1. Suppose that the level of savings varies positively with the level of income and that savings is identically equal to investment. Then the IS curve:

   (a) slopes positively.

   (b) slopes negatively.

   (c) is vertical.

   (d) does not exist.

2. Consider the Solow growth model without technological progress. Suppose that the rate of growth of the labor force is 2%. Then, in the steady-state equilibrium:

   (a) per capita income grows at the rate of 2%.

   (b) per capita consumption grows at the rate of 2%.

   (c) wage per unit of labor grows at the rate of 2%.

   (d) total income grows at the rate of 2%.

3. Consider a Simple Keynesian Model for a closed economy with government. Suppose there does not exist any public sector enterprise in the economy. Income earners are divided into two groups, Group 1 and Group 2, such that the saving propensity of the former is less than that of the latter. Aggregate planned investment is an increasing function of GDP (Y). Start with an initial equilibrium situation. Now, suppose the government imposes and collects additional taxes from Group 1 and uses the tax revenue so generated to make transfer payments to Group 2. Following this:

   (a) aggregate saving in the economy remains unchanged.

   (b) aggregate saving in the economy declines.

   (c) aggregate saving in the economy rises.

   (d) aggregate saving in the economy may change either way.

4. Suppose, in an economy, the level of consumption is fixed, while the level of investment varies inversely with the rate of interest. Then the IS curve is:

   (a) positively sloped.

   (b) negatively sloped.

(c) vertical.

(d) horizontal.

5. Suppose, in an economy, the demand function for labor is given by:

$$L^d = 100 - 5w,$$

whereas the supply function for labor is given by:

$$L^s = 5w;$$

where w denotes the real wage rate. Total labor endowment in this economy is 80 units. Suppose further that the real wage rate is flexible. Then involuntary unemployment in this economy is:

(a) 30.

(b) 50.

(c) 70.

(d) 0.

6. Consider again the economy specified in Question 5. Suppose now that the real wage rate is mandated by the government to be at least 11. Then total unemployment will be:

(a) 35.

(b) 0.

(c) 30.

(d) 10.

7. Consider a macro-economy defined by the following equations:

$$M = kPy + L(r),$$
$$S(r) = I(r),$$
$$y = \bar{y},$$

where M, P, y and r represent, respectively, money supply, the price level, output and the interest rate, while k and $\bar{y}$ are positive constants. Furthermore, $S(r)$ is the savings function, $I(r)$ is the investment demand function and $L(r)$ is the speculative demand for money function, with , $S'(r) > 0$, $I'(r) < 0$ and $L'(r) < 0$. Then, an increase in M must:

(a) increase P proportionately.

(b) reduce P.

(c) increase P more than proportionately.

(d) increase P less than proportionately.

8. Two individuals, X and Y, have to share Rs. 100. The shares of X and Y are denoted by x and y respectively, $x, y \geq 0$, $x + y = 100$. Their utility functions are $U_X(x, y) = x + \left(\frac{1}{4}\right) y$ and $U_Y(x, y) = y + \left(\frac{1}{2}\right) x$. The social welfare function is $W(U_X, U_Y) = \min\{U_X, U_Y\}$. Then the social welfare maximizing allocation is:

(a) $(44, 56)$.

(b) $(48, 52)$.

(c) $(50, 50)$.

(d) $(60, 40)$.

9. Consider two consumers. They consume one private good (X) and a public good (G). Consumption of the public good depends on the sum of their simultaneously and non-cooperatively chosen contributions towards the public good out of their incomes. Thus, if g_1 and g_2 are their contributions, then the consumption of the public good is $g = g_1 + g_2$. Let the utility function of consumer i $(i = 1, 2)$ be $U_i(x_i, g) = x_i g$. The price of the private good is $p > 0$ and the income of each consumer is $M > 0$. Then the consumers' equilibrium contributions towards the public good will be:

(a) $\left(\frac{M}{2}, \frac{M}{2}\right)$.

(b) $\left(\frac{M}{3}, \frac{M}{3}\right)$.

(c) $\left(\frac{M}{4}, \frac{M}{4}\right)$.

(d) $\left(\frac{M}{p}, \frac{M}{p}\right)$.

10. Consider two firms, 1 and 2, producing a homogeneous product and competing in Cournot fashion. Both firms produce at constant marginal cost, but firm 1 has a lower marginal cost than firm 2. Specifically, firm 1 requires one unit of labour and one unit of raw material to produce one unit of output, while firm 2 requires two units of labour and one unit of raw material to produce one unit of output. There is no fixed cost. The prices of labour and material are given and the market demand for the product is

determined according to the function $q = A - bp$, where q is the quantity demanded at price p and $A, b > 0$. Now, suppose the price of labour goes up, but that of raw material remains the same. Then, the equilibrium profit of firm 1 will:

(a) increase.

(b) decrease.

(c) remain unchanged.

(d) go up or down depending on the parameters.

11. Considered again the problem in Question 10. As before, suppose that the price of labour goes up, but that of raw material remains the same. Then, the equilibrium profit of firm 2 will:

(a) increase.

(b) decrease.

(c) remain unchanged.

(d) go up or down depending on the parameters.

12. Consider a firm which initially operates only in market A as a monopolist and faces market demand $Q = 20 - p$. Given its cost function $C(Q) = \frac{1}{4}Q^2$, it charges a monopoly price P_m in this market. Now suppose that, in addition to selling as a monopolist in market A, the firm starts selling its products in a competitive market, B, at price $\bar{p} = 6$. Under this situation the firm charges P_m^* in market A. Then:

(a) $P_m^* > P_m$.

(b) $P_m^* < P_m$.

(c) $P_m^* = P_m$.

(d) given the available information we cannot say whether $P_m^* > P_m$ or $P_m^* < P_m$.

13. Two consumers, A and B, have utility functions $U_A = \min\{x_A, y_A\}$ and $U_B = x_B + y_B$,respectively. Their endowments vectors are $e_A = (100, 100)$ and $e_B = (50, 0)$. Consider a competitive equilibrium price vector (P_X, P_Y). Then,

(a) $\left(\frac{1}{5}, \frac{2}{5}\right)$ is the unique equilibrium price vector.

(b) $\left(\frac{1}{5}, \frac{2}{5}\right)$ is one of the many possible equilibrium price vectors.

(c) $\left(\frac{1}{5}, \frac{2}{5}\right)$ is never an equilibrium price vector.

(d) an equilibrium price vector does not exist.

14. Suppose a firm is a monopsonist in the labor market and faces separate labor supply functions for male and female workers. The labor supply function for male workers is given by $l_M = (w_M)^k$, where l_M is the amount of male labor available when the wage offered to male workers is w_M, and k is a positive constant. Analogously, the labor supply function for female workers is given by $l_F = w_F$. Male and female workers are perfect substitutes for one another. The firm produces one unit of output from each unit of labor it employs, and sells its output in a competitive market at a price of p per unit. The firm can pay male and female workers differently if it chooses to. Suppose the firm decides to pay male workers more than female workers. Then it must be the case that:

(a) $k < \frac{1}{2}$.

(b) $\frac{1}{2} \leq k < 1$.

(c) $k = 1$.

(d) $k > 1$.

15. Consider the problem in Question 14, and assume that the firm pays male workers more than female workers. Suppose further that $p > 2$. Then the firm must:

(a) hire more male workers than female workers.

(b) hire more female workers than male workers.

(c) hire identical numbers of male and female workers.

(d) hire more females than males if $2 < p \leq 4$, but more males than females if $p > 4$.

16. Consider the system of linear equations:

$$\begin{aligned} (4a-1)\,x + y + z &= 0, \\ -y + z &= 0, \\ (4a-1)\,z &= 0. \end{aligned}$$

The value of a for which this system has a non-trivial solution (i.e., a solution other than $(0,0,0)$) is:

(a) $\frac{1}{2}$.

(b) $\frac{1}{4}$.

(c) $\frac{3}{4}$.

(d) 1.

17. Let $f : \mathbb{R} \to \mathbb{R}$ be a convex and differentiable function with $f(0) = 1$, where $\mathbb{R}$ denotes the set of real numbers. If the derivative of f at 2 is 2, then the maximum value of $f(2)$ is:

(a) 3.

(b) 5.

(c) 10.

(d) ∞.

18. Consider the equation $2x + 5y = 103$. Then how many pairs of positive integer values can (x, y) take such that $x > y$?

(a) 7.

(b) 8.

(c) 13.

(d) 14.

19. Let X be a discrete random variable with probability mass function (PMF) $f(x)$ such that

$$\begin{aligned} f(x) > 0 \quad & \text{if } x = 0, 1, ..., n, \text{ and} \\ f(x) = 0 \quad & \text{otherwise,} \end{aligned}$$

where n is a finite integer. If $Prob\,(X \geq m \mid X \leq m) = f(m)$, then the value of m is:

(a) 0.

(b) 1.

(c) $n - 1$.

(d) none of the above.

20. Consider the function $f(x) = 2ax \log_e x - ax^2$ where $a \neq 0$. Then

(a) the function has a maximum at $x = 1$.

(b) the function has a minimum at $x = 1$.

(c) the point $x = 1$ is a point of inflexion.

(d) none of the above.

21. Let $f : [0, 10] \to [10, 20]$ be a continuous and twice differentiable function such that $f(0) = 10$ and $f(10) = 20$. Suppose $|f'(x)| \leq 1$ for all $x \in [0, 10]$. Then, the value of $f''(5)$ is

(a) 0.

(b) $\frac{1}{2}$.

(c) 1.

(d) cannot be determined from the given information.

22. Consider the system of linear equations:

$$\begin{aligned} x + 2ay + az &= 0, \\ x + 3by + bz &= 0, \\ x + 4cy + cz &= 0. \end{aligned}$$

Suppose that this system has a non-zero solution. Then a, b, c

(a) are in arithmetic progression.

(b) are in geometric progression.

(c) are in harmonic progression.

(d) satisfy $2a + 3b + 4c = 0$.

23. Let a, b, c be real numbers. Consider the function $f(x_1, x_2) = \min\{a - x_1, b - x_2\}$ .Let (x_1^*, x_2^*) be the solution to the maximization problem

$$\max f(x_1, x_2) \text{ subject to } x_1 + x_2 = c.$$

Then $x_1^* - x_2^*$ equals

(a) $\frac{c+a-b}{2}$.

(b) $\frac{c+b-a}{2}$.

(c) $a-b$.

(d) $b-a$.

24. Suppose that you have 10 different books, two identical bags and a box. The bags can each contain three books and the box can contain four books. The number of ways in which you can pack all the books is

(a) $\frac{10!}{2!3!3!4!}$.

(b) $\frac{10!}{3!3!4!}$.

(c) $\frac{10!}{2!3!4!}$.

(d) none of the above.

25. Real numbers $a_1, a_2, ..., a_{99}$ form an arithmetic progression. Suppose that

$$a_2 + a_5 + a_8 + ... + a_{98} = 205.$$

Then the value of $\sum_{k=1}^{99} a_k$ is

(a) 612.

(b) 615.

(c) 618.

(d) none of the above.

26. A stone is thrown into a circular pond of radius 1 meter. Suppose the stone falls uniformly at random on the area of the pond. The expected distance of the stone from the center of the pond is

(a) $\frac{1}{3}$.

(b) $\frac{1}{2}$.

(c) $\frac{2}{3}$.

(d) $\frac{1}{\sqrt{2}}$.

27. Suppose that there are n stairs, where n is some positive integer. A person standing at the bottom wants to reach the top. The person can climb either 1 stair or 2 stairs at a time. Let T_n be the total number of ways in which the person can reach the top. For instance, $T_1 = 1$ and $T_2 = 2$. Then, which one of the following statements is true for every $n > 2$?

(a) $T_n = n$.

(b) $T_n = 2T_{n-1}$.

(c) $T_n = T_{n-1} + T_{n-2}$.

(d) $T_n = \sum_{k=1}^{n-1} T_k$.

28. Let $Y_1, Y_2, ..., Y_n$ be the income of n individuals with $E(Y_i) = \mu$ and $Var(Y_i) = \sigma^2$ for all $i = 1, 2, ...n$. These n individuals form m groups, each of size k. It is known that individuals within the same group are correlated but two individuals in different groups are always independent. Assume that when individuals are correlated, the correlation coefficient is the same for all pairs. Consider the random variable $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$. The limiting variance of $\bar{Y}$ when m is large but k is finite is

(a) 0.

(b) $\frac{1}{k}$.

(c) 1.

(d) $\frac{\sigma^2}{k}$.

29. A person makes repeated attempts to destroy a target. Attempts are made independently of each other. The probability of destroying the target in any attempt is 0.8. Given that he fails to destroy the target in the first five attempts, the probability that the target is destroyed in the 8-th attempt is

(a) 0.032.

(b) 0.064.

(c) 0.128.

(d) 0.160.

30. Let E and F be two events such that $0 < Prob\,(E) < 1$ and $Prob\,(E \mid F) + Prob\,(E \mid F^c) = 1$. Then

(a) E and F are mutually exclusive.

(b) $Prob\,(E^c \mid F) + Prob\,(E^c \mid F^c) = 1$.

(c) E and F are independent.

(d) $Prob\,(E \mid F) + Prob\,(E^c \mid F^c) = 1$.