Indian Statistical Institute, Bangalore
M.S (QMS):2016

Instructions

The test is divided into two sessions (i) Forenoon session and (ii) Afternoon session. Each session is for two hours. For the forenoon session question paper, the test code is QMA and for the afternoon session question paper, the test code is QMB. Candidates appearing for MS(QMS) should verify and ensure that they are answering the right question paper.

The test QMA is multiple choice type. For each question, exactly one of the four choices is correct. You get four marks for each correct answer, one mark for each unanswered question, and zero mark for each incorrect answer.

The test QMB is of short answer type. It has altogether 10 questions. A candidate has to answer minimum of 8 questions.

Questions will be set on the following and related topics.

Syllabus of QMA (Mathematics) and QMB (Mathematics)

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Equations.

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution. Elements of Probability and Probability distributions.

Sample questions for QMA (Mathematics)(2016)

1. Let $x_1, x_2, ...$ be positive integers which are in arithmetic progression (A.P)., such that $x_1 + x_2 + x_3 = 12$ and $x_4 + x_6 = 14$. Then x_5 is
   (a) 7
   (b) 1
   (c) 4
   (d) None of the above

2. I sold 2 books for Rs.30 each. My profit on one was 25% and the loss on the other was 25%. Then, on the whole, I
   (a) neither gained nor lost
   (b) lost Rs.5
   (c) lost Rs.4
   (d) gained Rs.4

3. For any real number x let $g(x)$ denote the determinant of the matrix $\begin{bmatrix} x & 2 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ 0 & 0 & 5 & x \\ 0 & 0 & x & 2 \end{bmatrix}$

   The solutions of $g(x) = 0$ are

   (a) $(\frac{4}{5}, -10, 10)$
   (b) $(\frac{5}{4}, \sqrt{10}, -\sqrt{10})$
   (c) $(4, \sqrt{10}, -\sqrt{10})$
   (d) $(\frac{4}{5}, \sqrt{10}, -\sqrt{10})$

4. A car whose original value is Rs.25,600 decreases in value by Rs.90 per month. In how many months the car's value falls below Rs.15,000?

   (a) 115 months
   (b) 117 months
   (c) 119 months
   (d) 121 months

5. A manufacturer determines that the number of drills it can sell is given by the form $D = -3P^2 + 180P - 285$ where P is the price of a drill in Rs. What is the maximum number of drills that can be sold?

   (a) 2145
   (b) 2415
   (c) 2225
   (d) 2445

6. A balloon takes off from a location that is 24ft above sea level. It rises 45ft/min. Choose an equation to model the balloon's elevation "h" as a function of time "t" (in minutes).

   (a) h = 24t + 45
   (b) h = 24t - 45
   (c) h = 45t + 24
   (d) h = 45t - 24

7. A multiple choice test has 10 questions. Each question has 4 choices; A, B, C, D. In how many ways can the test be answered?

   (a) 10 + 4
   (b) 10×4
   (c) 10^4
   (d) 4^{10}

8. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2$ and 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is

   (a) 2(p-q)(2q-p)/9
   (b) 2(q-p)(2p-q)/9
   (c) 2(2p-q)(2q-p)/9
   (d) 2(q-2p)(2q-p)/9

9. The value of $\lim_{x \to 0} \frac{\cos 3x - \cos x}{x^2}$ is equal to

   (a) 4
   (b) 1
   (c) -1
   (d) -4

10. If $\log 2, \log(2^x - 1)$ and $\log(2^x + 3)$ are in AP then x is equal to

   (a) $\frac{5}{2}$
   (b) $\log_2 5$
   (c) $\log_3 2$
   (d) $\frac{3}{2}$

11. The value of $\int_0^{1.5} [x^2] dx$, where [y] denotes the greatest integer less than or equal to y, is

   (a) 2
   (b) $2 - \sqrt{2}$
   (c) $2 + \sqrt{2}$
   (d) None of the above

12. Three prizes are to be distributed among six persons. The number of ways in which this can be done if no person gets all the prizes is

   (a) 120
   (b) 216
   (c) 1140
   (d) None of the above

13. Let $\widehat{a}$ and $\widehat{b}$ be unit vectors. If $\widehat{a} - \widehat{b}$ is a unit vector then the angle between $\widehat{a}$ and $\widehat{b}$ is

   (a) 60°
   (b) 120°
   (c) 90°
   (d) None of the above

14. If $A^2 - A + I = O$, then the inverse of the non singular square matrix A is

   (a) A - I
   (b) I - A
   (c) A + I
   (d) A

15. One side of an equilateral triangle is 24cm. The mid-points of its sides are joined to form another triangle whose mid-points are in turn joined to form still another equilateral triangle. This process continues indefinitely. The sum of the perimeters of all triangles is

   (a) 144cm
   (b) 169cm
   (c) 400cm
   (d) 625cm

16. Two boys A and B play a game where each is asked to select a number between 1 and 30. If the selected numbers match, they both win a prize; otherwise no prize is given. In a single trial of this game, the probability that no prize is won is

   (a) $\frac{1}{30}$
   (b) $\frac{29}{30}$
   (c) $\frac{2}{30}$
   (d) None of the above

17. Let $\{a_i\}$ be a sequence such that $\sum_{r=1}^{n} a_r = n^2$, then $\frac{1^3}{a_1} + \frac{1^3+2^3}{a_1+a_2} + \frac{1^3+2^3+3^3}{a_1+a_2+a_3} + ...$ upto 16 terms is

   (a) 346
   (b) 446
   (c) 546
   (d) None of the above

18. Mumbai Rajdhani Express going from New Delhi to Mumbai stops at four intermediate stations. Ten passengers enter the train during the journey with ten different tickets either of III AC or II AC. The number of different sets of tickets they may have is

   (a) ${}^{30}C_{10}$
   (b) ${}^{20}C_{10}$
   (c) ${}^{10}C_5$
   (d) ${}^{15}C_{10}$

19. Let $x_1 < -1$, and define $x_{n+1} = \frac{x_n}{1+x_n}$ for $n \geq 1$. Then

   (a) $x_n \to 1 \; as \; n \to \infty$
   (b) $x_n \to -1 \; as \; n \to \infty$
   (c) $x_n \to 0 \; as \; n \to \infty$
   (d) $x_n \to \infty \; as \; n \to \infty$

20. The value of $\int_1^\infty \frac{x^2-2}{x^3\sqrt{x^2-1}} dx$ is

   (a) $\frac{4}{3}$
   (b) $\frac{2}{3}$
   (c) 0
   (d) None of the above

Sample questions for QMB (Mathematics)(2016)

1. (a) Evaluate $\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{3^i 3^j 3^k}$ for $(i \neq j \neq k)$.

   (b) A restaurant offers 5 choices of appetizer, 10 choices of main meal and 4 choices of dessert. A customer can choose to eat just one course, or two different courses, or all three courses. Assuming all choices are available, how many different possible meals does the restaurant offer?

2. (a) Given a function g which has derivative $g'(x)$ for all x satisfying $g'(0) = 2$ and $g(x+y) = e^y g(x) + e^x g(y)$ for all $x, y \varepsilon R$.
   Show that $g'(x) + g(x) - 2e^x = 0$.

   (b) Let $A = \begin{bmatrix} \cos(\theta+\alpha) & \sin(\theta+\alpha) & 1 \\ \cos(\theta+\beta) & \sin(\theta+\beta) & 1 \\ \cos(\theta+\gamma) & \sin(\theta+\gamma) & 1 \end{bmatrix}$.
   Show that $|A|$ is independent of θ.

3. (a) Show that $1 = \int\int\int x^{l-1}y^{m-1}z^{n-1}(1-x-y-z)^{p-1}dx\ dy\ dz$; $(l,m,n,p) \geq 1$ taken over the tetrahedron bounded by the planes, $x = 0, y = 0, z = 0, x+y+z = 1$ is $\frac{(\Gamma l)(\Gamma m)(\Gamma n)(\Gamma p)}{\Gamma(l+m+n+p)}$.

   (b) Consider the following two experiments. In the first experiment a fair coin is tossed six times. Let p be the probability of getting exactly four heads in a row in this experiment. In the second experiment a pair of fair coins are simultaneously tossed three times. Let q be the probability of getting a pair of heads exactly twice in a row in the second experiment. then p and q are related as in

      i. $p = q$
      ii. $p < q$
      iii. $p > q$
      iv. $p = 2q$

4. (a) A rectangle is inscribed in an acute angled triangle ABC with one side of the rectangle along the base BC and one vertex in AB and another vertex in AC. Find the dimensions of the rectangle having the maximum area.

   (b) For all real numbers x, y show that

      i. $|x+y| \leq |x| + |y|$
      ii. $|x-y| \geq |x| - |y|$

5. (a) If $f(x) = (x) - x$, where $(x) =$ the least integer greater than or equal to x, then show that $f(x)$ is continuous at all non-integral values of x.

   (b) If one root of the equation $x^2 + ax + b = 0$ is also a root of $x^2 + mx + n = 0$, show that its other root is a root of
   $x^2 + (2a - m)x + a^2 - am + n = 0$.

6. (a) Solve the equation $2^{|x+2|} - |2^{x+1} - 1| = 2^{x+1} + 1$.

   (b) A spherical iron ball of radius 10cm coated with a layer of ice of uniform thickness melts at a rate of $100\pi cm^3/min$. Find the rate at which the thickness of the ice decreases when the thickness of ice is 5cm.

7. (a) For $n = 1, 2, 3, ...$ let A_n be a sequence of regular convex polygons with 2^{n+1} sides and radius (distance from any vertex to the center) 1. Thus A_1 is a square with diagonal length 2 and A_2 is an octagon with diagonal length 2. Let a_n be the area of the polygon A_n. Then

      i. Express a_n in terms of the angle θ_n made by the lines joining any two nearest vertices to the center of A_n,

      ii. Find the limit of the sequence a_n as $n \to \infty$.

   (b) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, Let B be a 3×3 matrix $[B_1\ B_2\ B_3]$

   such that $A^{50}B_1 = \begin{bmatrix} 1 \\ 25 \\ 25 \end{bmatrix}$, $A^{50}B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and $A^{50}B_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

   Find the value of $|B|$.

8. (a) Use the principle of mathematical induction to prove that,
   $^nC_0 +^n C_1 +^n C_2 + ... +^n C_n = 2^n$ for all $n \varepsilon N$.

   (b) Examine the following function for continuity at the origin
   $f(x) = \left\{ \begin{array}{cc} \frac{xe^{1/x}}{1+e^{1/x}} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{array} \right\}$.

9. (a) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then show that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

   (b) Evaluate $\int_{-2}^{3} |1 - x^2| dx$.

10. (a) Find the value of $\Sigma_{n=0}^{\infty} \frac{2n+3}{3^n}$.

    (b) If $m > 2$ and $t \in R$, find the integral part of
    $(\frac{4|t|}{16+t^2})^m$.