POST-GRADUATE DIPLOMA IN STATISTICAL METHODS AND ANALYTICS

TEST CODE: DST (Objective type) 2014

SYLLABUS

Algebra—Arithmetic, Geometric and Power series, sequences. Permutations and combinations. Binomial theorem. Theory of quadratic equations. Inequalities. Elementary set theory. Vectors and matrices. Determinant, rank and inverse of a matrix. Solutions of linear equations.

Coordinate geometry — Straight lines, circles, parabolas, ellipses and hyperbolas.

Calculus—Taylor and Maclaurin series. Limits and continuity of functions of one real variable. Differentiation and integration of functions of one real variable with applications. Definite integrals. Areas using integrals. Maxima and minima and their applications.

SAMPLE QUESTIONS

<u>Note:</u> *For each question there are four suggested answers of which only one is correct.*

1. Let $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, n being a positive integer. The value of

$$\left(1+\frac{C_0}{C_1}\right)\left(1+\frac{C_1}{C_2}\right)\cdots\left(1+\frac{C_{n-1}}{C_n}\right) \text{ is}$$

(A) $\left(\frac{n+1}{n+2}\right)^n$ (B) $\frac{n^n}{n!}$ (C) $\left(\frac{n}{n+1}\right)^n$ (D) $\frac{(n+1)^n}{n!}$.

2. Let $a_n = \left(1-\frac{1}{\sqrt{2}}\right)\cdots\left(1-\frac{1}{\sqrt{n+1}}\right)$, $n \geq 1$. Then $\lim_{n\to\infty} a_n$

(A) equals 1 (B) does not exist (C) equals $\frac{1}{\sqrt{\pi}}$ (D) equals 0.

3. $\lim_{x\to\infty}\left(\frac{3x-1}{3x+1}\right)^{4x}$ equals

(A) 1 (B) 0 (C) $e^{-8/3}$ (D) $e^{4/9}$.

4. $\lim_{n\to\infty} \frac{1}{n}\left(\frac{n}{n+1}+\frac{n}{n+2}+\cdots+\frac{n}{2n}\right)$ is equal to

(A) ∞ (B) 0 (C) $\log_e 2$ (D) 1.

5. Consider the sets defined by the real solutions of the inequalities

$$A=\{(x,y): x^2+y^4\le 1\} \qquad B=\{(x,y): x^4+y^6\le 1\}.$$

Then

(A) $B \subseteq A$

(B) $A \subseteq B$

(C) Each of the sets $A-B$, $B-A$ and $A\cap B$ is non-empty

(D) none of the above.

6. If $f(x)$ is a real valued function such that

$$2f(x)+3f(-x)=15-4x,$$

for every $x\in\mathbb{R}$, then $f(2)$ is

(A) -15 (B) 22 (C) 11 (D) 0.

7. If $f(x)=\frac{\sqrt{3}\sin x}{2+\cos x}$, then the range of $f(x)$ is

(A) the interval $[-1,\sqrt{3}/2]$ (B) the interval $[-\sqrt{3}/2,1]$

(C) the interval $[-1,1]$ (D) none of these.

8. If M is a 3×3 matrix such that $[0\ \ 1\ \ 2]M=[1\ \ 0\ \ 0]$ and $[3\ \ 4\ \ 5]M=[0\ \ 1\ \ 0]$ then $[6\ \ 7\ \ 8]M$ is equal to

(A) $[2\ \ 1\ \ -2]$ (B) $[0\ \ 0\ \ 1]$ (C) $[-1\ \ 2\ \ 0]$ (D) $[9\ \ 10\ \ 8]$.

9. The values of η for which the following system of equations

$$\begin{array}{rcrcrcl} x &+& y &+& z &=& 1\\ x &+& 2y &+& 4z &=& \eta\\ x &+& 4y &+& 10z &=& \eta^2 \end{array}$$

has a solution are

(A) $\eta=1,-2$ (B) $\eta=-1,-2$ (C) $\eta=3,-3$ (D) $\eta=1,2$.

10. The number of divisors of 6000, where 1 and 6000 are also considered as divisors of 6000 is

(A) 40 (B) 50 (C) 60 (D) 30.

11. Let x_1 and x_2 be the roots of the quadratic equation $x^2 - 3x + a = 0$, and x_3 and x_4 be the roots of the quadratic equation $x^2 - 12x + b = 0$. If x_1, x_2, x_3 and x_4 $(0 < x_1 < x_2 < x_3 < x_4)$ are in G.P., then ab equals

(A) 64 (B) 5184 (C) -64 (D) -5184.

12. The integral

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{50} x}{\sin^{50} x + \cos^{50} x}\, dx$$

equals

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) none of these.

13. Let the function $f(x)$ be defined as $f(x) = |x-1| + |x-2|$. Then which of the following statements is true?

(A) $f(x)$ is differentiable at $x = 1$

(B) $f(x)$ is differentiable at $x = 2$

(C) $f(x)$ is differentiable at $x = 1$ but not at $x = 2$

(D) none of the above.

14. $x^4 - 3x^2 + 2x^2y^2 - 3y^2 + y^4 + 2 = 0$ represents

(A) A pair of circles having the same radius

(B) A circle and an ellipse

(C) A pair of circles having different radii

(D) none of the above.

15. Let $\mathbb{N} = \{1, 2, 3, \ldots\}$ be the set of natural numbers. For each $n \in \mathbb{N}$, define $A_n = \{(n+1)k, k \in \mathbb{N}\}$. Then $A_1 \cap A_2$ equals

(A) A_3 (B) A_4 (C) A_5 (D) A_6.

16. The sum of the series $\frac{1}{1.2} + \frac{1}{2.3} + \ldots + \frac{1}{n(n+1)} + \ldots$ is

(A) 1 (B) 1/2 (C) 0 (D) non-existent.

17. $\lim_{x\to 2} \dfrac{1}{1+e^{\frac{1}{x-2}}}$ is

(A) 0 (B) 1/2 (C) 1 (D) non-existent.

18. ${}^nC_0 + 2\,{}^nC_1 + 3\,{}^nC_2 + \ldots + (n+1)\,{}^nC_n$ equals

(A) $2^n + n2^{n-1}$ (B) $2^n - n2^{n-1}$ (C) 2^n (D) none of these.

19. It is given that $e^a + e^b = 10$ where a and b are real. Then the maximum value of $(e^a + e^b + e^{a+b} + 1)$ is

(A) 36 (B) ∞ (C) 25 (D) 21.

20. If $A(t)$ is the area of the region bounded by the curve $y = e^{-|x|}$ and the portion of the x-axis between $-t$ and t, then $\lim_{t\to\infty} A(t)$ equals

(A) 0 (B) 1 (C) 2 (D) 4.

21. Suppose that the function $h(x)$ is defined as $h(x) = g(f(x))$ where $g(x)$ is monotone increasing, $f(x)$ is concave, and $g''(x)$ and $f''(x)$ exist for all x. Then $h(x)$ is

(A) always concave (B) always convex

(C) not necessarily concave (D) None of these.

22. The conditions on a, b and c under which the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, $b \neq 0$ and $c \neq 0$, are of unequal magnitude but of opposite signs, are the following:

(A) a and c have the same sign while b has the opposite sign;

(B) b and c have the same sign while a has the opposite sign; or a and b have the same sign while c has the opposite sign;

(C) a and c have the same sign;

(D) a, b and c have the same sign.

23. The sum of the series $3 + 11 + \ldots + (8n - 5)$ is

(A) $4n^2 - n$ (B) $8n^2 + 3n$ (C) $4n^2 + 4n - 5$ (D) $4n^2 + 2$.

24. Let $f(x) = \frac{2x}{x-1}$, $x \neq 1$. State which of the following statements is true.

(A) For all real y, there exists x such that $f(x) = y$;

(B) For all real $y \neq 1$, there exists x such that $f(x) = y$;

(C) For all real $y \neq 2$, there exists x such that $f(x) = y$;

(D) None of the above is true.

25. The determinant $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$ equals

(A) $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

(B) $2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

(C) $3\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

(D) None of these.

26. Let $x_1 > x_2 > 0$. Then which of the following is true?

(A) $\log(\frac{x_1+x_2}{2}) > \frac{\log x_1 + \log x_2}{2}$;

(B) $\log(\frac{x_1+x_2}{2}) < \frac{\log x_1 + \log x_2}{2}$;

(C) There exist x_1 and x_2 such that $x_1 > x_2 > 0$ and $\log(\frac{x_1+x_2}{2}) = \frac{\log x_1 + \log x_2}{2}$;

(D) None of these.

27. Let $y^2 - 4ax + 4a = 0$ and $x^2 + y^2 - 2(1+a)x + 1 + 2a - 3a^2 = 0$ be two curves. State which one of the following statements is true.

(A) These two curves intersect at two points;

(B) These two curves are tangent to each other;

(C) These two curves intersect orthogonally at one point;

(D) These two curves do not intersect.

28. The area enclosed by the curve $|x| + |y| = 1$ is

(A) 1 (B) 2 (C) $\sqrt{2}$ (D) 4.

29. If $f(x) = \sin\left(\frac{1}{x^2+1}\right)$, then

(A) $f(x)$ is continuous at $x = 0$, but not differentiable at $x = 0$;

(B) $f(x)$ is differentiable at $x = 0$, and $f'(0) \neq 0$;

(C) $f(x)$ is differentiable at $x = 0$, and $f'(0) = 0$;

(D) None of the above.

30. Consider the equation $P(x) = x^3 + px^2 + qx + r = 0$ where p, q and r are all real and positive. State which of the following statements is always correct.

(A) All roots of $P(x) = 0$ are real;

(B) The equation $P(x) = 0$ has at least one real root;

(C) The equation $P(x) = 0$ has no negative real root;

(D) The equation $P(x) = 0$ must have one positive and one negative real root.

31. For real α, the value of $\int_{\alpha}^{\alpha+1} [x]dx$, where $[x]$ denotes the largest integer less than or equal to x, is

(A) α (B) $[\alpha]$ (C) 1 (D) $\frac{[\alpha] + [\alpha+1]}{2}$.

32. Consider 30 multiple-choice questions, each with four options of which exactly one is correct. Then the number of ways one can get only the alternate questions correctly answered is

(A) 3^{15}, (B) 2^{31}, (C) $2 \times \binom{30}{15}$, (D) 2×3^{15}.

33. Let $f(x)$ be a continuous function from $[0, 1]$ to $[0, 1]$ satisfying the following properties.

(a) $f(0) = 0$,

(b) $f(1) = 1$, and

(c) $f(x_1) < f(x_2)$ for $x_1 < x_2$ with $0 < x_1, x_2 < 1$.

Then the number of such functions is

(A) 0 (B) 1 (C) 2 (D) ∞.

34. The following sum of $n+1$ terms

$$2 + 3 \times \binom{n}{1} + 5 \times \binom{n}{2} + 9 \times \binom{n}{3} + 17 \times \binom{n}{4} + \ldots$$

up to $n+1$ terms is equal to

(A) $3^{n+1} + 2^{n+1}$ (B) $3^n \times 2^n$ (C) $3^n + 2^n$ (D) 2×3^n.

35. Let A and B be disjoint sets containing m and n elements respectively, and let $C = A \cup B$. Then the number of subsets S (of C) which contains p elements and also has the property that $S \cap A$ contains q elements, is

(A) $\binom{m}{q}$ (B) $\binom{n}{q}$ (C) $\binom{m}{q} \times \binom{n}{p-q}$ (D) $\binom{m}{p-q} \times \binom{n}{q}$.

36. Consider any integer $I = m^2 + n^2$, where m and n are odd integers. Then

(A) I is never divisible by 2;

(B) I is never divisible by 4;

(C) I is never divisible by 6;

(D) None of the above.

37. Let $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ be a continuous function, $f(x) \to +\infty$ as $x \to \frac{\pi}{2}^-$ and $f(x) \to -\infty$ as $x \to -\frac{\pi}{2}^+$. Which one of the following functions satisfies the above properties of $f(x)$?

(A) $\cos x$ (B) $\tan x$ (C) $\tan^{-1} x$ (D) $\sin x$.

38. Suppose that A is a 3×3 real matrix such that for each $u = (u_1, u_2, u_3)' \in \mathbb{R}^3$, $u'Au = 0$ where u' stands for the transpose of u. Then which one of the following is true?

(A) $A' = -A$ (B) $A' = A$ (C) $AA' = I$ (D) None of these.

39. The function $f(x) = x^{1/x}$, $x \neq 0$ has

(A) a minimum at $x = e$;

(B) a maximum at $x = e$;

(C) neither a maximum nor a minimum at $x = e$;

(D) None of the above.

40. Let the following two equations represent two curves A and B.

$$A : 16x^2 + 9y^2 = 144 \qquad \text{and} \qquad B : x^2 + y^2 - 10x = -21$$

Further, let L and M be the tangents to these curves A and B, respectively, at the point $(3, 0)$. Then the angle between these two tangents, L and M, is

(A) 0° (B) 30° (C) 45° (D) 90°.

41. The number of permutations of the letters a, b, c and d such that b does not follow a, c does not follow b, and c does not follow d, is

(A) 11 (B) 12 (C) 13 (D) 14.

42. Let $f(x) = \sin x^2$, $x \in \mathbb{R}$. Then

(A) f has no local minima;

(B) f has no local maxima;

(C) f has local minima at $x = 0$ and $x = \pm\sqrt{(k + \frac{1}{2})\pi}$ for odd integers k and local maxima at $x = \pm\sqrt{(k + \frac{1}{2})\pi}$ for even integers k;

(D) None of the above.

43. Let

$$f(x) = \begin{cases} |x| + 1, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ |x| - 1, & \text{if } x > 0. \end{cases}$$

Then $\lim_{x \to a} f(x)$ exists

(A) if $a = 0$ (B) for all $a \in R$ (C) for all $a \neq 0$ (D) only if $a = 1$.

44. The function $f(x) = \sin x(1 + \cos x)$ which is defined for all real values of x

(A) has a maximum at $x = \pi/3$

(B) has a maximum at $x = \pi$

(C) has a minimum at $x = \pi/3$

(D) has neither a maximum nor a minimum at $x = \pi/3$.

45. Which of the following is true?

(A) $\log(1+x) < x - \frac{x^2}{2} + \frac{x^3}{3}$ for all $x > 0$
(B) $\log(1+x) > x - \frac{x^2}{2} + \frac{x^3}{3}$ for all $x > 0$
(C) $\log(1+x) > x - \frac{x^2}{2} + \frac{x^3}{3}$ for some $x > 0$
(D) $\log(1+x) < x - \frac{x^2}{2} + \frac{x^3}{3}$ for some $x > 0$.

46. The maximum value of the real valued function $f(x) = \cos x + \sin x$ is

(A) 2 (B) 1 (C) 0 (D) $\sqrt{2}$.

47. The value of the definite integral $\int_0^{\pi} \left| \frac{1}{2} + \cos x \right| dx$ is

(A) $\frac{\pi}{6} + \sqrt{3}$ (B) $\frac{\pi}{6} - \sqrt{3}$ (C) 0 (D) $\frac{1}{2}$.

48. If x is real, the set of real values of a for which the function

$$y = x^2 - ax + 1 - 2a^2$$

is always greater than zero is

(A) $-\frac{2}{3} < a \leq \frac{2}{3}$ (B) $-\frac{2}{3} \leq a < \frac{2}{3}$
(C) $-\frac{2}{3} < a < \frac{2}{3}$ (D) None of these.

49. Let $f(x) = \frac{x}{(x-1)(2x+3)}$, where $x > 1$. Then the 4^{th} derivative of f, $f^{(4)}(x)$ is equal to

(A) $-\frac{24}{5}\left[\frac{1}{(x-1)^5} - \frac{48}{(2x+3)^5}\right]$ (B) $\frac{24}{5}\left[-\frac{1}{(x-1)^5} + \frac{48}{(2x-3)^5}\right]$
(C) $\frac{24}{5}\left[\frac{1}{(x-1)^5} + \frac{48}{(2x+3)^5}\right]$ (D) $\frac{64}{5}\left[\frac{1}{(x-1)^5} + \frac{48}{(2x+3)^5}\right]$.

50. $\lim_{x\to 0} \frac{x \tan x}{1 - \cos tx}$ is equal to

(A) 0 (B) 1 (C) ∞ (D) 2.

51. The function $f(x)$ defined as $f(x) = x^3 - 6x^2 + 24x$, where x is real, is

(A) strictly increasing

(B) strictly decreasing

(C) increasing in $(-\infty, 0)$ and decreasing in $(0, \infty)$

(D) decreasing in $(-\infty, 0)$ and increasing in $(0, \infty)$.

52. The area under the curve x^2+3x-4 in the positive quadrant and bounded by the line $x = 5$ is equal to

(A) $59\frac{1}{6}$ (B) $61\frac{1}{3}$ (C) $40\frac{2}{3}$ (D) 72.

53. The value of the integral $\int_{-1}^{1} \frac{x^2}{1+x^2} \sin x \sin 3x \sin 5x dx$ is

(A) 0 (B) $\frac{1}{2}$ (C) -$\frac{1}{2}$ (D) 1.

54. The number of real roots of the equation $1 + \cos^2 x + \cos^3 x - \cos^4 x = 5$ is equal to

(A) 0 (B) 1 (C) 3 (D) 4.

55. If a, b, c are sides of a triangle ABC such that $x^2 - 2(a+b+c)x + 3\lambda(ab + bc + ca) = 0$ has real roots then

(A) $\lambda < \frac{4}{3}$ (B) $\lambda > \frac{5}{3}$ (C) $\lambda \in (\frac{4}{3}, \frac{5}{3})$ (D) $\lambda \in (\frac{1}{3}, \frac{5}{3})$.

56. Two opposite vertices of a rectangle are $(1, 3)$ and $(5, 1)$ while the other two vertices lie on the the straight line $y = 2x + c$. Then the value of c is

(A) 4 (B) 3 (C) -4 (D) -3.

57. If a focal chord of the parabola $y^2 = 4ax$ cuts it at two distinct points (x_1, y_1) and (x_2, y_2), then

(A) $x_1x_2 = a^2$ (B) $y_1y_2 = a^2$ (C) $x_1x_2^2 = a^2$ (D) $x_1^2x_2 = a^2$.

58. Consider a circle with centre at origin and radius $2\sqrt{2}$. A square is inscribed in the circle whose sides are parallel to the X and Y axes. The coordinates of one of the vertices of this square are

(A) $(2, -2)$ (B) $(2\sqrt{2}, -2)$ (C) $(-2, 2\sqrt{2})$ (D) $(2\sqrt{2}, -2\sqrt{2})$.

59. The equation $5x^2 + 9y^2 + 10x - 36y - 4 = 0$ represents

(A) an ellipse with the coordinates of foci being $(\pm 3, 0)$

(B) a hyperbola with the coordinates of foci being $(\pm 3, 0)$

(C) an ellipse with the coordinates of foci being $(\pm 2, 0)$

(D) an hyperbola with the coordinates of foci being $(\pm 2, 0)$.

60. The equation of any circle passing through the origin and with its centre on the X-axis is given by

(A) $x^2 + y^2 - 2ax = 0$ where a must be positive

(B) $x^2 + y^2 - 2ax = 0$ for any given $a \in \mathbb{R}$

(C) $x^2 + y^2 - 2by = 0$ where b must be positive

(D) $x^2 + y^2 - 2by = 0$ for any given $b \in \mathbb{R}$.

61. If $l = 1 + a + a^2 + \ldots$, $m = 1 + b + b^2 + \ldots$, and $n = 1 + c + c^2 + \ldots$, where $|a| < 1$, $|b| < 1$, $|c| < 1$ and a, b, c are in arithmetic progression, then l, m, n are in

(A) arithmetic progression (B) geometric progression

(C) harmonic progression (D) none of these.

62. If the sum of first n terms of an arithmetic progression is cn^2, then the sum of squares of these n terms is

(A) $\dfrac{n(4n^2-1)c^2}{6}$ (B) $\dfrac{n(4n^2+1)c^2}{3}$

(C) $\dfrac{n(4n^2-1)c^2}{3}$ (D) $\dfrac{n(4n^2+1)c^2}{6}$.

63. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then r is equal to

(A) 1 (B) 2 (C) 3 (D) none of these.

64. The value of λ such that the system of equation

$$\begin{array}{rcrcrcr} 2x & - & y & + & 2z & = & 2 \\ x & - & 2y & + & z & = & -4 \\ x & + & y & + & \lambda z & = & 4 \end{array}$$

has no solution is

(A) 3 (B) 1 (C) 0 (D) -3.

65. The sum $\dfrac{n}{n^2} + \dfrac{n}{n^2+1^2} + \dfrac{n}{n^2+2^2} + \ldots + \dfrac{n}{n^2+(n-1)^2} + \ldots\ldots$ is

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{8}$ (C) $\dfrac{\pi}{6}$ (D) 2π.

66. Consider all possible words obtained by arranging all the letters of the word **AGAIN**. These words are now arranged in the alphabetical order, as in a dictionary. The fiftieth word in this arrangement is

(A) IAANG (B) NAAGI (C) NAAIG (D) IAAGN.

67. Let $y = [\log_{10} 3245.7]$ where $[a]$ denotes the greatest integer less than or equal to a. Then

(A) $y = 0$ (B) $y = 1$ (C) $y = 2$ (D) $y = 3$.

68. The number of integer solutions for the equation $x^2 + y^2 = 2011$ is

(A) 0 (B) 1 (C) 2 (D) 3.

69. The number of ways in which the number 1440 can be expressed as a product of two factors is equal to

(A) 18 (B) 720 (C) 360 (D) 36.

70. For the matrices $A = \begin{pmatrix} a & a \\ 0 & a \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $(B^{-1}AB)^3$ is equal to

(A) $\begin{pmatrix} a^3 & a^3 \\ 0 & a^3 \end{pmatrix}$ (B) $\begin{pmatrix} a^3 & 3a^3 \\ 0 & a^3 \end{pmatrix}$

(C) $\begin{pmatrix} a^3 & 0 \\ 3a^3 & a^3 \end{pmatrix}$ (D) $\begin{pmatrix} a^3 & 0 \\ -3a^3 & a^3 \end{pmatrix}$

71. Five letters A, B, C, D and E are arranged so that A and C are always adjacent to each other and B and E are never adjacent to each other. The total number of such arrangements is

(A) 24 (B) 16 (C) 12 (D) 32.

72. The sum $\sum_{k=1}^{n}(-1)^k \, {}^nC_k \sum_{j=0}^{k}(-1)^j \, {}^kC_j$ is equal to

(A) -1 (B) 0 (C) 1 (D) 2^n.