Test Code: PQB (Short Answer Type) 2016

M.Tech. in Quality, Reliability and Operations Research (Kolkata)

The candidates applying for M. Tech. in Quality, Reliability and Operations Research will have to take two tests: **Test MMA** (objective type) in the forenoon session and **Test PQB** (short answer type) in the afternoon session.

For Test **MMA**, see a different Booklet. For Test **PQB**, refer to this Booklet **ONLY.**

If you are from **Statistics / Mathematics Stream,** you will be required to **ANSWER PART I.**

If you are from **Engineering Stream**, you will be required to **ANSWER PART II.**

In **PART I**, a **TOTAL of TEN [10]** questions, are divided into **TWO Groups**: **S1**: **Statistics and S2: Probability – each group carrying FIVE [5]** questions. You will be required to answer a **TOTAL of SIX [6]** questions, taking **AT LEAST TWO [2]** from **each group**.

In **PART II**, a **TOTAL of TEN [10]** questions, are divided into **TWO Groups**: **E1**: **Mathematics** and **E2: Engineering and Technology. E1** will contain **FOUR [4]** questions and **E2** will contain **SIX [6]** questions. You will be required to answer a **TOTAL of SIX [6]** questions, taking **AT LEAST TWO [2]** from **each group**.

Syllabus

PART I: STATISTICS / MATHEMATICS STREAM

Statistics (S1)

- Descriptive statistics for univariate, bivariate and multivariate data.
- Standard univariate probability distributions [Binomial, Poisson, Normal] and their fittings, properties of distributions. Sampling distributions.
- Theory of estimation and tests of statistical hypotheses.
- Simple and Multiple linear regression, linear statistical models, ANOVA.
- Principles of experimental designs and basic designs [CRD, RBD & LSD].
- Elements of non-parametric inference.
- Elements of sequential tests.
- Sample surveys – simple random sampling with and without replacement, stratified and cluster sampling.

Probability (S2)

- Classical definition of probability and standard results on operations with events, conditional probability and independence.
- Distributions of discrete type [Bernoulli, Binomial, Multinomial, Hypergeometric, Poisson, Geometric and Negative Binomial] and continuous type [Uniform, Exponential, Normal, Gamma, Beta] random variables and their moments.
- Bivariate distributions (with special emphasis on bivariate normal), marginal and conditional distributions, correlation and regression.
- Multivariate distributions, marginal and conditional distributions, regression, independence, partial and multiple correlations.
- Order statistics [including distributions of extreme values and of sample range for uniform and exponential distributions].
- Distributions of functions of random variables.
- Multivariate normal distribution [density, marginal and conditional distributions, regression].

- Weak law of large numbers, central limit theorem.
- Basics of Markov chains and Poisson processes.

PART II: ENGINEERING STREAM

Mathematics (E1)

- Elementary theory of equations, inequalities, permutation and combination, complex numbers and De Moivre's theorem.
- Elementary set theory, functions and relations, matrices, determinants, solutions of linear equations.
- Trigonometry [multiple and sub-multiple angles, inverse circular functions, identities, solutions of equations, properties of triangles].
- Coordinate geometry (two dimensions) [straight line, circle, parabola, ellipse and hyperbola], plane geometry, Mensuration.
- Sequences, series and their convergence and divergence, power series, limit and continuity of functions of one or more variables, differentiation and its applications, maxima and minima, integration, definite integrals areas using integrals, ordinary and partial differential equations (up to second order)

Engineering and Technology (E2)

Engineering Mechanics and Thermodynamics

- Forces in plane and space, analysis of trusses, beams, columns, friction, principles of strength of materials, work-energy principle, moment of inertia, plane motion of rigid bodies, belt drivers, gearing.
- Laws of thermodynamics, internal energy, work and heat changes, reversible changes, adiabatic changes, heat of formation, combustion, reaction, solution and dilution, entropy and free energy and maximum work function, reversible cycle and its efficiency, principles of internal combustion engines. Principles of refrigeration.

Electrical and Electronics Engineering

- DC circuits, AC circuits (1-ϕ), energy and power relationships, Transformer, DC and AC machines, concepts of control theory and applications.
- Network analysis, 2 port network, transmission lines, elementary electronics (including amplifiers, oscillators, op-amp circuits), analog and digital electronic circuits.

Engineering Drawing

- Concept of projection, point projection, line projection, plan, elevation, sectional view (1st angle / 3rd angle) of simple mechanical objects, isometric view, dimensioning, sketch of machine parts.
  (Use of set square, compass and diagonal scale should suffice).

PART I (FOR STATISTICS / MATHEMATICS STREAM)

GROUP S1
Statistics

1. (a) Drug A is known to be quite effective in curing headache. The performance of drug A in terms of time taken to cure headache has been obtained for 100 randomly chosen patients and the results are given below.

Time to Cure for Drug A (in Minutes)	0 - 5	5 – 10	10 - 15	15 - 20	20 - 25	25 - 30	> 30
Number of Patients	35	24	18	12	5	4	2

Suppose XYZ Pharmaceuticals has come out with drug B and claims that it is superior to drug A. In order to verify the claim of XYZ Pharmaceuticals, data were collected on 100 patients selected randomly and the results are given below.

Time to Cure for Drug B (in Minutes)	0 - 5	5 – 10	10 - 15	15 - 20	20 - 25	25 - 30	> 30
Number of Patients	37	24	19	11	5	3	1

Draw the ogives for the time to cure headache for both the drugs (make free hand sketches on your answer sheet). Observe the ogives closely and offer your comments on the claim made by XYZ Pharmaceuticals about drug B.

(b) Suppose that median software development productivity is typically 23 units or less. A software tool vendor has developed a tool and claims that its usage will improve productivity. In order to check the claim made by the vendor you have used the tool on 10 software development projects and the productivity numbers obtained were 26.4, 24.3, 21.7, 22.4, 24.8, 25.2, 25.7, 26.1, 23.5 and 25.1.

Do you think that the data supports the vendor's claim assuming that you will believe the vendor if the chance of this kind or a worse sample is less than 1% under the assumption of no improvement? Clearly state the hypotheses and the assumptions made by you to carry out the computations.

2. Let $x_1, x_2, \ldots, x_n$ be a random sample from the population whose probability density function is given by

$$f(x) = \begin{cases} e^{-(x-\theta)} \; if x > \theta, & -\infty < \theta < \infty \\ 0 & Otherwise \end{cases}.$$

Show that the statistic $x_{(1)} = \min_i x_i$ is a consistent estimator of θ and is sufficient for θ.

3. Let $(y_{1j}; j = 1,2,\ldots n)$, $(y_{2j}; j = 1,2,\ldots n)$ and $(y_{3j}; j = 1,2,\ldots n)$ be three random samples from $N(\mu_1, \sigma^2)$, $N(\mu_2, \sigma^2)$ and $N(\mu_3, \sigma^2)$ populations respectively, where $\mu_1 = \beta_1$, $\mu_2 = \beta_1 + \beta_2$, and $\mu_3 = \beta_1 + \beta_2 + \beta_3$.

Formulate the above problem as a standard linear model and find the following

(a) BLUEs for β_i $i = 1, 2, 3$, state the necessary results,
(b) Error (residual) sum of squares, and
(c) Dispersion matrix of $\hat{\beta} = \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{pmatrix}$.

Note: $\begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

4. (a) Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be a random sample from the discrete distribution with joint probability mass function

$$f_{X,Y}(x,y) = \begin{cases} \frac{\alpha}{4} & for \quad (x,y) = (0,0) \text{ and } (1,1) \\ \frac{2-\alpha}{4} & for \quad (x,y) = (0,1) \text{ and } (1,0) \end{cases} \text{, where} \quad 0 \leq \alpha \leq 2$$

Find the maximum likelihood estimator of α.

(b) A straight line regression E(y) = $\alpha + \beta x$ is to be fitted using four observations. Assume Var($y|x$) = σ^2 for all x. The values of x at which observations are to be made, lie in the closed interval [−1, 1].

The following choices of the values of x where observations are to be made are available

i) two observations each at $x = -1$ and $x = 1$,

ii) one observation each at $x = -1$ and $x = 1$ and two observations at $x = 0$, and

iii) one observation each at $x = -1, -½, ½, 1$.

If the interest is to estimate the slope with least variance, which of the above data collection plans would you choose and why?

5. Suppose a population of size N is divided into L strata and the h^{th} stratum consists of N_h units, $h = 1, 2, \ldots, L$. From the h^{th} stratum an SRSWOR sample of size n_h is drawn, $h = 1, 2, \ldots, L$.

Obtain an unbiased estimator of the population variance based on the stratified random sample.

6. Let $[\{x_i ; i = 1, 2, \ldots, p\}; \{y_j ; j = 1,2,\ldots, q\}; \{z_k ; k = 1, 2, \ldots, r\}]$ represent random samples from $N(\alpha + \beta, \sigma^2)$, $N(\beta + \gamma, \sigma^2)$ and $N(\gamma + \alpha, \sigma^2)$ populations respectively. The populations are to be treated as independent.

(a) Display the set of complete sufficient statistics for the parameters $(\alpha, \beta, \gamma, \sigma^2)$.

(b) Find unbiased estimator for β based on the sample means only. Is it unique?

(c) Show that the estimator in (b) is uncorrelated with all error functions.

(d) Suggest an unbiased estimator for σ^2 with maximum d.f.

(e) Suggest a test for H_0: $\beta = \beta_0$.

7. If X_1, X_2, X_3 constitute a random sample from a Bernoulli population with mean for p , show why $[X_1 + 2X_2 + 3X_3] / 6$ is *not* a sufficient statistic for p.

8. Life distributions of two independent components of a machine are known to be exponential with means μ and λ respectively. The machine fails if at least one of the components fails. Compute the chance that the machine will fail due to the second component. Out of n independent prototypes of the machine m of them fail due to the second component. Show that $m/(n-m)$ approximately estimates the odds ratio $\theta = \lambda/\mu$.

9. Suppose in a randomised block design for v treatments in b blocks, an observation under treatment i in block j is missing. Use the missing plot technique to give an "estimate" for the missing cell. Obtain $\mathrm{Var}(\hat{\tau}_1 - \hat{\tau}_2)$ where $(\hat{\tau}_1 - \hat{\tau}_2)$ is the BLUE of the elementary treatment contrast $(\tau_1 - \tau_2)$ of the effects of the treatments *1* and *2*. Also, give the analysis of variance for testing the equality of the treatment effects under the missing data situation.

GROUP S2
Probability

1. (a) Find the probability of choosing 3 numbers a, b and c from the set of integers $\{1, 2, 3,\ldots,(2k+1)\}$, where k is an positive integer, such that a, b and c are in Arithmetic Progression.

   (b) One of the sequences of letters XXX, YYY, ZZZ is transmitted over a communication channel with respective probabilities p_1, p_2, p_3, where ($p_1 + p_2 + p_3 = 1$). The probability that each transmitted letter will be correctly understood is α and the probabilities that the letter will be confused with two other letters are $(1-\alpha)/2$and $(1-\alpha)/2$. It is assumed that the letters are distorted independently.

   i) Find the probability that XXX was transmitted if XYZ was received.
   ii) Find the probability that XYZ was received if XXX was transmitted.

2. (a) Let X and Y be independent standard normal variables. Obtain the moment generating function of XY.

   (b) Let X and Y be two independent random variables following $N(5, 1)$ and $N(15, 1)$ respectively. Draw rough sketch of the density of the random variable Z constructed as follows:

   i) Two observations, say x and y are drawn randomly from X & Y, and Z is constructed as $(x + y)/2$.
   ii) An unbiased coin is tossed. If a head turn up Z is constructed by drawing a random observation from X, otherwise Z is constructed by drawing a random observation from Y.

3. Determine k so that

$$f(x, y) = \begin{cases} ky\exp(-xy) & if\ 0 \le x < \infty\ \ and\ \ 1 \le y \le 4 \\ 0 & Otherwise \end{cases}$$

   is a joint probability density function of the two random variables X and Y. Compute their covariance. Derive the conditional probability density of X given $y = 4$.

4. Let Z_1, Z_2, Z_3 and Z_4 be four independent standard normal variables. Find the distribution of

$$U = \frac{Z_1 - Z_2 - Z_3 + Z_4}{Z_1 + Z_2 - Z_3 - Z_4}.$$

5. Let $[X_n \mid n \in \{0, 1, 2, \ldots\}]$ be a discrete time parameter Markov Chain with state space $I = \{i_0, i_1, i_2, \ldots, i_n\}$ and one step transition matrix $P = \left((p_{ij})\right)_{i,j \in I}$.

   Show that $\mathrm{P}(X_0 = i_0 | X_1 = i_1, X_2 = i_2, \ldots, X_n = i_n) = \mathrm{P}(X_0 = i_0 \mid X_1 = i_1)$, $\forall n = 0, 1, 2, \ldots, \forall \{i_0, i_1, i_2, \ldots, i_n\} \in I$, and assuming that the left hand side is defined.

6. A boy goes to his school either by bus or on foot. If one day he goes to the school by bus, then the probability that he goes by bus the next day is 7/10. If one day he walks to the school, then the probability that he goes by bus the next day is 2/5.

   (a). Given that he walks to the school on a particular Tuesday, find the probability that he will go to the school by bus on Thursday of that week.

   (b). Given that the boy walks to the school on both Tuesday and Thursday of that week, find the probability that he will also walk to the school on Wednesday.

   [You may assume that the boy will not be absent from the school on Wednesday or Thursday of that week.]

7. A coin is tossed an odd number of times. If the probability of getting more heads than tails in these tosses is equal to the probability of getting more tails than heads then show that the coin is unbiased.

8. Suppose in a big hotel there are N rooms with single occupancy and also suppose that there are N boarders. In a dinner party to celebrate the marriage anniversary of one of the boarders they start drinking alcohol to their hearts' content and as a consequence they become unable to identify their own rooms. What is the probability that at the end of the dinner party none of the boarders occupies the room originally assigned to them? What is the limiting value of this probability as $N \to \infty$.

9. A manufacturer sells a bottle of mineral water at a fixed price of Rs.10. If the volume of water in the bottle is less than 800 ml then he is unable to sell it and it represents a total loss. The filled bottles have a normally distributed volume with mean μ ml and standard deviation 100 ml. The cost of filling per bottle is Rs. c, where $c = 0.002\mu + 1$. Determine the mean volume μ which will maximize the expected profit of the manufacturer. [Use $\sqrt{-\ln(0.0008\pi)} = 2.447$].

10. Consider a Markov Chain with state space $I = \{1,2,3,4,5,6\}$ and transition probability matrix P given by

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{8} & \frac{7}{8} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{8} & \frac{1}{8} \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 \end{bmatrix}$$

Find the various classes of this chain and classify them as recurrent or transient.

PART II (FOR ENGINEERING STREAM)

GROUP E1
Mathematics

1. (a) Find the number of ordered pairs (x, y) such that that $x^2 + 2y^2 = 1$ and x, y are prime numbers.

   (b) Prove that

$$\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{\tan x - x} = 1$$

   (c) In ΔABC, $\angle B = \angle C$, the points D, E, F are taken on AB, BC and CA such that ΔDEF is an equilateral triangle. Suppose that the measures of the angles $\angle DEB, \angle ADF$ and $\angle CFE$ are α, β and γ respectively. Show that $BD : CF = \sin\left(\frac{\beta+\gamma}{2}\right) : \sin(120° - \alpha)$.

2. (a) Let (h, k) be a fixed point where $h, k > 0$. A straight line passing through this point cuts the positive direction of the coordinate axes at the points P and Q. Show that the minimum area of the triangle OPQ; O being the origin, is $2hk$ sq unit.

   (b) Consider n distinct real numbers: $a_1, a_2, \ldots, a_n$. A permutation $([1], [2], \ldots, [n])$ of the indices $\{1, 2, \ldots, n\}$ is said to V-shaped if there exists an integer r $(1 \leq r \leq n)$ such that $a_{[1]} \geq a_{[2]} \geq \ldots \geq a_{[r-1]} \geq a_{[r]} \leq a_{[r+1]} \leq \ldots \leq a_{[n]}$. Find the total of such V-shaped permutations.

   (c) Find minimum and maximum values of the function $f(x) = 2x^3 - 15x^2 + 36x$ over the interval [1, 5] with corresponding values of x, where these extrema occur.

3. (a) Suppose all the integers from 1 to 3333 are listed. Find the number of occurrences of the digit '0' in the list.

(b) Find the values of k ($k > 0$) for which the equation

$$|Z-i| + |Z+i| = k$$

represents an ellipse?

4.(a) Let $f(x)$ be a polynomial in x and let a, b be two real numbers where $a \neq b$. Show that if $f(x)$ is divided by $(x - a)(x - b)$ then the remainder is

$$\frac{(x-a)f(b)-(x-b)f(a)}{b-a}.$$

(b) Find $\frac{dy}{dx}$ when $y=\left(x^{\log x}\right)(\log x)^{x}$, $x > 1$

5. Evaluate the value of $3.9^{1/2}.27^{1/4}.\ 81^{1/8}.$ ...up to infinity.

6. (a) If ω is a complex cube root of unity then show that

$$a^3+b^3+c^3-3abc=(a+b+c)\left(a+b\omega+c\omega^2\right)\left(a+b\omega^2+c\omega\right).$$

(b) Show that $\left[\frac{\sum_{r>s} x^r}{r!}\right] \div \left[\frac{\sum_{r>s} y^r}{r!}\right] > \frac{x^s}{y^s}$, whenever $x > y > 0$.

7. (a) Cable of a suspension bridge hangs in the form of a parabola and is attached to the supporting pillars 200 m apart. The lowest point of the cable is 40 m below the point of suspension. Find the angle between the cable and the supporting pillars. State all the assumptions involved.

(b) Let A, B and C be the angles of a triangle with angle C as the smallest of them. Show that

(i) $\sin\left(\frac{C}{2}\right) \leq \frac{1}{2}$

(ii) Hence, or otherwise, show that $\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) < \frac{1}{4}$.

8.(a) Evaluate the following two integrals directly and compare them.

$$\iint_{ax^2+by^2\le 1} dxdy \qquad \text{and} \qquad \iint_{\sqrt{a}|x|\le 1, \sqrt{b}|y|\le 1} dxdy.$$

(b) Determine x, y and z so that the 3×3 matrix with the following row vectors is orthogonal : $(1 / \sqrt{3}, 1 / \sqrt{3}, 1 / \sqrt{3})$, $(1/ \sqrt{2}, -1/ \sqrt{2}, 0)$, (x, y, z).

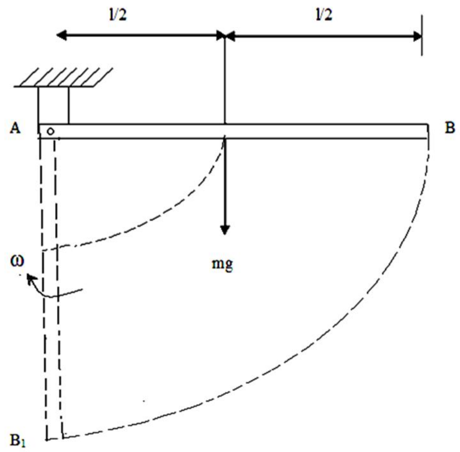
9. (a) A variable line through the point (a, b) cuts the axes of reference at A and B respectively. The lines through A and B parallel to the y-axis and x-axis respectively meet at P. Find the locus of P.

(b) Inside a square ABCD with sides of length 12 cm, segment AE is drawn where E is the point on DC such that DE = 5 cm. The perpendicular bisector of AE is drawn and it intersects AE, AD and BC at the points M, P and Q respectively. Find the ratio PM : MQ.

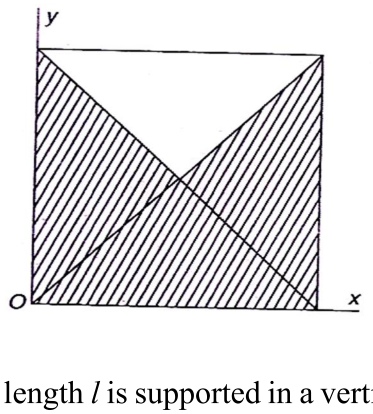
GROUP E2: Engineering and Technology

Engineering Mechanics and Thermodynamics

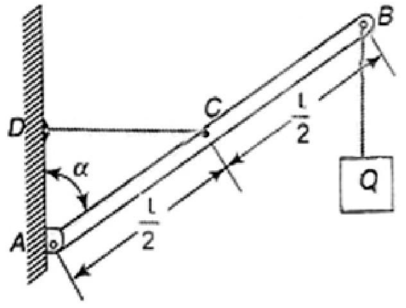
1 .(a) If the slender prismatic bar in the following figure is released from rest in the horizontal position AB and allowed to fall under the influence of gravity, show that the angular velocity it will acquire by the time it reaches the vertical position AB_1 is $\varpi = \sqrt{\frac{3g}{l}}$.



(b) Locate the centroid of the shaded three-quarters of the area of a square of dimension *a* as shown in the figure below.

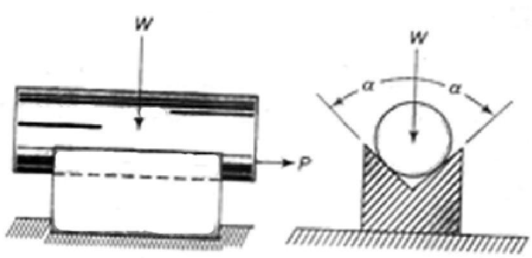


(c) A rigid bar AB of length *l* is supported in a vertical plane and carries a load *Q* at its free end as shown in the figure below. Neglecting the weight of the bar itself, compute the magnitude of the tension induced in the horizontal string CD.



2. (a) A particle of weight W moves rectilinearly under the action of a force $P\cos\omega t$. Develop the velocity-time and displacement-time equation if at the beginning the displacement and the velocity are zero.

(b) A short right circular cylinder of weight W rests in horizontal 'V' notch having the angle 2α as shown in the figure. If the coefficient of friction is μ, find the horizontal force P necessary to cause slipping to impend.



(c) A beam of rectangular cross-section is to be cut from a circular log of diameter D. Determine the ratio of the depth to the width of the beam to resist maximum bending moment.

3. (a) It is proposed that solar energy be used to warm a large collector plate. This energy would, in turn, be transferred as heat to a fluid within a heat engine, and the engine would reject energy as heat to the atmosphere. Experiments indicate that about 1880 kJ/m^{2}h of energy can be collected when the plate is operating at 90^{0}C. Estimate the minimum collector area that would be required for a plate producing 1 kW of useful shaft power. The atmospheric temperature may be assumed to be 20^{0}C.

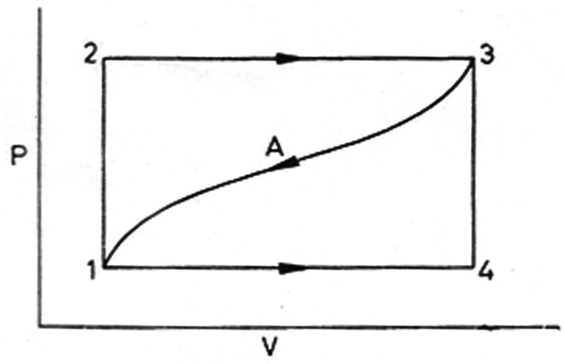
(b) A mass of m_1 kg of a certain perfect gas at a temperature T_1^0 K is mixed at constant pressure with m_2 kg of mass of the same gas at a temperature T_2^0 ($T_1>T_2$). The system is thermally insulated.

(i) Find the change in entropy of the mixture,

(ii) Show that the change in entropy of the mixture for equal masses of the gas ($m_1 = m_2 = m$) is $\Delta S_m = 2mC_P \ln\left[\frac{\frac{(T_1+T_2)}{2}}{\sqrt{T_1T_2}}\right]$,

(iii)What will be the rationale to claim that the change in entropy for equal masses of the gas is necessarily positive?

4. (a)When a system changes from state 1 to state 3 along the path 1-2-3 as shown in the figure below, 40 kCal of heat flows in the system, which in turn does 20 kCal of work.



(i) How much work will be done by the system if the heat flowing into the system along the path 1-4-3 is 25 kCal?,

(ii) 15 kCal of work is done on the system while returning from state 3 to state 1 along the curved path 3-A-1. How much heat is transferred from or to the system?,

(iii) What is the value of the internal energy at state 3 if the internal energy at state 1 is 15 kCal?,

(iv) If the value of the internal energy at state 4 is 20 kCal, how much heat will be transferred from or to the system for the paths 1-4 and 4-3? Indicate the direction of flow of heat.

(b) The specific heat of certain gas under constant pressure is related to the temperature by $C_P = 6.55 - \frac{1.5 \times 10^2}{T} + \frac{1.1 \times 10^4}{T^2} \text{ kCal/mole - K}^0$.

Where T is in K^0. The molecular weight of the gas is 30. Determine the heat transferred per kg of gas if the system is heated from 400^0K to 1000^0K

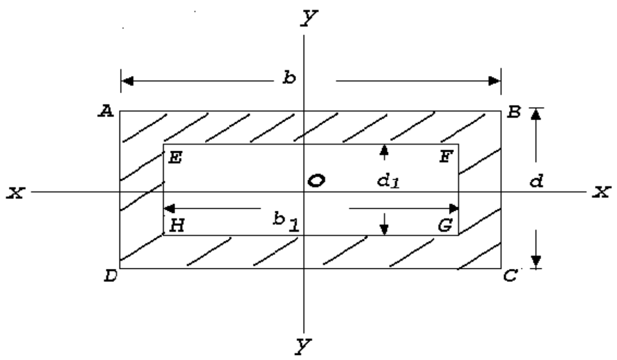
(i) Under constant pressure,

(ii) Under constant volume.

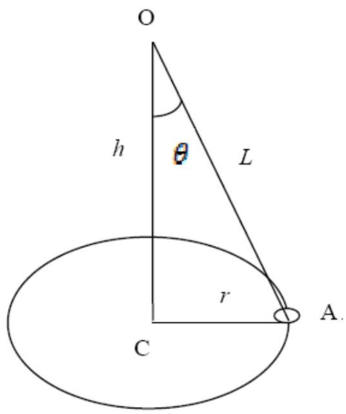
Take R= 1.98 kcal/kg-mol-K.

5. (a) A screw jack has a thread of 12 mm pitch. What effort needs to be applied the end of a handle of 450 mm to lift a load of 2.5 kN, if the corresponding efficiency is 50%?

(b) Derive the expression for moment of inertia I_{YY} of the shaded hollow rectangular section in the figure below.



(c) As illustrated in the following figure, a particle of weight W attached to a fixed point O by a



string of length L whirls in a horizontal circular path of radius r with uniform speed V so that the string generates a height h. Show that the relation between V, r, h and the tensile force T in the string is

$$T = W\sqrt{\left(1 + \frac{r}{h}\right)^2}.$$

6. (a) A uniform ladder 5 m long and 14 kg mass is placed against a vertical wall at an angle 50^0 to the horizontal ground. The co-efficient of friction between ladder and wall is 0.2 and between ladder and ground in 0.5. Calculate how far up the ladder a man of 63 kg. can climb before the ladder shifts.

(b) Determine the diameter of a steel shaft rotating at an angular velocity of 300 rpm transmitting 500 HP. The allowable stress = 800 kg/cm^2. The allowable angle of twist = 0.5^0 per m, G = 8×10^5 kg/cm^2 . What would be the savings if a hollow shaft is used to transmit the same power under the same condition, the ratio of diameters being 0.9?

7. (a) The approximated equation for adiabatic flow of superheated steam through a nozzle is given by $pv^n = constant.$ Show that

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{n/(n-1)},$$

where p_1 = pressure of steam at entry ; p_2 = pressure of steam at throat and p_2 / p_1 is the critical pressure ratio.

(b) The dry saturated steam is expanded in a nozzle from pressure of 10 bar to pressure of 4 bar. If the expansion is super saturated, find the degree of under-cooling.

Electrical & Electronics Engineering

8. (a) Design the circuit for the Boolean function $Y = AB + \overline{A}C$ using NOR gates only.

(b) Design the circuit for a synchronous counter using JK flip-flops, which will generate the outputs in the order 000, 011, 101, 100, 111, 010, 001, 110 and 000.

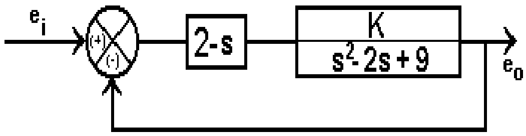
9. (a) The induced emf in a dc machine is 200 V at a speed of 1200 rpm. Calculate the electromagnetic torque developed at an armature current of 15 A.

(b) A 500 kVA, 11000 V/400 V, 50 Hz, single-phase transformer has 100 turns on the secondary winding. Calculate (i) the approximate number of turns in the primary winding, (ii) the approximate value of the primary and the secondary currents and (iii) the approximate maximum value of flux in the core.

(c) The stator of a three-phase, 8-pole synchronous generator driven at 750 rpm has 72 slots. The winding has been made with 36 coils having 10 turns per coil. Calculate approximately the rms value of the induced emf per phase if the flux per pole is 0.15 Wb, which is sinusoidally distributed. Assume that full-pitch coils have been used and Sin 10^0 = 0.174.

10. (a) On full-load unity power factor test, a meter having specification of 235 V and 5A makes 60 revolutions in 6 minutes, but its normal speed is 520 revolution/KWh. Does the meter have any inaccuracy? If so, find the percentage error.

(b) Write down the transfer function of the given system (as shown in the following figure) and find the values of K for which the system will be stable but under damped.



11. (a) A series ac circuit that resonates at 48 Hz consists of a coil (having R = 30 Ω and L = 500 mH) and a capacitor. If the supply voltage is 100 volt determine the value of the capacitor.

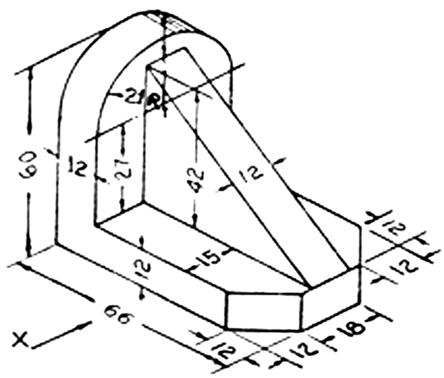
(b). Calculate the value of a capacitor which when connected across the circuit (as of Q. 4 (a) above), enhances the resonant frequency to 60 Hz. Compare the value of the source current in both the cases.

12. A centrifugal pump, which is gear-driven by a DC motor, delivers 810 kg of water per minute to a tank of height 11 meter above the level of the pump. Draw the block diagram of the overall arrangement.

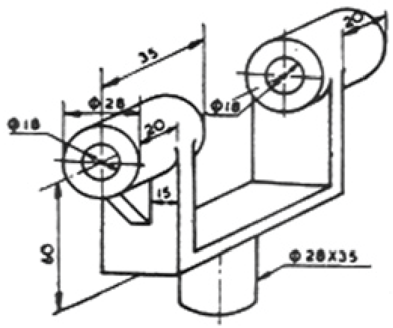
Determine input power across the gearing and current taken by the motor operated at 220 volt provided the efficiency of the pump, gearing and motor respectively be 70%, 70% and 90% only. (Take g = 9.8 ms-2).

Engineering Drawing

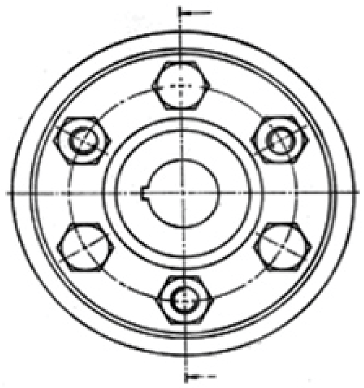
14. (a) Draw the (a) front view, (b) side view from the right, and (c) top view of the given object mentioning dimensions and angle of projection (first angle/ third angle).



(b) Sketch the two views (top view and front view) of the object shown in the figure. Put the dimensions.



15. The front view of the rigid flange shaft coupling is given below. Sketch the sectional side view of the flange shaft coupling. No dimension is to be put.



16. (a). A hollow cube of 5cm side is lying on HP and one of its vertical face is touching VP. A slim rod, to be taken as its solid diagonal, is placed within it. Draw top and front / side views of solid diagonal and, from the drawn figure determine its true length.

    (b). Two balls are vertically erected to 18 cm and 30 cm respectively above the flat ground. These balls are away from a 3 cm thick wall (on the ground) by 12 cm and 21 cm respectively but on either side of the wall. The distance between the balls, measured along the ground and parallel to the wall is 27 cm. Determine their approximate distance.

17. Sketch the profile of a square thread, knuckle thread and a white-worth thread showing all relevant dimensions in terms of the pitch.

18. Sketch
    (a) single riveted lap joint,
    (b) double riveted lap joint chain-riveting,
    (c) double riveted lap joint zigzag-riveting, and
    (d) single cover single riveted butt joint.

19. Draw the isometric view of an octahedron erected vertically up on one of its vertices. (Distinct free hand sketch only.)

20. A parallelepiped of dimension $100 \times 60 \times 80$ is truncated by a plane which passes through 85, 45 and 65 unit distance on the associated edges from the nearest top point of the object. Draw the isometric view of the truncated solid object. In third angle projection method, draw its plan. (All dimensions are in mm).