Test Code: STB (Short Answer Type) 2015

Junior Research Fellowship for Research Course in Statistics

The candidates for research course in Statistics will have to take two short-answer type tests – STA and STB. Each test is of two-hour duration. Test STA will have about 10 questions of equal value, set from selected topics in Mathematics and Statistics at the undergraduate level. Test STB will have roughly 8 questions of equal value, on topics in Statistics at Master's level.

Syllabus for STB

(a) *Probability*: Basic concepts, elementary set theory and sample space, conditional probability and Bayes theorem. Standard univariate and multivariate distributions. Transformations of variables. Moment generating functions, characteristic functions, weak and strong laws of large numbers, convergence in distribution and central limit theorem. Markov chains.
(b) *Inference*: Sufficiency, minimum variance unbiased estimation, Bayes estimates, maximum likelihood and other common methods of estimation. Optimum tests for simple and composite hypotheses. Elements of sequential and non-parametric tests. Analysis of discrete data - contingency chi-square.
(c) *Multivariate Analysis*: Standard sampling distributions. Order statistics with applications. Regression, partial and multiple correlations. Basic properties of multivariate normal distribution, Wishart distribution, Hotelling's T^2 and related tests.
(d) *Design of Experiments*: Inference in linear models. Standard orthogonal and non-orthogonal designs. Analysis of general block designs. Factorial experiments. One and two-way ANOVA.
(e) *Sample Surveys*: Simple random sampling, Systematic sampling, PPS sampling, Stratified sampling. Ratio and regression methods of estimation. Non-sampling errors, Non-response bias.

Sample Questions : STB

1. Let $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$ be the unit circle in $\mathbb{R}^2$. Let (X_1, Y_1), (X_2, Y_2) be independent, both having uniform distribution over S. Let D denote the Euclidean distance between (X_1, Y_1) and (X_2, Y_2). Show that $\mathrm{E}(D^2) = 1$.

2. Let $X_1, \ldots, X_n$ be independent N(0,1) variables and let
$$U_n = \frac{X_1 + \ldots + X_n}{X_1^3 + \ldots + X_n^3}.$$
Show that U_n converges in distribution to $a + bU$ where $a, b \in \mathbb{R}$ and U is a standard Cauchy random variable.

3. Suppose $X_1, \ldots, X_n$ are independent $\mathrm{N}(0,1)$ variables. Let $2 \leq m < n$. Let S_m^2 and S_n^2 be given by $S_m^2 \overset{def}{=} \sum_{i=1}^m (X_i - \bar{X}_m)^2$ and $S_n^2 \overset{def}{=} \sum_{i=1}^n (X_i - \bar{X}_n)^2$, respectively, where $\bar{X}_m \overset{def}{=} \sum_{i=1}^m X_i/m$ and $\bar{X}_n \overset{def}{=} \sum_{i=1}^n X_i/n$. Let $T = S_n^2 - S_m^2$. Show that $T \sim \chi_{n-m}^2$.

4. Let $X_1, \ldots, X_n$ be a random sample of size n from a uniform distribution on $(0, 1)$. Let $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be the corresponding order statistics. Define
$$R(X_1) = r \text{ if } X_{(r)} = X_1,\ r = 1, 2, \ldots, n.$$
Find the correlation coefficient between X_1 and $R(X_1)$.

5. Let $X \sim N(\theta, 1)$. Let $Y_i = Z_i + X$, where Z_i, $i = 1, \ldots, n$ are independent $N(0, 1)$ variables and $Z_1, Z_2, \ldots, Z_n$ are independent of X. Suppose $Y_1, \ldots, Y_n$ are observed. Is $\bar{Y} = \sum_{i=1}^n Y_i/n$ sufficient for θ ? Justify.

6. Suppose $\theta \sim \mathrm{Uniform}\{1, 2, \ldots, N\}$ and the conditional distribution of U given θ is $\mathrm{Uniform}\{1, 2, \ldots, \theta\}$. Based on the observation U, find the predictor of θ with minimum Mean Squared Error.

7. Suppose we observe the random variable X that has a density
$$f(x) = \begin{cases} 2(1-\theta)x + \theta & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$
where $0 \leq \theta \leq 2$ and θ is unknown. Find the Uniformly Most Powerful test for $H_0 : \theta \leq \frac{1}{2}$ vs. $H_1 : \theta > \frac{1}{2}$ based on X.

8. A 2^5 factorial experiment with factors A, B, C, D, E, is conducted in 4 blocks each of size 8. Four of the treatment combinations in the principal block are : *ab*, *acd*, *de*, *bce*.

   (i) What will be the remaining treatment combinations in the principal block?

   (ii) Identify the confounded effects.

9. A block design with 8 treatments $1, \ldots, 8$, is planned with the following 10 blocks:

Block No.	Treatments		Block No.	Treatments
1	1, 2, 5		6	3, 4, 8
2	3, 7		7	4, 8
3	7, 8, 8		8	2, 5
4	5, 6		9	1, 5, 6
5	3, 3, 7		10	7, 8

Let τ_i denote the effect of treatment i, $i = 1, \ldots, 8$.

(i) Are the following contrasts estimable: $\tau_1 - \tau_2$, $\tau_2 - \tau_6$, $\tau_5 - 2\tau_7 + \tau_8$, $2\tau_3 - \tau_7 - \tau_8$?

(ii) What is the rank of the C-matrix of this design?

For more sample questions, visit
http://www.isical.ac.in/~deanweb/JRFSTATSQ.html