SYLLABUS FOR MSQE
(Program Code: MQEK and MQED) 2016

Syllabus for PEA (Mathematics), 2016

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations; (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency, dispersion, correlation and regression, probability distributions, standard distributions-Binomial and Normal.

Syllabus for PEB (Economics), 2016

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

PEB (2016)

Answer any 6 questions. All questions carry equal marks.

1. Consider an exchange economy consisting of two individuals 1 and 2, and two goods, X and Y. The utility function of individual 1 is $U_1 = X_1 + Y_1$, and that of individual 2 is $min\{X_2, Y_2\}$, where X_i (resp. Y_i) is the amount of X (resp. Y) consumed by individual i, where $i = 1, 2$. Individual 1 has 4 units of X and 8 units of Y, and individual 2 has 6 units of X and 4 units of Y to begin with.

(i) What is the set of Pareto optimal outcomes in this economy? Justify your answer.

(ii) What is the competitive equilibrium in this economy? Justify your answer.

(iii) Are the perfectly competitive equilibria Pareto optimal?

(iv) Now consider another economy where everything is as before, apart from individual 2's preferences, which are as follows: (a) among any two any bundles consisting of X and Y, individual 2 prefers the bundle which has a larger amount of commodity X irrespective of the amount of commodity Y in the two bundles, and (b) between any two bundles with the same amount of X, she prefers the one with a larger amount of Y. Find the set of Pareto optimal outcomes in this economy. [6]+[6]+[2]+[6]

2. Consider a monopolist who can sell in the domestic market, as well as in the export market. In the domestic market she faces a demand $p_d = 10 - q_d$, where p_d and q_d are domestic price and demand respectively. In the export market she can sell unlimited quantities at a price of 4. Suppose the monopolist has a single plant with cost function $\frac{q^2}{4}$.

(a) Solve for total output, domestic sale and exports of the monopolist.

(b) Solve for the domestic and world welfare at this equilibrium. [10]+[10]

3. A consumer consumes electricity, denoted by E, and butter, denoted by B. The per unit price of B is 1. To consume electricity the consumer has to pay a fixed charge R, and a per unit price of p. If consumption of $E \leq \frac{1}{2}$ then $p = 1$; otherwise $p = 2$. The utility function of the consumer is $3E + B$, and her income is $I > R$.

(i) Draw the consumer's budget line.

(ii) If $R = 0$ and $I = 1$, find the consumer's optimal consumption of E and B.

(iii) Consider a different pricing scheme where there is a rental charge of R and the price of E is 1 for any $X \leq 1/2$, and every *additional* unit beyond $\frac{1}{2}$ is priced at $p = 2$. Find the optimal consumption of B and E when $R = 1$ and $I = 3$. [7]+[7]+[6]

4. A monopoly publishing house publishes a magazine, earning revenue from selling the magazine, as well as by publishing advertisements. Thus $R = q.p(q) + A(q)$, where R is total revenue, q denotes quantity, $p(q)$ is the inverse demand function, and $A(q)$ is the advertising revenue. Assume that $p(q)$ is decreasing and $A(q)$ is increasing in q. The cost of production $c(q)$ is also increasing in the quantity sold. Assume all functions are twice differentiable in q.

(i) Derive the profit-maximising outcome.

(ii) Is the marginal revenue curve necessarily negatively sloped?

(iii) Can the monopolist fix the price of the magazine below the marginal cost of production?[7]+[7]+[6]

5. Consider a Solow style growth model where the production function is given by

$$Y_t = A_t F(K_t, H_t)$$

where Y_t = output of the final good, K_t is the capital stock, A_t = the level of technology, and H_t = the quantity of labor used in production (the labor force). Assume technology is equal to $A_t = A_0(1+\alpha)^t$ where $\alpha > 0$ is the growth rate of technology, A_0 is the time 0 level of technology, and $H_{t+1} = (1+n)H_t$, where $n > 0$ is the labour force growth rate. The production function is homogenous of degree 1 and satisfies the usual properties. (Assume that inputs are essential and Inada conditions hold). Assume that capital evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t$$

where I_t is the level of investment.

(i) Define $y_t = \frac{Y_t}{H_t}$.Show that

$$y_t = A_t f(k_t)$$

where $f(k) = F(k, 1)$.

(ii) Define $k_t = \frac{K_t}{H_t}$ and $i_t = \frac{I_t}{H_t}$. Show that

$$k_{t+1} = \frac{(1-\delta)k_t + i_t}{1+n}$$

(iii) Suppose the savings rate is given by $s_t = \sigma y_t$ where $\sigma \in [0, 1]$. Derive the condition that determines the steady state capital stock when $\alpha = 0$. How many non-zero steady states are there ?

(iv) Let $\gamma_t = \frac{k_{t+1}}{k_t}$ be the gross growth rate. Suppose $\alpha = 0$. Derive an expression for γ_t and evaluate and discuss the sign for $\frac{d\gamma_t}{dk_t}$.

(v) Let $f(k_t) = k_t^{\theta}$, $A_0 = 1$, and $\alpha > 0$. Along a balanced growth path show that $\frac{k_{t+1}}{k_t}$ and $\frac{y_{t+1}}{y_t}$ grow at the same rate. [2]+[3]+[5]+[5]+[5]

6. Consider the aggregate supply curve for an economy given by

$$P_t = P_t^e(1+\mu)F(u_t, z)$$

where P_t = actual price level at time period t, P_t^e = expected prices at time t, and the function, F, given by,

$$F(u_t, z) = 1 - \alpha u_t + z$$

captures the effects of the unemployment rate (u_t) at time t and the level of unemploment benefits (z) on the price level (through their effects on wages). Assume $\mu > 0$ denotes the monopoly markup. Assume μ and z are constant.

(i) Show that the aggregate supply curve can be transformed to be written in terms of π_t (the inflation rate) and the expected inflation rate, π_t^e, i.e. $\pi_t = \pi_t^e + (\mu + z) - \alpha u_t$, where $\pi_t = \frac{P_{t-1} - P_t}{P_t}$ and $\pi_t^e = \frac{P_{t+1}^e - P_t}{P_t}$. What is this equation called ? Briefly interpret it.

(ii) Now assume that $\pi_t^e = \theta \pi_{t-1}$ where $\theta > 0$. What is this equation called ? Re-write the equation in the above bullet and interpret when $\theta = 1$ and $\theta \neq 1$.

(iii) Let $\pi_t^e = \pi_{t-1}$ Derive the natura rate of unemployment, and express the change in the inflation rate in terms of the natural rate. Briefly interpret this equation.

(iv) How would you think about wage indexation in this model ? Does wage indexation increase the effect of unemployment on inflation? Assume $\pi_t^e = \pi_{t-1}$. [8]+[3]+[6]+[6]

7. Consider an inter-temporal choice problem in which a consumer maximises utility,

$$U(c_1, c_2) = u(c_1) + \frac{u(c_2)}{1+\delta},$$

where c_i is the consumption in period i, $i = 1, 2$, and δ is the discount factor (measure of the consumer's impatience), subject to

$$c_1 + \frac{c_2}{1+\delta} = Y_1 + \frac{Y_2}{1+r} \equiv W,$$

where Y_i is the consumer's income in period $i = 1, 2$, and r is the rate of interest. Assume $c_i > 1\ \forall i$.

(i) Let $u(c_i) = log(c_i)$. Find a condition such that there is consumption smoothing.

(ii) Plot the two cases where (a) the consumer biases its consumption towards the future, and (b) where the consumer biases it consumption towards the present. Put c_2 on the vertical axis and c_1 on the horizontal axis.

(iii) Suppose there is consumption smoothing. Solve for $c_1^* = c_1(r, Y_1, Y_2)$. Interpret this equation.

(iv) Define Y_P, the permanent income, as that constant stream of income (Y_P, Y_P) which gives the same lifetime income as does the fluctuating income stream (Y_1, Y_2). What does this imply about the optimal choice of c_1, c_2, and Y_P? Interpret your result graphically. [5]+[6]+[4]+[5]

8. Consider a cake of size 1 which can be divided between two individuals, A and B. Let α (resp. β) be the amount allocated to A (resp. B), where $\alpha + \beta = 1$ and $0 \leq \alpha, \beta \leq 1$. Agents A's utility function is $u_A(\alpha) = \alpha$ and that of agent B is $u_B(\beta) = \beta$.

(i) What is the set of Pareto optimal allocations in this economy?

(ii) Suppose A is asked to cut the cake in two parts, after which B can choose which of the two segments to pick for herself, leaving the other segment for agent A. How should A cut the cake?

(iii) Suppose A is altruistic, and his utility function puts weight on what B obtains, i.e. $u_A(\alpha, \beta) = \alpha + \mu\beta$, where μ is the weight on agent B's utility. (a) If $0 < \mu < 1$, does the answer to either 8(i) or 8(ii) change? (b) What if $\mu > 1$? [5]+[5]+[10]

9. A firm uses four inputs to produce an output with a production function

$$f(x_1, x_2, x_3, x_4) = \min\{x_1, x_2\} + \min\{x_3, x_4\}.$$

(i) Suppose that 1 unit of output is to be produced and factor prices are $1, 2, 3$ and 4 for x_1, x_2, x_3 and x_4 respectively. Solve for the optimal factor demands.

(ii) Derive the cost function.

(iii) What kind of returns to scale does this technology exhibit? [6]+[8]+[6]

10. Consider an IS-LM model where the sectoral demand functions are given by

$$C = 90 + 0.75Y,$$
$$G = 30, I = 300 - 50r,$$
$$(\frac{M}{P})_d = 0.25Y - 62.5r, (\frac{M}{P})_s = 500.$$

Any disequilibrium in the international money market is corrected instantaneously through a change in r. However, any disequilibrium in the goods market, which is corrected through a change in Y, takes much longer to be eliminated.

(a) Now consider an initial situation where $Y = 2500, r = 1/5$. What is the change in the level of I that must occur before there is any change in the level of Y?

(b) Draw a graph to explain your answer.

(c) Calculate the value of (r, Y) that puts both the money and goods market in equilibrium. What is the value of investment at this point compared to $(r = 2, Y = 2500)$?

[10]+[5]+[5]