Test Code : PHB : (Short Answer Type) 2015

Junior Research Fellowship in Theoretical Physics and Applied Mathematics

The candidates for Junior Research Fellowships in Applied Mathematics and Theoretical Physics will have to write two papers − Test MMA (objective type) in the forenoon session and Test PHB (short answer type) in the afternoon session.

The PHB test booklet will consist of three parts. The candidates are required to answer Part I and only one of the remaining parts II & III.

The syllabi and sample questions for the test are as follows.

PART-I

Mathematical and logical reasoning

Syllabus

B.Sc. Pass Mathematics syllabus of Indian Universities.

Sample Questions

1. Let
$$f(x) = \frac{xe^{1/x} - x}{e^{1/x}}; \quad x \in IR$$
Find $\lim_{x\to\infty} f(x)$.

2. Let f be a real valued function defined on the interval $[-2, 2]$ as:
$$f(x) \;=\; \begin{cases} (x+1)\,2^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & \text{for} \quad x \neq 0 \\ 0 & \text{for} \quad x = 0 \end{cases}$$
   (a) Find the range of the function.
   (b) Is f continuous at every point in $(-2, 2)$? Justify your answer.

3. Let
$$A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$$
Find the eigenvalue of A^{2014}.

4. The position of a particle moving in a plane is given by $x = \sin\omega t$, $y = \cos\alpha\omega t$. Show that the trajectory repeats itself periodically, only if α is a rational number.

5. It is given that $\phi(1) = 2$ and $f(x) = \int\limits_{x^2}^{x} \phi(t)dt$. Find $f'(1)$.

6. X is a uniformly distributed random variable with probability density function

$$f(x) = \begin{cases} \frac{5}{a} & \text{for} \quad -\frac{a}{10} \leq x \leq \frac{a}{10} \\ 0 & \text{for} \quad \text{otherwise} \end{cases}$$

where a is a non-negative constant. If $P(|x| < 2) = 2P(|x| > 2)$, then find a.

7. Find the roots of the equation $z^5 = -i$, and indicate their locations in the complex plane.

8. A point is chosen randomly from a triangle with sides of lengths 3 cm, 4 cm and 5 cm, respectively. What is the probability that the point will lie outside the in-circle?

9. Let $f : R \to R$ and $f(x) = \frac{d}{dx}f(x) + \int\limits_{0}^{x} f(t)\,dt$ where it is given that $f(x) \geq 0$ for all x. Find the possible values of $f(1)$.

10. Displacement of a particle executing periodic motion is given by $y = 4\cos^2(t)\sin(5t)$. How many harmonic waves need to be superposed to get the above displacement?

11. Evaluate $\int\limits_{0}^{\pi/2} \frac{dx}{1 + (\tan x)^{100}}$.

12. Let p, q be two prime numbers each greater than or equal to 5 and $p > q$. Show that $p^2 - q^2$ is divisible by 24.

13. Show that the area of the triangle formed by z, iz, and $z + iz$ is $\frac{r^2}{2}$, where $r = |z|$ and $z = a + ib$, with a, b being real non-zero numbers.

14. A particle sliding on a smooth inclined plane requires 4 sec to reach the bottom starting from rest at the top. How much time does it take to cover $\frac{1}{4}$th distance starting from rest at the top?

15. Given any polynomial $A(x)$ with coefficients in R, show that there exists a polynomial $B(x)$ such that $A(x).B(x) = C(x^2)$, where $C(y)$ is some polynomial in y with coefficients in R.

16. Find the maximum possible value of xy^2z^3 subject to the conditions x, y, $z \geq 0$ and $x + y + z = 3$.

17. A ball of unit mass is dropped from a height h. The frictional force of air is proportional to the velocity of the ball (with a constant of proportionality α). Show that the height of the ball in time t is

$$y(t) = h - \frac{g}{\alpha}\left[t - \frac{1}{\alpha}\left(1 - e^{-\alpha t}\right)\right]$$

where g is the acceleration due to gravity.

18. A 1.5 Kg mass is attached to the end of a 90 cm string. The system is whirled in a horizontal circular path. The maximum tension that the string can withstand is 400 N. What is the maximum number of revolutions per minute allowed if the string is not to break?

19. A particle is constrained to move along the X-axis under the influence of the net force $F = -kx$ with amplitude A and frequency f, where k is a positive constant. What is the speed of the particle at $x = A/2$?

20. A particle of mass m that moves along the X-axis has potential energy $V = a + bx^2$, where a and b are positive constants. Its initial velocity at $x = 0$ is $v = 0$. Find the frequency with which it will execute simple harmonic motion.

21. Suppose a, b, c are positive integers such that

$$abc + ab + bc + ca + a + b + c = 1000$$

Find the value of $a + b + c$.

22. Determine the greatest and least values of the function

$$f(x) = x^3 - 3x^2 + 2x + 1$$

in the interval $[2, 3]$.

23. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the same circle, evaluate the radius of the circle.

PART-II

Applied Mathematics

Syllabus

1. *Linear algebra* : Matrices.
2. *Abstract algebra* : Groups, Rings, Fields.
3. *Real analysis* : Functions of single and several variables, Metric space, Normed linear space, Riemann integral, Fourier series, Integral transform.
4. *Differential equations* : ODE – Existence of solution, Fundamental system of integrals, Elementary notions, Special functions. PDE upto second order, Equations of parabolic, Hyperbolic and elliptic types.
5. *Dynamics of particles and rigid bodies* : Motion of a particle in a plane and on a smooth curve under different laws of resistance, Kinematics of a rigid body, Motion of a solid body on an inclined smooth or rough plane.
6. *Functions of complex variables* : Analytic function, Cauchy's theorem, Taylor and Laurent series, Singularities, Branch-point, Contour integration, Analytic continuation.
7. *Fluid Mechanics* : Kinematics of fluid, Equation of continuity, Irrotational motion, Velocity potential, Dynamics of ideal fluid, Eulerian and Lagrangian equations of motion, Stream function, Sources, Sinks and doublets, Vortex, Surface waves, Group velocity, Viscous flow – Navier Stokes equation, Boundary layer theory, Simple problems.
8. *Probability and statistics* : Probability axioms, Conditional probability, Probability distribution, Mathematical expectations, Characteristic functions, Covariance, Correlation coefficient, Law of large numbers, Central limit theorem. Random samples, Sample characteristics, Estimation, Statistical hypothesis, Neyman Pearson theorem, Likelihood ratio testing.

Sample Questions

1. Consider a concave mirror in the shape of a parabola with *focus* F whose equation is given by $y^2 = 4x$. Let P be a point source of light *inside* the parabola. Find Q on the parabola such that the ray PQ on reflection passes through the focus F.
2. Let $\phi : (\mathbf{Q}, +) \to (\mathbf{Q}, +)$ be a homomorphism of the additive group of rationals into itself. Show that for some $\lambda \in \mathbf{Q}$

$$\phi(x) = \lambda x, \quad \text{for all } x \in \mathbf{Q}.$$

3. (a) If G is a group of even order, then prove that it has an element $a \neq e$, satisfying $a^2 = e$.

   (b) Let $A = \{a \ : \ a = 9x + 15y, \ \ x, y$ are integers and $|a| \leq 1000\}$. Find the cardinality of A.

4. (a) Does there exist a hexagon with sides of lengths 2, 2, 3, 3, 4, 4 (with certain order) and with each angle equal? Justify your answer.

   (b) Let a, b be positive integers with a odd. Define the sequence $\{u_n\}$,

$$\begin{aligned} u_{n+1} &= \frac{1}{2} u_n, && \text{if } u_n \text{ is even} \\ &= u_n + a && \text{otherwise} \end{aligned}$$

   Show that $u_n \leq a$ for some $n \in N$.

5. (a) A body of mass M is suspended from a fixed point O by a light inextensible string of length l and mass m.

      i. Find the tension in the rope at a distance z below O.

      ii. If the point of support now begins to rise with velocity $2g$, what is the tension in the string?

   (b) A stone of mass m is thrown vertically upwards with initial speed V. If the air resistance at speed v is mkv^2, where k is a positive constant, show that the stone returns to its starting point with speed $V\sqrt{1 + kV^2/g}$.

6. (a) Show that

$$\frac{1 - t^2}{(1 - 2tx + t^2)^{3/2}} = \sum_{n=0}^{\infty} (2n + 1)\, P_n(x)\, t^n$$

   where $P_n(x)$ denotes Legendre polynomial of degree n.

   (b) Use the generating function for the Hermite polynomials to find $H_0(x)$, $H_1(x)$ and $H_2(x)$.

7. (a) Show that $x_n^3 + y_n^3 \to 0$ implies $x_n + y_n \to 0$. Is the reverse implication true?

   (b) A function is defined as follows:

$$\begin{aligned} f(x) &= 0, && \text{where } x \text{ is irrational} \\ &= \frac{1}{q}, && \text{where } x = \frac{p}{q}. \end{aligned}$$

   p and q are two positive integers prime to each other. Show that at $x = a$, $f(x)$ is continuous if a is irrational and $f(x)$ is discontinuous if a is rational.

8. Show that if the solution of the ODE

$$2xy'' + (3 - 2x)\,y' + 2y = 0$$

is expressed in the form $y = \sum_{n=0}^{\infty} a_n\, x^{n+\sigma}$, where σ can take two possible values. Find the relation between a_n and a_{n+1}, and show that one solution reduces to a polynomial.

9. (a) Using Laplace transformation, solve the differential equation

$$\frac{d^2y(t)}{dt^2} + at\frac{dy(t)}{dt} - 2ay(t) = 1$$

subject to the conditions $y(0) = y'(0) = 0$, where $a > 0$ being a constant.

(b) Using $f(x) = x^2$, $-\pi < x < \pi$, show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$.

10. Expand the function

$$f(x) = x^2, \qquad 0 < x < 2\pi$$

in a Fourier Series when the period is 2π. Hence show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \ldots = \frac{\pi^2}{6}.$$

11. Show that

$$\frac{\sin x}{x} = \prod_{r=1}^{\infty} \left(1 - \frac{x^2}{r^2\pi^2}\right)$$

(Hint: If n is an even integer and $n = 2m$, $x^n - 1 = 0$ has two real roots and $m - 1$ complex conjugate pairs)

12. (a) Find the set of all possible z in C when it is given that the group (with respect to multiplication) generated by the complex number $z = re^{i\theta}$ is finite.

(b) Let $A : k \times k$ be real symmetric matrix and x_n be a sequence in R^k. Show that if A is positive definite then $x_n'\, A\, x_n \to 0 \Rightarrow x_n \to 0$.

13. Consider the following upper-triangular matrix A over Z_5, the field of integers modulo 5:

$$A = \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 4 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$$

Show that A is invertible and find its inverse over Z_5.

14. Find the integral surface of the equation

$$(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$$

passing through the curve $xz=a^3$, $y=0$, where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$.

15. Using a suitable contour evaluate $\int_0^{2\pi}\frac{d\theta}{3-2\cos\theta+\sin\theta}$.

16. (a) Evaluate $\int\int\sqrt{4x^2-y}\,dx\,dy$ over the triangle formed by the straight lines $y=0$, $x=1$, $y=x$.

(b) Use Laplace transform to solve the following differential equation

$$Y''(t)-Y(t)=1+e^{3t}$$

given $Y(0)=-\frac{7}{8}$, $Y'(0)=0$. [Here, $Y'(t)=\frac{dY}{dt}$ and $Y''(t)=\frac{d^2Y}{dt^2}$]

17. (a) The velocity along the centre line of a nozzle of length L is given by

$$u=2t\left(1-\frac{0.5x}{L}\right)^2$$

where u is the velocity in m/s, t is the time in seconds from the commencement of flow and x is the distance of the inlet from the nozzle. Find the convective acceleration and the local acceleration when $t=3$, $x=L/2$, and $L=0.8\,\text{m}$.

(b) Identify the type of the partial differential equation

$$\frac{\partial^2 f}{\partial x^2}-\frac{\partial^2 f}{\partial y^2}=x-y$$

and find its generalized solution including the particular integral.

18. Water flows through a circular pipe. At one section, diameter of the pipe is 0.3 m, static pressure is 260 KPa gauge, velocity is 3 m/sec and the elevation is 10 m. The pipe diameter at the other section is 0.15 m with zero elevation. Find the pressure at the downstream section neglecting the frictional effect. Density of water may be assumed as 999 Kg/m^3.

19. (a) Let a number be drawn at random from $\{1,2,\dots n\}$. Call it X. A number is drawn at random from $\{1,2,\dots x\}$. Call it Z. Find $E(z)$ and $\text{Var}(z)$.

(b) Let $X\sim\exp(\lambda)$ with $\lambda>0$. Show that for all $t>0$, the value of $E(X/X>t)-t$ does not depend on t.

PART-III

Theoretical Physics

Syllabus

1. **Classical Mechanics**

   Mechanics of a particle and system of particles, Scattering in a central field, Lagrange's equation and their applications, Hamilton's equation, Canonical transformation, Special theory of relativity, Small oscillation, Vibration and acoustics.

2. **Electromagnetic theory**

   Electrostatics, Magnetostatics, Classical electrodynamics, Maxwell's equations, Gauge transformation, Poynting's theorem, Wave equation and plane waves, Radiating system and scattering.

3. **Statistical Physics and Condensed Matter Physics**

   Statistical basis of thermodynamics, Ensembles – microcanonical, canonical and grand canonical, Quantum statistics, Phase transitions, Statistical fluctuations, Free electron theory, Band theory of electrons, Semiconductor physics, Transport phenomena, Magnetism, Superconductivity.

4. **Quantum Mechanics and Quantum Field Theory**

   Inadequacy of classical physics, Schrödinger wave equation, General formalism of wave mechanics, Exactly solvable eigenvalue problems, Approximation methods, Scattering theory, Time dependent perturbation theory, Symmetries and conservation laws, Relativistic quantum mechanics, Quantum field theory – scalar and spinor fields, Quantum electrodynamics.

5. **Elementary Particles**

   Elementary particles, Weak and strong interactions, Selection rules, CPT theorem, Symmetry principles in particle physics.

Sample Questions

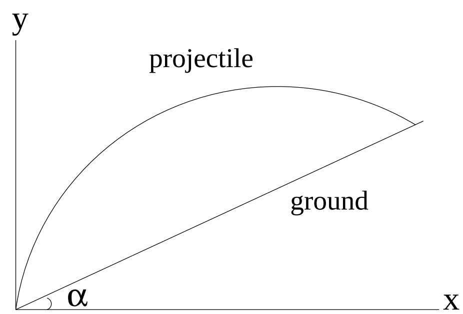
1. Consider a particle of mass m constrained to move on a frictionless circular loop of radius R. The loop is rotated with angular frequency ω about a vertical axis passing through its center. [Assume that at any instant of time the mass is at a position $\theta(t)$]

   (a) Set up the Lagrangian for this system (upto a constant).

   (b) Write down the equation of motion for the particle.

(c) Find out the equilibrium positions where the mass would settle down when ω changes.

2. A projectile is fired uphill over the ground which slopes at an angle α to the horizontal (as shown in the figure). Find the direction in which it should be aimed to achieve the maximum range. (Hint: Use a relation between x and y co-ordinates where the projectile touches the ground)



3. Two pendulums of mass m and length l are coupled by a massless spring of spring constant k. The unstretched length of the spring is equal to the distance between the supports of the two pendulums. Set up the Lagrangian in terms of generalized coordinates and velocities and derive the equations of motion.

4. A uniform flat disc of mass M and radius r rotates about a horizontal axis through its center with angular speed ω_0. A chip of mass m breaks off the edge of the disc at an instant such that the chip rises vertically above the point at which it broke off. How high does the chip rise above the point before it starts to fall off? What is the final angular momentum of the disc?

5. (a) A photon of energy E_i is scattered by an electron of mass m_e that is initially at rest. Final energy of the photon is E_f. Let θ be the angle between the directions of the incident photon and the scattered photon. Using the principles of Special Theory of Relativity find θ.

   (b) A person standing at the rear of a railroad car fires a bullet towards the front of the car. The speed of the bullet, as measured in the frame of the car, is 0.5 c (where c is the speed of light in vacuum) and the proper length of the car is 400 m. The train is moving at 0.6 c as measured by observers in the ground. For the ground observers, find

      i. the length of the railroad car,
      ii. the speed of the bullet,
      iii. the time required for the bullet to reach the front of the car.

6. (a) A sphere of radius R_1 has charge density ρ uniform within its volume, except for a small spherical hollow region of radius R_2 located at a distance a from the centre.

    i. Find the field $\mathbf{E}$ at the centre of the hollow sphere.

    ii. Find the potential ϕ at the same point.

   (b) An electric charge Q is uniformly distributed over the surface of a sphere of radius r. Show that the force on a small charge element dq is radial and outward and is given by

$$d\mathbf{F} = \frac{1}{2}\mathbf{E}\,dq$$

   where $\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$ is the electric field at the surface of the sphere and ε_0 is the permittivity of the free space.

7. (a) Consider a possible solution to Maxwell's equation given by

$$\vec{A}(\vec{x},t) = A_0\,e^{i(\vec{k}.\vec{x}-\omega t)},\ \ \phi(\vec{x},t) = 0$$

   where $\vec{A}$ is the vector potential and ϕ is the scalar potential. Further, suppose $\vec{A}_0$, $\vec{k}$ and ω are constants in space-time. Give and interpret the constraints on $\vec{A}_0$, $\vec{k}$ and ω imposed by each of the Maxwell's equations given below

   (i) $\vec{\nabla}.\vec{B} = 0$ (ii) $\vec{\nabla}\times\vec{\mathrm{E}} + \frac{1}{\mathrm{c}}\frac{\partial\vec{\mathrm{B}}}{\partial \mathrm{t}} = 0$

   (iii) $\vec{\nabla}.\vec{E} = 0$ (iv) $\vec{\nabla}\times\vec{\mathrm{B}} - \frac{1}{\mathrm{c}}\frac{\partial\vec{\mathrm{E}}}{\partial \mathrm{t}} = 0$

   (b) A parallel plate capacitor with plate separation d is filled with two layers of dielectric material a and b. The dielectric constant and conductivity of materials a and b are ϵ_a, σ_a and ϵ_b, σ_b respectively. The thicknesses of the materials a and b are d_a and d_b, respectively.

    i. Calculate the electric fields in the materials a and b.

    ii. Find the current flowing through the capacitor.

8. (a) Consider a gas in a container obeying Van der Waals gas equation

$$\left(P + \frac{a}{v^2}\right)(V - b) = nRT$$

   where a and b are constants. The initial volume is V and then isothermally it is compressed to one half of its volume. Find the work done by the gas.

   (b) Find the energy gap at the Brillouin zone boundary for nearly free electrons in a linear lattice with potential energy $U(x) = U_0\cos\left(\frac{2\pi x}{a}\right)$, where a is the lattice spacing.

9. (a) A one dimensional lattice consists of a linear array of N particles ($N >> 1$) interacting via spring-like nearest neighbor forces. The normal mode frequencies (radians/sec) are given by

$$\omega_n = \bar{\omega}\sqrt{2(1 - \cos(\frac{2\pi n}{N}))}$$

where $\bar{\omega}$ is a constant and n an integer ranging from $-N/2$ to $+N/2$. The system is in thermal equilibrium at temperature T.

   i. Compute C_v, the specific at constant volume for the regime $T \to \infty$,
   ii. Show that for $T \to 0$, $C_v \to A\,\omega^{-\alpha}\,T^{\gamma}$, where A is a constant. Determine the exponents α and γ. (Treat the problem quantum mechanically)

   (b) Consider the $2p \to 1s$ electromagnetic transition in an atom formed by a muon and a strontium nucleus ($Z = 38$). Given the lifetime of the $2p$ state of hydrogen is 10^{-9} second, calculate the fine structure splitting energy.

10. A hypothetical semi-conductor has a conduction band (cb) that can be described by $E_{cb} = E_1 - E_2\cos(ka)$ and valence band (vb) which is represented by $E_{vb} = E_3 - E_4\sin^2\left(\frac{ka}{2}\right)$ where $E_3 < (E_1 - E_2)$ and $-\pi/a < k < \pi/a$. Find out expressions for

   (a) the band widths of the conduction band and the valence band,
   (b) the band gap of the material,
   (c) the effective mass of the electrons at the bottom of the conduction band.

11. Consider a simple harmonic oscillator in one dimension with the Hamiltonian

$$H = \hbar\omega\left(a^{\dagger}a + \frac{1}{2}\right)$$

where a and $a^{\dagger}$ are the annihilation and creation operators respectively and the other symbols have their usual meanings. The ket vector of the harmonic oscillator at $t = 0$ is given by

$$|\psi(0)\rangle = N\left(|0\rangle + 2|1\rangle + 3|2\rangle\right)$$

where N is the normalization constant and $|n\rangle$ is the eigenket of corresponding energy eigenvalue $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$.

   (a) Find the normalization constant N.
   (b) Calculate the probability of finding the energy to be $\frac{3}{2}\hbar\omega$ on energy measurement.

(c) Find the ket vector $|\psi(t)\rangle$ at time t and calculate expectation value of the energy for this ket vector.

12. A particle, initially (i.e., $t \to -\infty$) in its ground state in an infinite potential well whose walls are located at $x = 0$ and $x = a$, is subjected at time $t = 0$ to a time-dependent perturbation $\hat{V}(t) = \epsilon \hat{x} e^{-t^2}$ where ϵ is a small real number. Find the probability that the particle will be found in its first excited state after a sufficiently long time (i.e., $t \to \infty$).

13. (a) A particle is initially in its ground state in a one-dimensional harmonic oscillator potential $V(x) = \frac{1}{2}\omega x^2$. If the coupling constant ω is suddenly doubled, calculate the probability of finding the particle in the ground state of the new potential.

    (b) Let $S_\pm = S_x \pm i S_y$ where S_x, S_y and S_z are Pauli spin matrices. If $|\pm, \frac{1}{2}\rangle$ are eigenvectors of S_z, then find $S_\pm|\pm, \frac{1}{2}\rangle$.

14. (a) Consider the Dirac Hamiltonian $H = c\vec{\alpha}.\vec{p} + \beta mc^2 + V(r)$ where the symbols have their usual meaning. Show that $[H, \vec{L}] = -i\hbar c(\vec{\alpha} \times \vec{p})$.

    (b) Consider a state $|j_1, j_2, j, m\rangle$ which is common eigenstate of the angular momentum operators J_1^2, J_2^2 and J^2 where $J = J_1 + J_2$. Show that this state is also an eigenstate of $J_1.J_2$ and find the eigenvalue.

15. Consider a Klein-Gordon theory for the scalar field $\phi(x)$ with a $\lambda\phi^4$ interaction with λ and m being the coupling constant and mass parameter, respectively.

    (a) Write down the Lagrangian and the equation of motion.

    (b) Draw Feynman diagrams for a process of two particles scattering into two particles in the lowest order *and* next to lowest order in λ. Consider the initial momenta and final momenta of the particles to be p_1, p_2 and q_1, q_2 respectively. Indicate the momentum assignments in the Feynman diagrams. Write down the scattering amplitudes in momentum space. (Detailed numerical factors are not necessary)

16. (a) Explain why the following processes are not observed in nature. Discuss any four of the seven options. (The symbols carry their usual meanings)

$$
\begin{aligned}
\mathrm{p} &\to \mathrm{e}^+ + \pi^0 \\
\Lambda^0 &\to \mathrm{K}^0 + \pi^0 \\
\mathrm{p} + \bar{\mathrm{p}} &\to \Lambda^0 + \Lambda^0 \\
\Lambda^0 &\to \mathrm{K}^+ + \mathrm{K}^- \\
\mathrm{n} &\to \mathrm{p} + \mathrm{e}^- \\
\mathrm{p} &\to \mathrm{e}^+ + \nu_\mathrm{e} \\
\mu^+ &\to \mathrm{e}^+ + \gamma
\end{aligned}
$$

(b) Consider a reaction $p + p \rightarrow x + K^+ + K^+$, where x is an unknown particle.

   i. What are the values of electric charge, strangeness and baryon number of the unknown particle x?

   ii. If the mass of x is $2.15\,\text{GeV}$, what is the minimum value of the incident proton momentum required for the reaction to take place? Given the mass of K^+ and proton are respectively $0.494\,\text{GeV}$ and $0.938\,\text{GeV}$ and the target protons are at rest.