Wave and Traversal Algorithms

Wave Algorithms

- A *wave algorithm* is a distributed algorithm that satisfies the following three requirements:
  - Termination: *Each computation is finite*
  - Decision: *Each computation contains at least one decide event*
  - Dependence: *In each computation each decide event is causally preceded by an event in each process*

The Echo Algorithm – a wave algorithm

var rec_p : integer init 0; // Counts no of recvd mesgs
$father_p$: process init *udef*;

For the initiator

begin forall $q \in Neigh_p$ do send ⟨ tok ⟩ to q ;
while $rec_p < \#Neigh_p$ do
begin receive ⟨ tok ⟩ ; $rec_p = rec_p + 1$ end ;
decide
end

For non-initiators

begin receive ⟨ tok ⟩ from neighbor q ; $father_p = q$; $rec_p = rec_p + 1$;
forall $q \in Neigh_p$, $q \neq father_p$ do send ⟨ tok ⟩ to q ;
while $rec_p < \#Neigh_p$ do
begin receive ⟨ tok ⟩ ; $rec_p = rec_p + 1$ end ;
send ⟨ tok ⟩ to $father_p$
end

Traversal Algorithms

- A *traversal algorithm* is an algorithm with the following three properties:
  - *In each computation there is one initiator, which starts the algorithm by sending out exactly one message*
  - *A process, upon receipt of a message, either sends out one message or decides*
  - *The algorithm terminates in the initiator and when this happens, each process has sent a message at least once*

Sequential Polling – a traversal algorithm

var rec_p : integer init 0; // For initiator only

For the initiator

begin while $rec_p < \#Neigh_p$ do

begin send ⟨ tok ⟩ to q_{recp+1} ;

receive ⟨ tok ⟩ ; $rec_p = rec_p + 1$

end ;

decide

end

For non-initiators

begin receive ⟨ tok ⟩ from q ; send ⟨ tok ⟩ to q ; end

Classical Depth-first Search

var $used_p[q]$: boolean init false for each $q \in Neigh_p$;
$father_p$: process init $udef$;

// For the initiator only – execute once
begin $father_p = p$; choose $q \in Neigh_p$;
$used_p[q]$ = true ; send $\langle$ tok $\rangle$ to q ;
end

Classical Depth-first Search contd..

// For each process, upon receipt of $\langle$ tok $\rangle$ from q_0:

begin if $father_p = udef$ then $father_p = q_0$;

if $\forall q \in Neigh_p$: $used_p[q]$

then *decide*

else if $\exists q \in Neigh_p$: $(q \neq father_p \wedge \neg used_p[q])$

then begin if $father_p \neq q_0 \wedge \neg used_p[q_0]$

then $q = q_0$

else choose $q \in Neigh_p \setminus \{ father_p \}$ with $\neg used_p[q]$;

$used_p[q] = true$; send $\langle$ tok $\rangle$ to q

end

else begin $used_p[\, father_p \,]$ = true ;

send $\langle$ tok $\rangle$ to $father_p$

end

end

Classical Depth-first Search Algorithm

- *The classical depth-first search algorithm computes a depth-first search spanning tree using* 2|E| *messages and* 2|E| *time units*

Awerbuch's DFS Algorithm

- Prevents the transmission of the token through a frond edge
- When process *p* is first visited by the token
  - *p* informs each neighbor *r,* except its father, of the visit by sending a ⟨vis⟩ message to *r*
  - The forwarding of the token is suspended until *p* has received an ⟨ack⟩ message from each neighbor
- When later, the token arrives at *r*, *r* will not forward the token to *p,* unless *p* is *r*'s father
- Awerbuch's algorithm computes a depth-first search tree in $4N - 2$ time units and uses $4.|E|$ messages

Cidon's DFS Algorithm

- The token is forwarded immediately
- The following situation is important:
  - Process p has been visited by the token and has sent a $\langle$vis$\rangle$ message to its neighbor r
  - The token reaches r before the $\langle$vis$\rangle$ message from p
  - In this case r may forward the token to p along a frond edge
- The situation is handled as follows:
  - Process p records to which neighbor it most recently sent the token – normally it expects to get it back from the same
  - If it gets it back from some other neighbor it *ignores the token*, but marks the edge rp as used, as if it received a $\langle$vis$\rangle$ message from p
  - When r eventually receives the $\langle$vis$\rangle$ message from p it behaves as if it never had sent the token to p
- Cidon's algorithm computes a DFS tree in $2N - 2$ time units and uses $4.|E|$ messages