

# INDIAN STATISTICAL INSTITUTE

## Assignment Set IV

M. Tech (CS) - I Year, 2013-2014 (Semester - I)

*Probability and Stochastic Processes*

Deadline for submission: 29.11.2013

Total Marks : 50

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Note: This is a programming assignment. Clearly mark out your name, course, semester and roll number on the programs that you submit. Stick to the naming convention that we followed in the DFS Lab.

The assignments are given to aid your understanding of the subject. So, it is desired that you solve them yourself. In case you have taken any help, acknowledge it properly. Any instance of malpractice would be dealt with appropriately.

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- (Q1) Choose independently 100 random numbers uniformly from  $\{0, 1\}$ , and let  $X$  represent their sum. Carry out this experiment 10000 times. Make a plot such that the values of  $X$  are on the horizontal axis and the proportion of their occurrence is on the vertical axis. Also plot the pdf of  $N(50, \frac{100}{3})$ . [10]
- (Q2) Remember the Buffon's needle problem done in the class? Take the needle length to be 1 and the parallel lines to be unit distance apart. Simulate it by simulating the distributions of the needle's angle from the parallel lines and how far its centre (or farthest end) is from the nearest parallel line and run it for 10000 times and give an estimate of  $\pi$ . [10]
- (Q3) The time spent waiting for the next car to cross a highway follows exponential distribution with an average time between cars of 30 seconds. For  $t = 1000, 10000, 100000$  count the number of cars that arrive in time  $[0, t)$ , denote the count by  $X$ . Run this experiment 10000 times for each  $t$  and calculate
- (a) the average number of cars that arrive for each  $t$  and compare it with  $\frac{t}{30}$ .
  - (b) the proportion of times 10 cars arrive.
  - (c) the proportion of times 33 cars arrive
- [10]
- (Q4) A new born infant dies within one year with probability 0.006990. Table 1 indicates the probability of a person dying within 1 year. Use the law of large numbers to

Age	Prob. of death	Age	Prob. of death	Age	Prob. of death	Age	Prob. of death	Age	Prob. of death
1	0.000447	21	0.001250	41	0.002420	61	0.011835	81	0.068153
2	0.000301	22	0.001342	42	0.002628	62	0.012728	82	0.075349
3	0.000233	23	0.001382	43	0.002860	63	0.013743	83	0.083230
4	0.000177	24	0.001382	44	0.003117	64	0.014885	84	0.091933
5	0.000161	25	0.001370	45	0.003396	65	0.016182	85	0.101625
6	0.000150	26	0.001364	46	0.003703	66	0.017612	86	0.112448
7	0.000139	27	0.001362	47	0.004051	67	0.019138	87	0.124502
8	0.000123	28	0.001373	48	0.004444	68	0.020752	88	0.137837
9	0.000105	29	0.001393	49	0.004878	69	0.022497	89	0.152458
10	0.000091	30	0.001419	50	0.005347	70	0.024488	90	0.168352
11	0.000096	31	0.001445	51	0.005838	71	0.026747	91	0.185486
12	0.000135	32	0.001478	52	0.006337	72	0.029212	92	0.203817
13	0.000217	33	0.001519	53	0.006837	73	0.031885	93	0.223298
14	0.000332	34	0.001569	54	0.007347	74	0.034832	94	0.243867
15	0.000456	35	0.001631	55	0.007905	75	0.038217	95	0.264277
16	0.000579	36	0.001709	56	0.008508	76	0.042059	96	0.284168
17	0.000709	37	0.001807	57	0.009116	77	0.046261	97	0.303164
18	0.000843	38	0.001927	58	0.009723	78	0.050826	98	0.320876
19	0.000977	39	0.002070	59	0.010354	79	0.055865	99	0.336919
20	0.001118	40	0.002234	60	0.011046	80	0.061620	100	0.353765

Table 1: The probability of a person dying within one year at a certain age.

estimate the expected amount of children out of 100000 that live to see their 101st birthday. [10]

(Q5) Write a function which samples from  $\{0, 1, 2, 3, 4\}$  with uniform probability (using the random function provided by the programming language).

- (a) Using only (don't use the random function provided by the programming language) the random function above write a function to sample from  $\{0, 1, 2, 3\}$  with uniform probability.
- (b) Using the random function in (a) above write a function to sample from  $\{0, 1, 2, \dots, 15\}$  with uniform probability.
- (c) Using the function in (b) write a function to sample from  $\{0, 1, 2, \dots, 7\}$  with uniform probability.

[10]