1. Let S be the set of students who have taken a course in Spanish, F the set of students who have taken a course in French, and R the set of students who have taken a course in Russian. Then $|S| = 1232$, $|F| = 879$, $|R| = 114$, $|S \cap F| = 103$, $|S \cap R| = 23$, $|F \cap R| = 14$, and $|S \cup F \cup R| = 2092$. When we insert these quantities into the equation

$$|S \cup F \cup R| = |S| + |F| + |R| - |S \cap F| - |S \cap R| - |F \cap R| + |S \cap F \cap R|$$

we obtain

$$2092 = 1232 + 879 + 114 - 103 - 23 - 14 + |S \cap F \cap R|.$$

We now solve for $|S \cap F \cap R|$. We find that $|S \cap F \cap R| = 7$. Therefore, there are seven students who have taken courses in Spanish, French, and Russian.

2. To find the number of freshmen who are not taking a course in either mathematics or computer science, subtract the number that are taking a course in either of these subjects from the total number of freshmen. Let A be the set of all freshmen taking a course in com- puter science, and let B be the set of all freshmen taking a course in mathematics. It follows that $|A| = 453$, $|B| = 567$, and $|A \cap B| = 299$. The number of freshmen taking a course in either computer science or mathematics is

$$|A \cup B| = |A| + |B| - |A \cap B| = 453 + 567 - 299 = 721.$$

Consequently, there are $1807 - 721 = 1086$ freshmen who are not taking a course in computer science or mathematics.

3. Let A be the set of positive integers divisible by 5 and not exceeding 100 and B be the set of positive integers divisible by 7 and not exceeding 100. Then $|A| = 20$, $|B| = 14$ and $|A \cap B| = 2$. But we want the cardinality of $|(A \cup B)^c|$.

$$|(A \cup B)^c| = 100 - |(A \cup B)|$$

and

$$|A \cup B| = |A| + |B| - |A \cap B| = 20 + 14 - 2 = 32$$

give us

$$|(A \cup B)^c| = 100 - 32 = 68.$$

4. $\frac{\text{Total number of derangements}}{n!}$

5. $\binom{20+5-1}{20}$

6. 26^6

7. 2520 (multinomial)

8. Partition of 11 into 4 variables. 11 indistinguishable objects into 4 distinguishable bins. $\binom{11+4-1}{11}$.

9. $\binom{5}{2}$

10. $\binom{n+m-1}{n}$ (index must total n over m variables. i.e solution of $y_1 + y_2 + \ldots + y_m = n$ with $y_i \geq 0$ for $i = 1, 2, \ldots, m$).

11. 3 (fill 2 in each box. 3 ways to partition 3)

12. 6^4

13. 12

14. (a) $\frac{1}{26!}$

    (b) $\frac{1}{26}$

    (c) $\frac{1}{2}$ (Choose 2 positions for z and a in $\binom{26}{2}$ ways and permute the rest in 24! ways or just permute 24 alphabets into 26 positions and then fit z and a into the remaining positions)

    (d) $\frac{1}{26}$

    (e) $\frac{1}{650}$

    (f) $\frac{1}{26\times25\times24} = \frac{1}{15600}$

15. $\frac{s\times(s-1)\ldots(s-n+1)}{s^n}$ for $s \geq n$ and 0 otherwise.

16. $\frac{\binom{10+3-1}{10}}{\binom{10+4-1}{10}}$