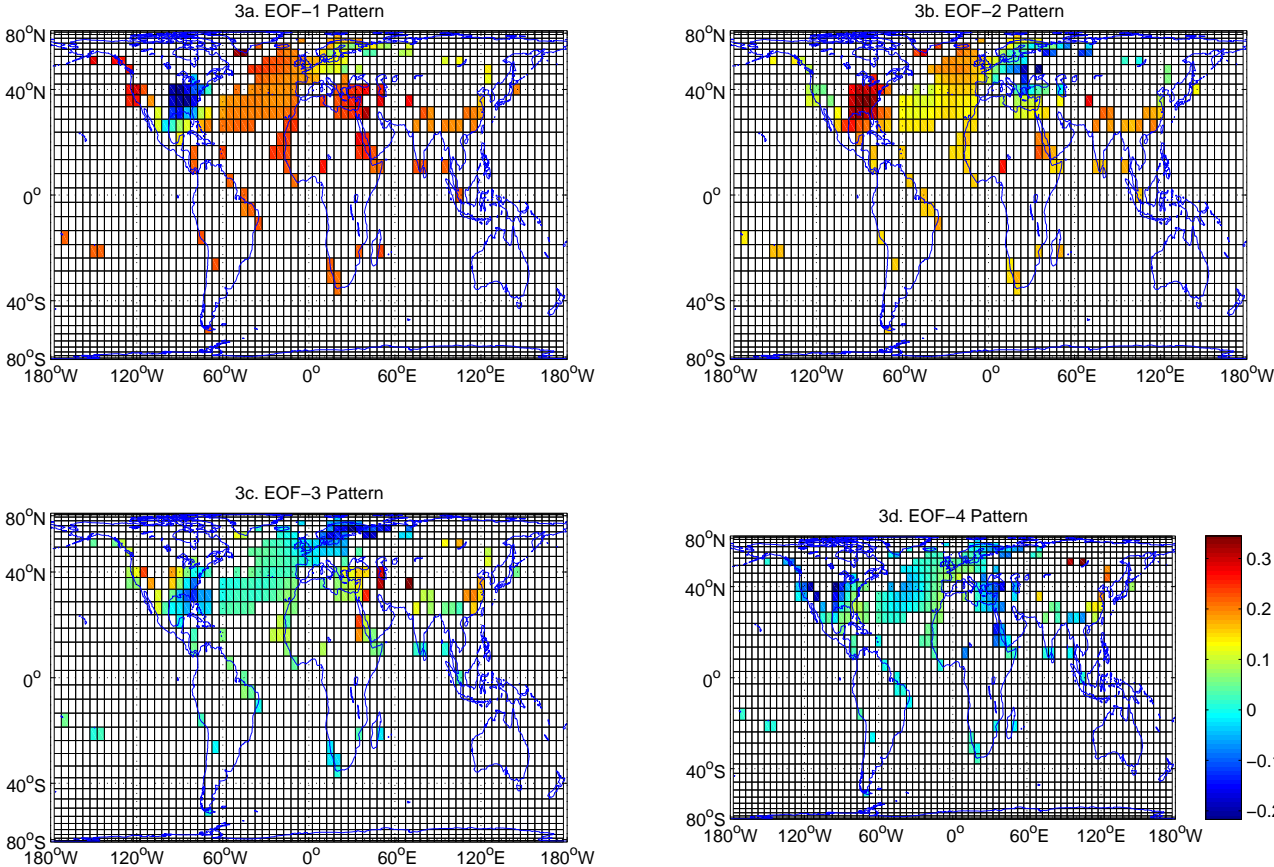


Figure 4: Eigen value trace

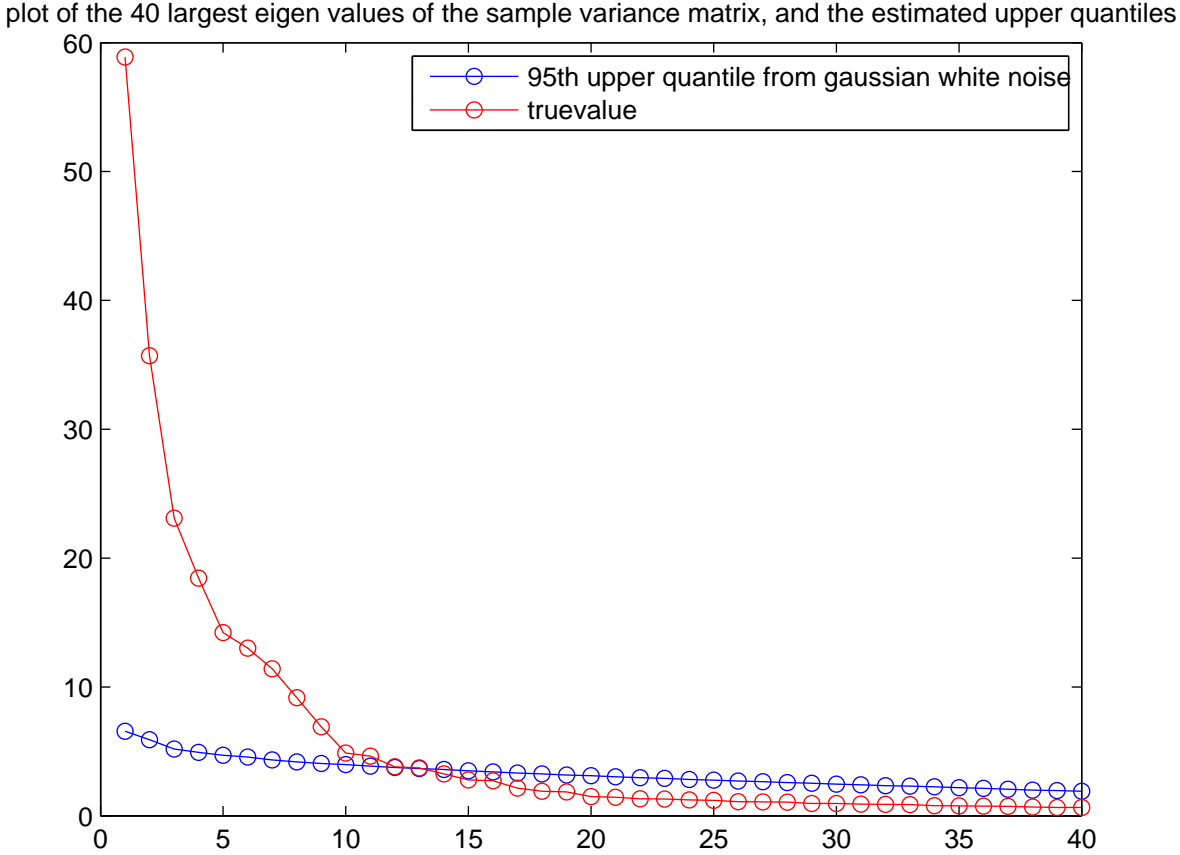


Figure 5: The first 4 PC series

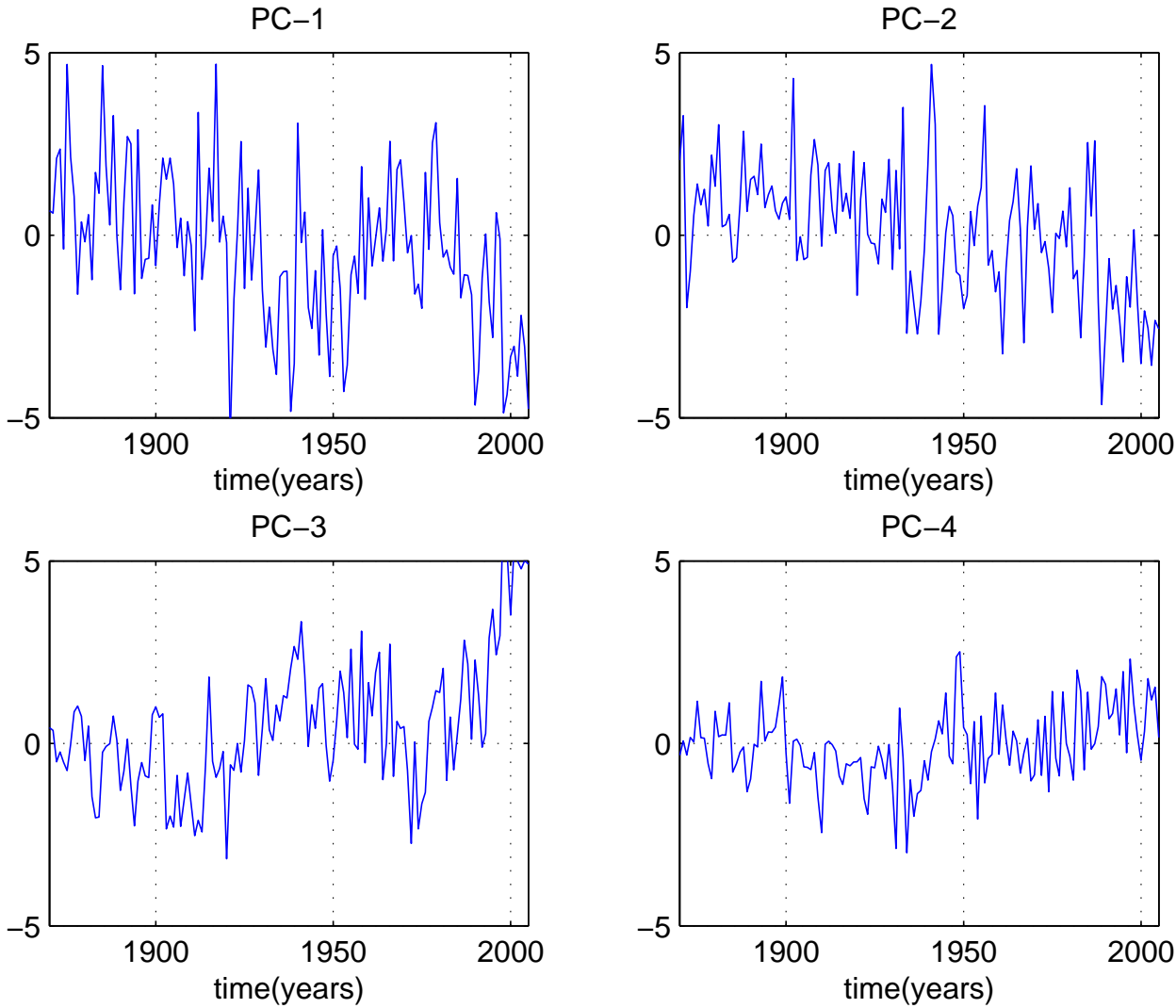
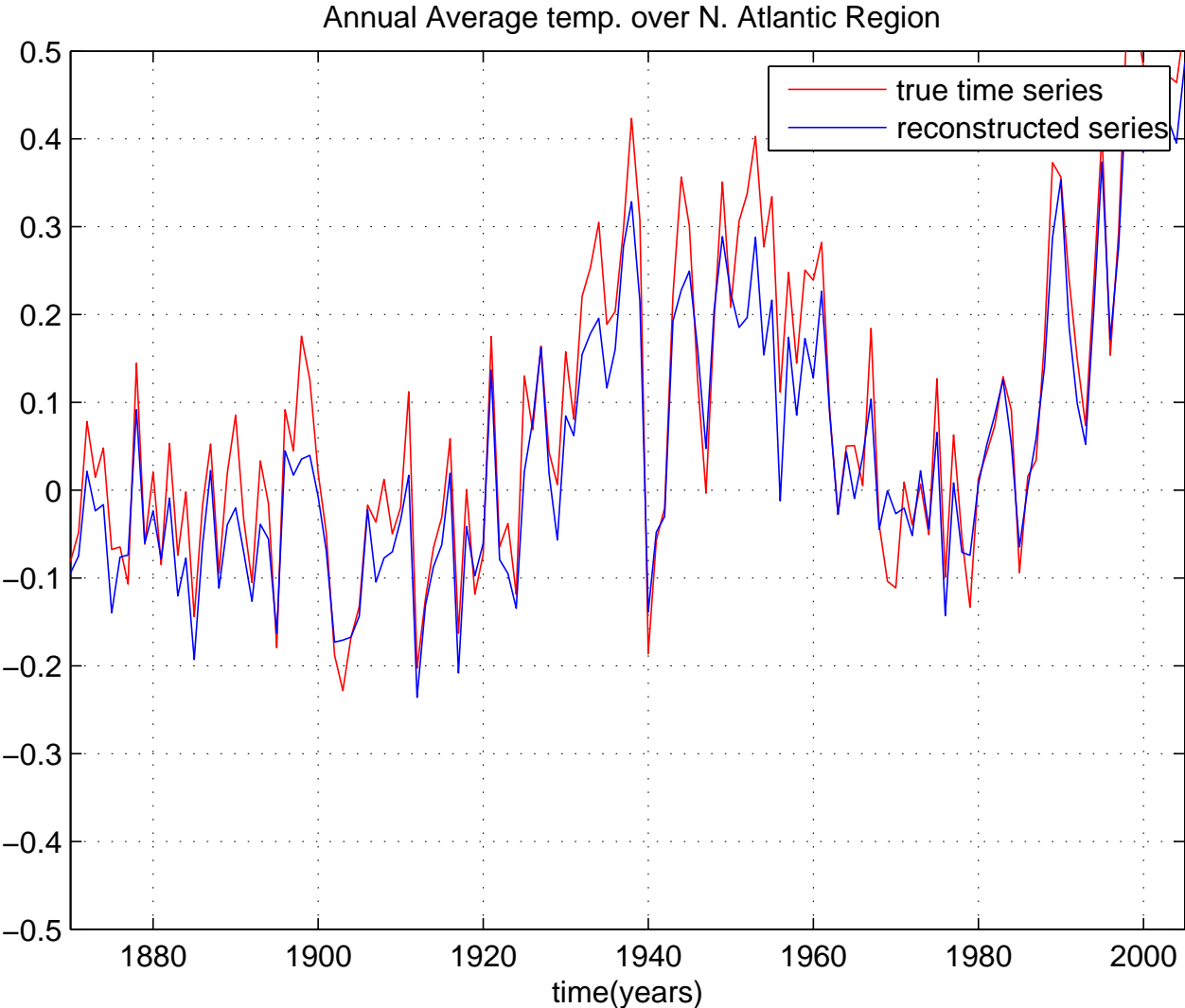


Figure 6: The true vs reconstructed annual temperatures over NA



Singular Spectrum Analysis (SSA) of annual temperatures over NA

- Now I develop a SSA of the reconstructed time series of annual temperature data over NA from 01/1870 to 12/2005.
- I choose an embedding dimension, m , compute the covariance matrix of the embedded data set, calculate its eigen values, eigen vectors and reconstructed components for the time series.
- I compare the eigen values with those from a white noise process to check their significance at level 95%.

- Shown in Figure 7 are the plots from SSA with $m = 20$ years.
- Figure 7a shows EOFs 1 and 2 patterns over time. EOF 1 shows a declining trend while EOF 2 shows a rising trend. Together they explain about 63% of the variation. The plot suggests that these EOFs correspond to the trend in the temperature record.
- Figure 7b shows EOFs 3 and 4 patterns. They are an oscillatory pair in quadrature with each other and have a period of about 6 years. They explain about 10% of the variation.

- Figure 7c shows the EOFs 5 and 6. They also form oscillatory pairs in quadrature with each other and have a period of about 10 years. They explain about 8% of the variation.
- In Figure 7d is shown the time series reconstructed out of the first two EOFs. The trend is flat until 1910 and then a rise after that.
- Figure 8 is the plot of the eigen values of covariance matrix and the corresponding white noise estimates. The plot suggests that only the first two are significantly different from noise values.

m = 20 Plots

Figure 7: SSA for $m = 20$.

7a: EOFs 1 and 2 Pattern; 7b: EOFs 3 and 4 patterns; 7c: EOFs 5 and 6 patterns; 7d: Time Series reconstructed out of EOFs 1 and 2.

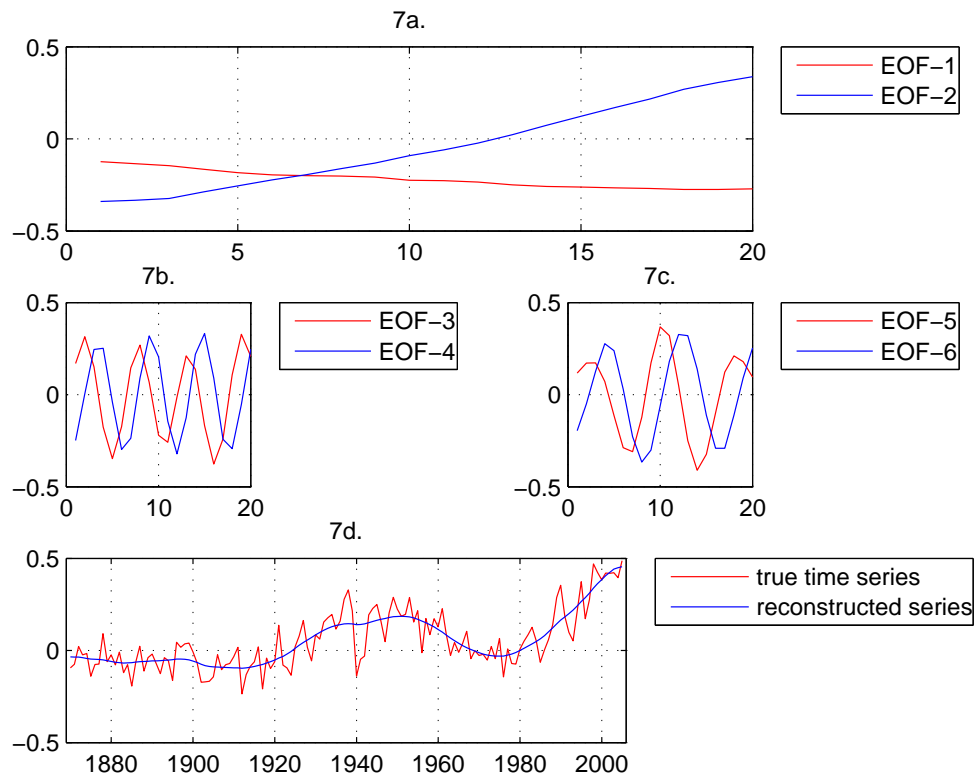
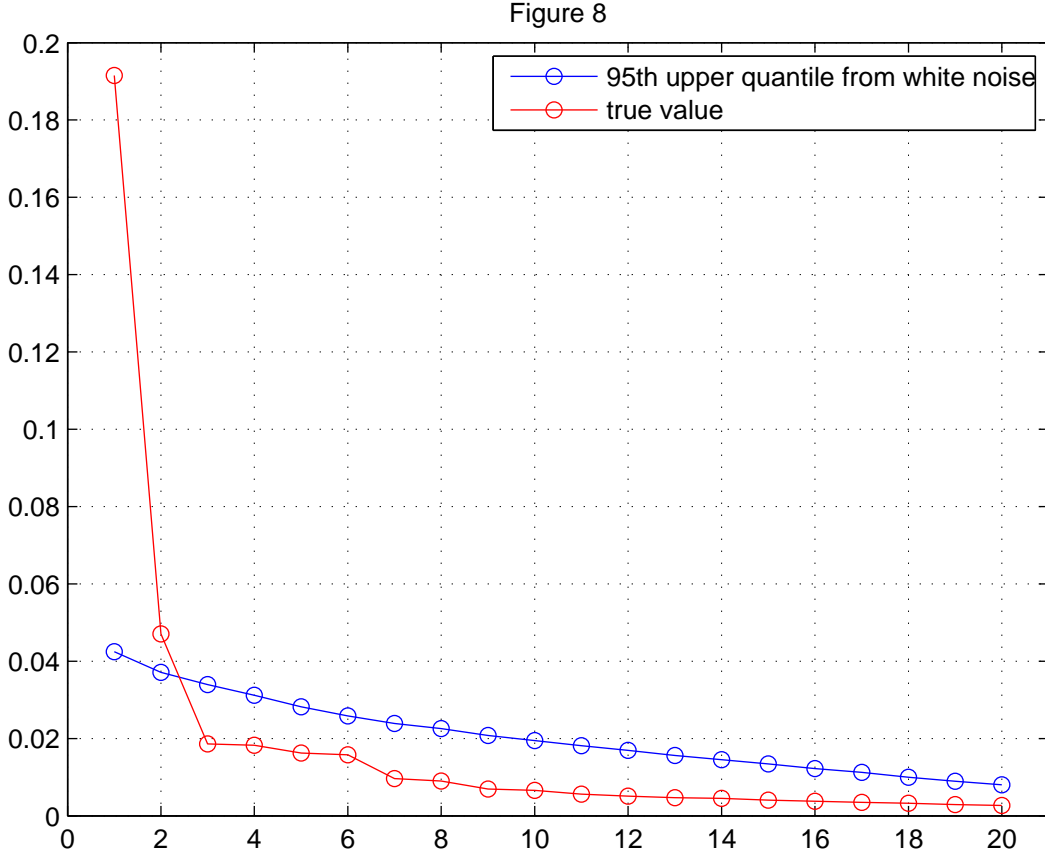


Figure 8: Eigen value trace for m=20



Other embedding dimensions analysis.

- Figure 9 shows the plots for SSA Analysis with $m = 6$.
- Again EOFs 1 and 2 correspond to trend with similar pattern as in the $m = 20$ case (Figure 7a). EOF pairs 3 & 4 and 5 & 6 are in quadrature and show oscillations with period of about 5 to 6 years.
- However as Figure 10 shows, apart from EOF 1, the other ones are closer to noise floor.

- When $m = 10$ (Figure 11), EOFs 1 and 2 correspond to trend; EOFs 3 & 4 are in quadrature with period around 6 years; EOF 5 has period of 10 years while EOF 6 shows a noisy pattern.
- In Figure 12, again only EOF 1 is significantly above noise floor.
- When $m = 30$ (Figure 13), EOFs 1 and 2 correspond to trend; EOFs 3 & 4 are in quadrature with period around 6 years; EOFs 5 & 6 are in quadrature with period around 6 years. In Figure 14, EOFs 1 & 2 are significantly above noise level.

m = 6, 10, 30 Plots.

Figure 9: SSA for m = 6

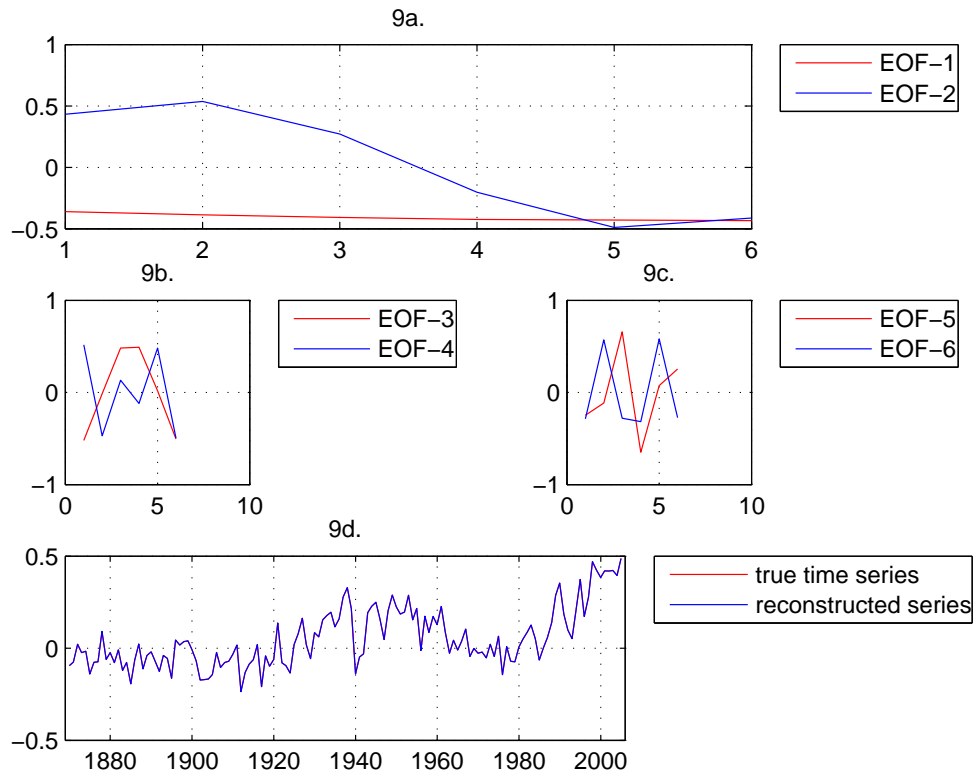


Figure 10: Eigen value trace for $m=6$

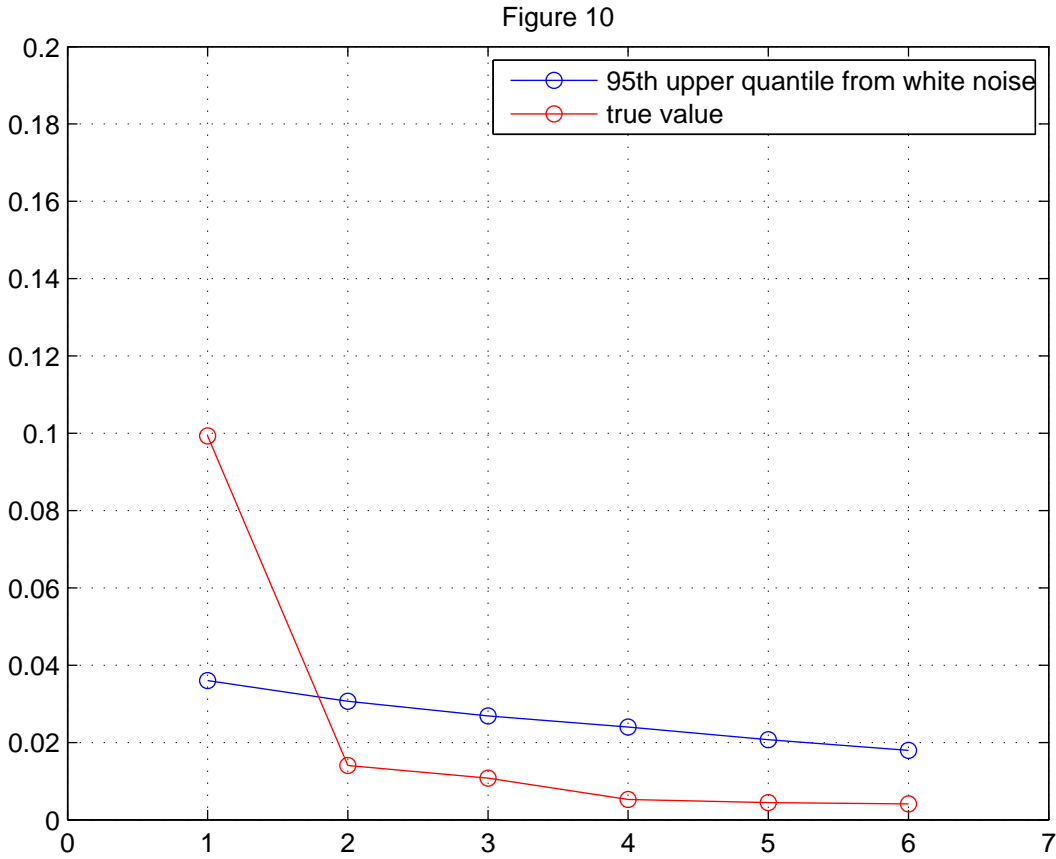


Figure 11: SSA for $m = 10$

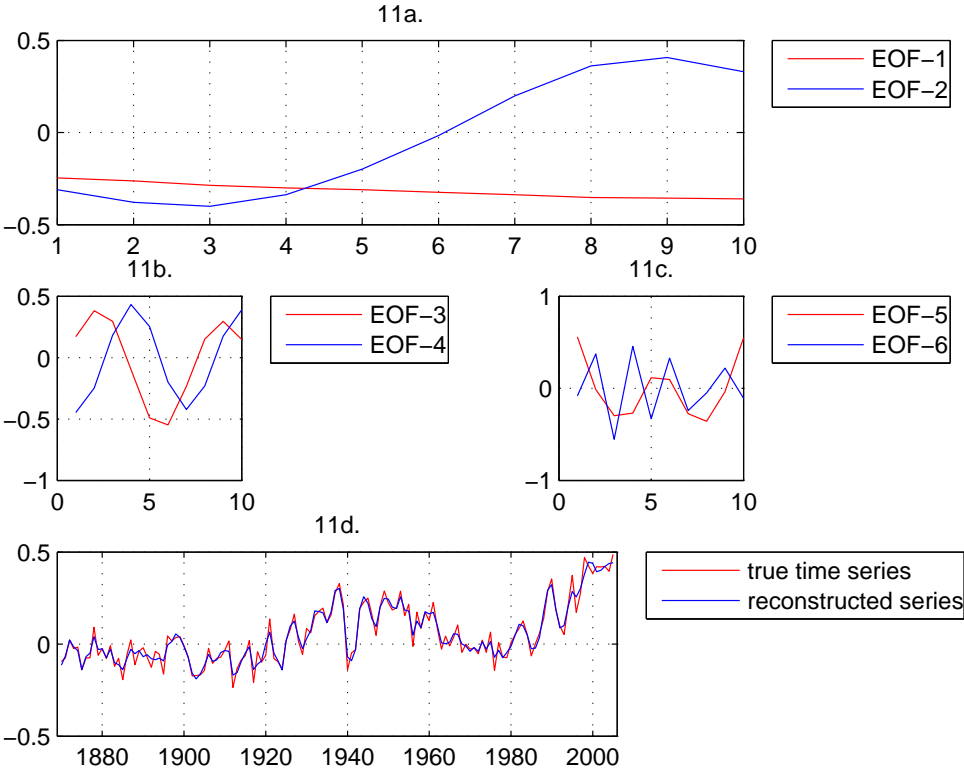


Figure 12: Eigen value trace for m=10

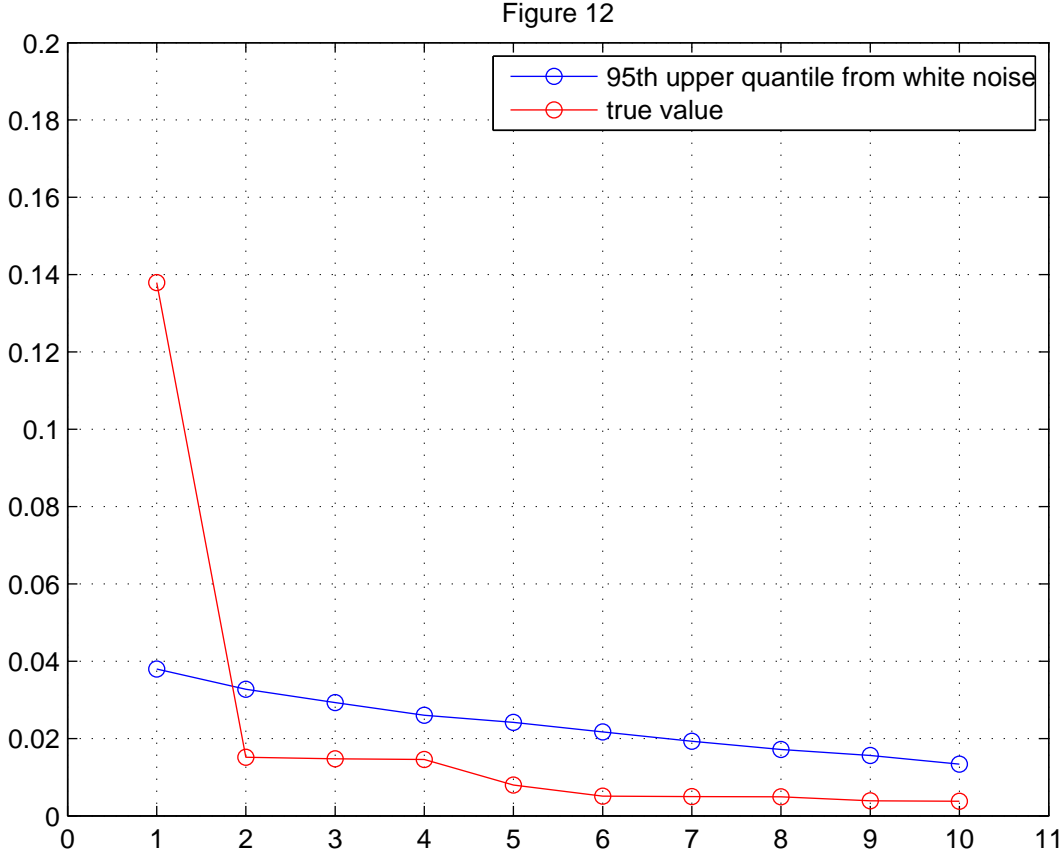


Figure 13: SSA for $m = 30$

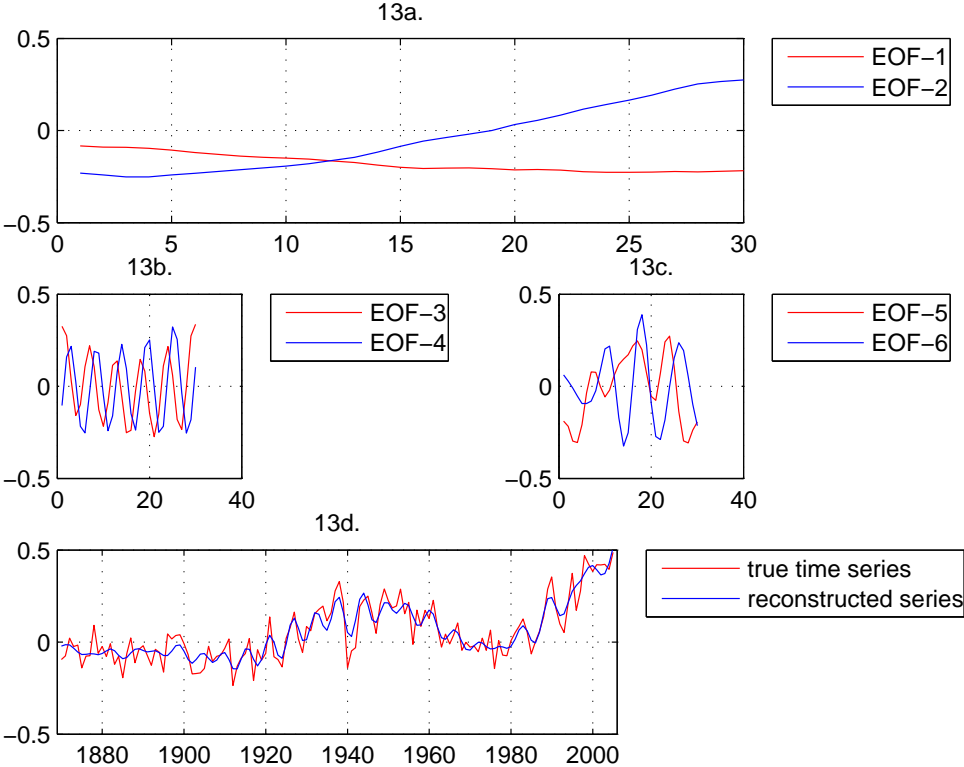
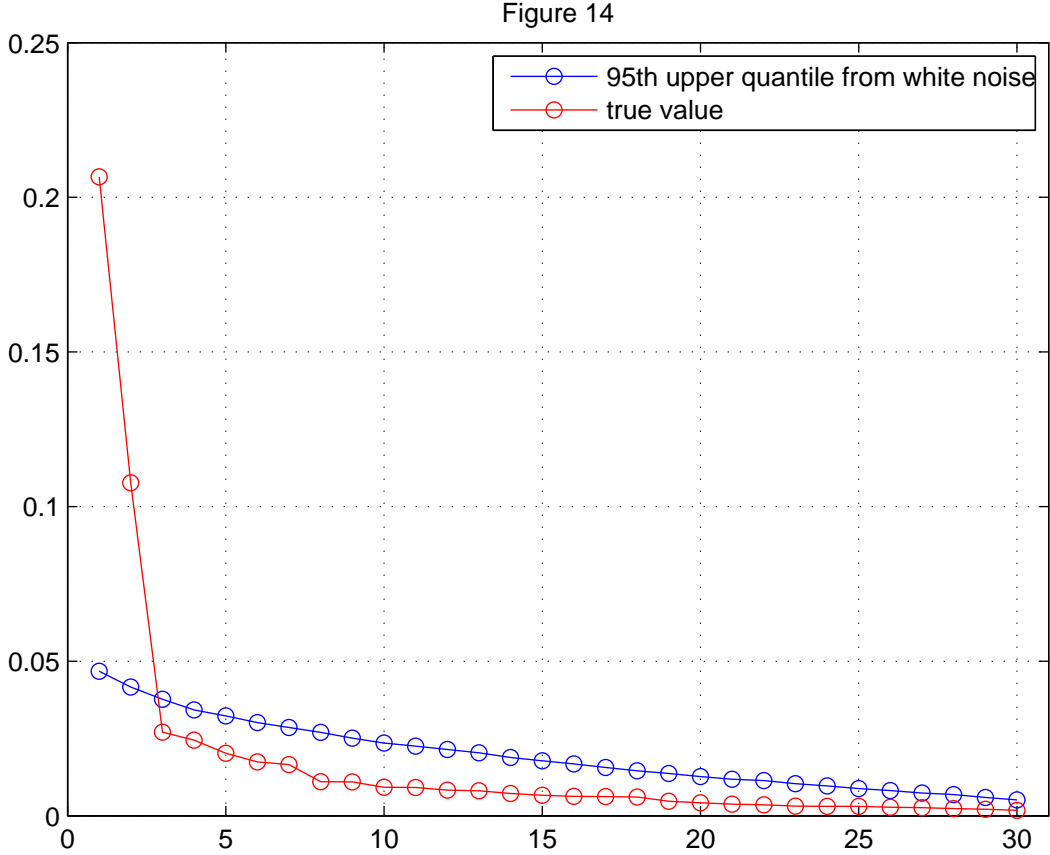


Figure 14: Eigen value trace for m=30



Sensitivity of SSA to choice of embedding dimension, m

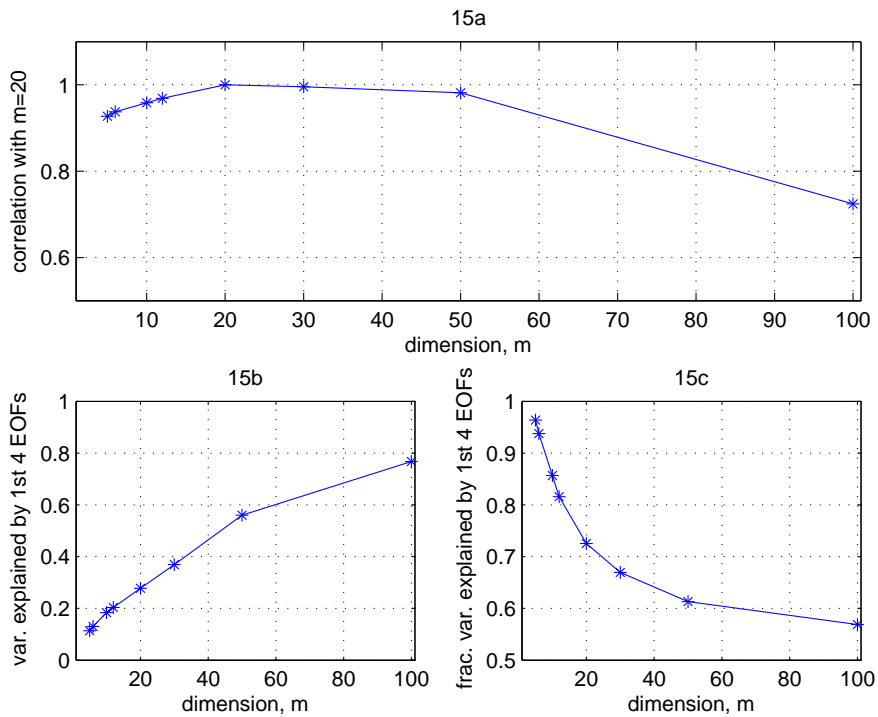
- Now I check how robust is the SSA analysis.
- As all eigen value trace plots in Figures 10 ($m = 6$), 12 ($m = 10$), 8 ($m = 20$) and 14 ($m = 30$) show, there is a sharp fall from eigen value 1 to 2, another less sharper fall from eigen values 2 to 3 after which the trace gets flat.

- Also we see that as m increases, the trace gets steeper. This suggests stationarity in the data. If the series were truly stationary, the covariance matrix for a smaller m can be "roughly" embedded into that for a larger m . Then the latter will have its largest eigen values bigger, while the least eigen values smaller, leading to a steeper eigen value trace.
- Since the first two EOFs represent trend and the next two periodicity for all dimensions, I reconstruct the time series using these EOFs for $m = 5, 6, 10, 12, 20, 30, 50$ and 100 years. In each case I compute the correlation of the summed components with that for $m = 20$.

- Figure 15a shows the plot of the correlations. The plot shows that the correlation is over 0.8 for $m < 80$. This suggests that the different values of m less than 80 do not substantially alter the SSA analysis results.
- In Figure 15b is plotted the variance explained by the first four EOFs for the above values of m . The plot is rising which supports my earlier claim about stationarity, and the rise is fairly smooth till $m = 60$ suggesting robustness of the analysis for $m < 60$.
- Figure 15c shows the plot of the fractional variance explained by the first 4 EOFs vs the embedding dimension, m . This plot is declining. The values are over 50% in all case, and over 60% for $m < 60$.

Figure 15

15a: Correlation between time series constructed using embedding dimension, m and 20; 15b: Variance explained by 1st 4 EOFs; 15c: Fractional Variance explained by the 1st 4 EOFs.



Conclusions

- From different dimensional SSA analysis, we observe that the EOFs 1 and 2 represent the trend in the data. As the true and reconstructed timeseries show (Figure 7a), there has been a rise in surface air temperatures since 1910. The estimation of EOFs 1 and 2 is robust, in the sense that their trend changes little with varying dimension, m .
- EOFs 3 and 4 explain the major periodicity in the data. They have a period between 5 and 6 years. However their estimation is not so robust. Their trace pattern varies with m . This is justified by the fact that the corresponding eigen values are very near to the noise floor for all m .

- The analysis shows no sign of bidecadal oscillations as reported by Ghil and Vautard [2].
- This raises the question whether a minimum sample size is required to detect them. However a sample size of 136 years seems fairly large.
- Another reason could be, as Elsner and Tsonis [1] point out, the years before 1881 made the difference. I get the leading patterns from the EOF analysis of the covariance matrix constructed using the past 100 years: 1906 to 2005 and use those to reconstruct the annual average temperature over NA time series for the span of past 136 years (1870-2005). Perhaps including the earliest years in calculating the EOFs would make a difference.

- However that would mean that the data is non stationary while SSA is designed mainly for stationary data.
- This makes one doubt the existence of bidecadal oscillations in the temperature records, and the tools used to detect them.

References

- [1] ELSNER, J.B. and TSONIS, A.A.(1991) Do bidecadal oscillations exist in the global temperature record? *Nature*353 551-553
- [2] VAUTARD, R. and GHIL, M.(1991) Inter-decadal oscillations and the warming trend in global temperature time series. *Nature*350 324-327
- [3] JONES, P.D. and MOBERG, A.(2003) Hemispheric and large-scale surface air temperature variations: an extensive revision and an update to 2001. *J.Climate*16 206-223