INDIAN STATISTICAL INSTITUTE

Tutorial-4

M. Tech (CS) - I Year, 2016-2017 (Semester - II)

Design and Analysis of Algorithms

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(Q1) Let G be an undirected weighted graph. No two edges in G have the same weight. Prove or disprove the following statement. G has a unique minimum cost spanning tree.

(Q2) Let e be an edge of minimum weight in an undirected graph G. Then prove that e belongs to some minimum spanning tree.

(Q3) Consider the following algorithm to find a minimum spanning tree of an undirected and connected graph $G(V, E)$. First sort all the edges in non-increasing order, then process the edge set in sorted order as follows. Delete e from the graph if and only if removing e doesnot disconnect the graph. Prove that the above algorithm will give a minimum spanning tree of the graph.

(Q4) Let A be the adjacency matrix of a directed graph $G(V, E)$. What does the matrix A^k denote? Prove your claim.

(Q5) Let f be a flow in the network $G(V, E)$ and α be a real number. The *scalar flow product*, denoted by αf, is a function from $V \times V$ to $\mathbb{R}$ defined as $(\alpha f)(u, v) = \alpha f(u, v)$. Prove that the flows in the network form a *convex set*. That is, show that if f_1 and f_2 are flows, then so is $\alpha f_1 + (1 - \alpha) f_2$ for all $\alpha \in [0, 1]$.

(Q6) What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?

Symbol	a	b	c	d	e	f	g	h
Frequency	1	1	2	3	5	8	13	21

Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers?

(Q7) Suppose we store n keys in a hash table of size $m = n^2$ using a hash function h chosen from a *2-universal* hash family. What can you say about the probability of any collision?