Sample Questions (PCA)

1. A person throws a pair of fair dice. If the sum of the numbers on the dice is a perfect square, then the probability that the number 3 appeared on at least one of the dice is

   (A) 1/9 (B) 4/7 (C) 1/18 (D) 7/36

2. Consider the system of linear equations: $x+y+z=5,\quad 2x+2y+3z=4$. Then

   (A) the system is inconsistent
   (B) the system has a unique solution
   (C) the system has infinitely many solutions
   (D) none of the above is true

3. If $g'(x)=f(x)$ then $\int x^3 f(x^2)dx$ is given by

   (A) $x^2g(x^2)-\int xg(x^2)dx+C$ (B) $\frac{1}{2}x^2g(x^2)-\int xg(x^2)dx+C$
   (C) $2x^2g(x^2)-\int xg(x^2)dx+C$ (D) $x^2g(x^2)-\frac{1}{2}\int xg(x^2)dx+C$

4. If $({}^nC_0+{}^nC_1)({}^nC_1+{}^nC_2)\cdots({}^nC_{n-1}+{}^nC_n)=k\ {}^nC_0\ {}^nC_1\ \cdots\ {}^nC_{n-1}$, then k is equal to

   (A) $\frac{(n+1)^n}{n!}$ (B) $\frac{n^n}{n!}$ (C) $\frac{(n+1)^n}{nn!}$ (D) $\frac{(n+1)^{n+1}}{n!}$

5. Let S be a set of n elements. The number of ways in which n distinct non-empty subsets $X_1,...,X_n$ of S can be chosen such that $X_1 \subseteq X_2 \cdots \subseteq X_n$, is

   (A) $\binom{n}{1}\binom{n}{2}\cdots\binom{n}{n}$ (B) 1 (C) $n!$ (D) 2^n

6. Let A be a 4×4 matrix such that both A and $\mathrm{Adj}(A)$ are non-null. If $\det A=0$, then the rank of A is

   (A) 1 (B) 2 (C) 3 (D) 4

7. Suppose a,b,c are in A.P. and a^2,b^2,c^2 are in G.P. If $a<b<c$ and $a+b+c=\frac{3}{2}$, then the value of a is

   (A) $\frac{1}{2\sqrt{2}}$ (B) $-\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

8. The set of all a satisfying the inequality

$$\frac{1}{\sqrt{a}}\int_1^a \left(\frac{3}{2}\sqrt{x}+1-\frac{1}{\sqrt{x}}\right)dx < 4$$

is equal to the interval

(A) $(-5,-2)$ (B) $(1,4)$ (C) $(0,2)$ (D) $(0,4)$

9. Let C_0 be the set of all continuous functions $f:[0,1]\to\mathbb{R}$ and C_1 be the set of all differentiable functions $g:[0,1]\to\mathbb{R}$ such that the derivative g' is continuous. (Here, differentiability at 0 means right differentiability and differentiability at 1 means left differentiability.) If $T: C_1\to C_0$ is defined by $T(g)=g'$, then

(A) T is one-to-one and onto
(B) T is one-to-one but not onto
(C) T is onto but not one-to-one
(D) T is neither one-to-one nor onto.

10. The number of distinct even divisors of

$$\prod_{k=1}^{5} k!$$

is

(A) 24 (B) 32 (C) 64 (D) 72

11. A straight line passes through the intersection of the lines given by $3x-4y+1=0$ and $5x+y=1$ and makes equal intercepts of the same sign on the coordinate axes. The equation of the straight line is

(A) $23x-23y+11=0$ (B) $23x-23y-11=0$
(C) $23x+23y+11=0$ (D) $23x+23y-11=0$

12. Suppose A and B are two square matrices such that the largest eigenvalue of $(AB-BA)$ is positive. Then the smallest eigen value of $(AB-BA)$

(A) must be positive (B) must be negative
(C) must be 0 (D) is none of the above

13. A fair die is rolled five times. What is the probability that the largest number rolled is 5?

(A) 5/6 (B) 1/6 (C) $1-(1/6)^6$ (D) $(5/6)^5-(2/3)^5$

14. The series

$$\sum_n \frac{3\cdot 6\cdot 9\cdots 3n}{7\cdot 10\cdot 13\cdots(3n+4)}x^n, \quad x>0$$

(A) converges for $0<x\le 1$ and diverges for $x>1$
(B) converges for all $x>0$
(C) converges for $0<x<\frac{1}{2}$ and diverges for $x\ge\frac{1}{2}$
(D) converges for $\frac{1}{2}<x<1$ and diverges for $0<x\le\frac{1}{2}$, $x\ge 1$.

15. Given a real number $\alpha\in(0,1)$, define a sequence $\{x_n\}_{n\ge 0}$ by the following recurrence relation:

$$x_{n+1}=\alpha x_n+(1-\alpha)x_{n-1}, \quad n\ge 1.$$

If $\lim_{n\to\infty}x_n=\ell$ then the value of ℓ is

(A) $\frac{\alpha x_0+x_1}{1-\alpha}$ (B) $\frac{(1-\alpha)x_0+x_1}{2-\alpha}$
(C) $\frac{\alpha x_0+x_1}{2-\alpha}$ (D) $\frac{(1-\alpha)x_1+x_0}{2-\alpha}$

16. Suppose that a 3×3 matrix A has an eigen value -1. If the matrix $A+I$ is equal to

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

then the eigen vectors of A corresponding to the eigenvalue -1 are in the form,

(A) $\begin{bmatrix} 2t \\ 0 \\ t \end{bmatrix}$, $t\in\mathbb{R}$ (B) $\begin{bmatrix} 2t \\ s \\ t \end{bmatrix}$, $s,t\in\mathbb{R}$

(C) $\begin{bmatrix} t \\ 0 \\ -2t \end{bmatrix}$, $t\in\mathbb{R}$ (D) $\begin{bmatrix} t \\ s \\ 2t \end{bmatrix}$, $s,t\in\mathbb{R}$

17. The number of real roots of the polynomial $x^3 - 2x + 7$ is

(A) 0 (B) 1 (C) 2 (D) 3

18. Two rows of n chairs, facing each other, are laid out. The number of different ways that n couples can sit on these chairs such that each person sits directly opposite to his/her partner is

(A) $n!$ (B) $n!/2$ (C) $2^n n!$ (D) $2n!$.

19. Consider two real valued functions f and g given by

$$f(x) = \frac{x}{x-1} \text{ for } x > 1, \quad \text{and} \qquad g(x) = 7 - x^3 \text{ for } x \in \mathbb{R}.$$

Which of the following statements about inverse functions is true?

(A) Neither f^{-1} nor g^{-1} exists
(B) f^{-1} exists, but not g^{-1}
(C) f^{-1} does not exist, but g^{-1} does
(D) Both f^{-1} and g^{-1} exist.

20. Let

$$A = \begin{bmatrix} a & 1 & 1 \\ b & a & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

The number of elements in the set

$$\Big\{(a, b) \in \mathbb{Z}^2 : 0 \leq a, b \leq 2021, \ \text{rank}(A) = 2\Big\}$$

is

(A) 2021 (B) 2020 (C) $2021^2 - 1$ (D) 2020×2021

21. Amir, Bhola, Chaitali and Deepak are invited to a party. If Bhola and Chaitali attend, then Deepak will attend too. If Bhola does not attend, then Amir will not attend. If Deepak does not attend, which of the following is necessarily *true*?

(A) Chaitali does not attend (B) Amir does not attend
(C) Either (a) or (b), or both (D) None of the above

22. Let $G = (V, E)$ be an undirected simple graph, and s be a designated vertex in G. For each $v \in V$, let $d(v)$ be the length of a shortest path between s and v. For an edge (u, v) in G, what can not be the value of $d(u) - d(v)$?

(A) 2 (B) -1 (C) 0 (D) 1

23. Which of the following degree sets is NOT possible for a graph of order 5?

(A) 0, 1, 2, 2, 3 (B) 2, 2, 2, 2, 2 (C) 2, 2, 2, 3, 3 (D) 2, 2, 3, 3, 3

24. Transposing the adjacency matrix of an undirected graph with n connected components gives the adjacency matrix of a graph with

(A) $n/2$ components
(B) $(n - 1)$ components
(C) $(n + 1)$ components
(D) n components

25. Your college has sent a contingent to take part in a cultural festival at a neighbouring institution. Several team events are part of the program. Each event takes place through the day with many elimination rounds. Your contingent is multi-talented and each individual has the skills to take part in a subset of the events. However, the same individual cannot be part of the team for two different events because of a possible clash in timings. Your aim is to create teams to take part in as many events as possible.

To do this, you decide to model the problem as a graph where the nodes are the events and edges represent pairs of events where the team that you plan to send shares a member. In this setting, the graph theoretic question to be answered is:

(A) Find a spanning tree with maximum number of edges.
(B) Find an independent set of maximum cardinality.
(C) Find a vertex cover of minimum cardinality.
(D) Find a vertex colouring with minimum number of colours.

26. Which of the words below matches the regular expression $a(a + b)^*b + b(a + b)^*a$?

(A) *aba* (B) *bab* (C) *abba* (D) *aabb*

27. Let the regular expression of all the strings generated from English lowercase letters, which starts with a vowel, and has odd number of letters can be expressed as $[a+e+i+o+u][a+b+\cdots+z]^n$. Which of the following expressions is correct about n?

(A) $n \geq 0$ and n is odd (B) $n \geq 0$ and n is even
(C) $n > 0$ and n is odd (D) $n > 0$ and n is even

28. Which of the following properties is always applicable to the numbers generated by the regular expression $1[3+6+9]^*8$?

(A) Odd (B) Divisible by 3 (C) Divisible by 8 (D) Divisible by 9

29. Assume that the Boolean operator `&` performs bitwise AND on the 8-bit unsigned variables associated with it. If for some $n > 0$, we have n `&` $(n-1) = 0$, which of the following statements is necessarily true?

(A) n is even (B) n is odd
(C) n is a power of 2 (D) n is a power of 3

30. What does the following function compute in terms of n and d, for integer values of n and d, $n > 1, d > 1$? Note that $a//b$ denotes the quotient (integer part) of $a \div b$, for integers a and b. For instance $7//3$ is 2.

```
function foo(n,d){
  x := 0;
  while (n >= 1) {
    x := x+1;
    n := n//d;
  }
  return(x);
}
```

(A) The number of ways of choosing d elements from a set of size n.
(B) The number of ways of rearranging d elements from a set of size n.
(C) The number of digits in the base d representation of n.
(D) The number of ways of partitioning n elements into groups of size d.

31. Suppose a machine generates an integer uniformly at random from $\{0, 1, \ldots, n\}$ and whenever it is less than an integer k it prints "OK". If it prints "OK" with a probability 0.5, what is the guaranteed relation between n and k?

(A) $k = n/2$ (B) $k = (n-1)/2$
(C) $k = (n+1)/2$ (D) Nothing can be inferred

32. Let the variables x and y vary over the set of natural numbers, $\mathbb{N}$. Consider the formulas expressing properties about $\mathbb{N}$: (i) $\forall x \exists y (x < y)$, and (ii) $\exists y \forall x (x \leq y)$. Which of the following statements is correct?

(A) Both (i) and (ii) hold.
(B) (i) holds but (ii) does not hold.
(C) (ii) holds but (i) does not hold.
(D) Both (i) and (ii) do not hold.

33. Let L be a regular language and F be a finite language over $\{0, 1\}$. Consider the statements: (i) $L \cup F$ is regular, and (ii) $L \cup F^c$ is regular, where F^c denotes the complement of F. Which of the following statements is correct?

(A) Both (i) and (ii) are true.
(B) (i) is true but (ii) is false.
(C) (ii) is true but (i) is false.
(D) Both (i) and (ii) are false.

34. Let a, b, c be members of a Boolean algebra B. Define $a \to b \stackrel{\text{def}}{=} \neg a \vee b$. Which of the following statements is correct?

(A) $a \to (b \to c) = b \to (a \to c)$
(B) $a \to (b \to c) = a \to (c \to b)$
(C) $a \to (b \to c) = b \to (c \to a)$
(D) $a \to (b \to c) = c \to (a \to b)$

35. Let P be a set of propositional variables. Let $\Gamma = \{p \to q, \neg r \to \neg q, \neg s \to \neg r, s \to t \mid p, q, r, s, t \in P\}$ be a set of propositional logic formulas. Which of the following formulas is derivable from Γ in classical propositional logic?

(A) $s \to q$ (B) $r \to p$ (C) $p \to t$ (D) $s \to p$

36. Let us consider the language $\{\epsilon, a, a^2, \ldots, a^{10}\}$, where ϵ denotes the empty string, and a^n denotes $\underbrace{aa \cdots a}_{n \text{ times}}$. What is the minimum number of states required by a DFA to accept this language?

(A) 5 (B) 10 (C) 11 (D) 12

37. Let A be the adjacency matrix of a directed acyclic graph G. Then A^k_{ij} (the value at position (i, j) of the matrix A^k) denotes:

(A) the number of paths from i to j in G of length exactly k

(B) the number of paths from i to j in G of length at most k

(C) the number of vertices whose removal breaks all paths of length at most k from i to j

(D) the number of vertices that are both reachable from i as well from whom j is reachable using paths of length at most k

38. The diameter of a tree on n vertices in which every vertex has degree either 1 or 3 is:

(A) at least $2\log_3(n+1)$ and at most $\frac{n}{2}$.

(B) at least $2\log_2(\frac{n+2}{3})$ and at most $\frac{n}{3}$.

(C) at least $2\log_2(\frac{n+2}{3})$ and at most $\frac{n}{2}$.

(D) at least $2\log_3(n+1)$ and at most $\frac{n}{3}$.

39. Consider the following program fragment:

```
1. i = 1;
2. while (i <= n)  do
3.      begin
4.           sum = sum + a[i];
5.           i = i + 1;
6.      end
```

Let (i) $\boldsymbol{A}$ represent the initialization in line 1, (ii) $\boldsymbol{T}$ represent the test implied by line 2, (iii) $\boldsymbol{B}$ represent the statement in line 4, and (iv) $\boldsymbol{I}$ represent the increment in line 5. Which of the following regular expressions represents all possible sequences of steps taken by this program?

(A) $AT(BIT)^+$ (B) $AT(BIT)^*$ (C) $A(TBI)^+$ (D) $A(TBI)^*$

40. What is the minimum number of elementary Boolean operations (AND / OR / NOT) required to construct an equivalent Boolean expression of $AB + A\overline{B} + \overline{A}C$ ($\overline{X}$ denotes the complement of X)?

(A) 1 (B) 3 (C) 5 (D) 7