**JRF (Quality, Reliability and Operations Research): 2009**
**INDIAN STATISTICAL INSTITUTE**

## OUTLINE OF THE SYLLABUS

The syllabus for JRF (QROR) will include the following subject areas: 1) Statistics , 2) Statistical Quality Control, 3) Quality Management and Quality Assurance Systems, 4) Operations Research , 5) Reliability, and 6) Mathematics. A broad coverage for each of the above subject areas is given below.

1. **Statistics (STAT)**: Elementary probability and distribution theory , Linear models, Estimation and test of hypothesis , Design of experiments( block design, full and fractional factorial designs), Bivariate distributions, Markov chain

2. **Statistical Quality Control(SQC)**: Attribute and variable control charts, CUSUM and EWMA charts, Process capability analysis, Acceptance sampling

3. **Quality Management and Quality Assurance Systems (QMAS)**: Total Quality management, Six Sigma, Quality assurance systems like ISO 9000 etc., Environmental management systems

4. **Operations Research (OR)**: Linear programming ( basic theory, simplex algorithms and its variants, duality theory, transportation and assignment problem), Non-linear programming ( basic theory, convex function and its generalization, unconstrained and constrained optimization), Inventory theory (EOQ models, dynamic demand model, deterministic models with price-breaks, concept of probabilistic models), Queuing Theory ( (M/M/S) and (M/G/1)), Game theory ( two persons zero-sum game, bimatrix game)

5. **Reliability (REL)**: Coherent Systems and System reliability, Failure time modelling, Estimation and Testing in reliability

6. **Mathematics(MATH)**: Permutations and combinations, Set Theory, Calculus, Linear and matrix algebra, Difference equations - all at B.Sc level.

## INSTRUCTIONS

The test will be divided into two sessions (i) Forenoon session and (ii) Afternoon session. Each session is for two hours. The forenoon session question paper will be identified by test code *RQRI*, whereas the afternoon session will have test code as *RQRII*. Candidates appearing for JRF (QROR) should verify and ensure that they are answering the right question paper.

The test **RQRI** will be of multiple choice type. Each question will have four alternatives, only one of which is correct/best possible answer. There will be thirty such questions, out of which 20 questions will be from Mathematics, and 2 questions each from five other areas, as given in the syllabus above. All questions have to be answered.

The test **RQRII** will have altogether 18 questions, 3 from each of the six areas(including Mathematics), as given in the syllabus above. A candidate has to choose three areas (out of six) and answer two questions from each area.

Some sample questions from each of the above mentioned six areas are given below.

## Multiple Choice

1. Consider a count variable $X$ following a Poisson distribution such that the zero count (that is, $X = 0$) is not observable. Based on $n$ observations $x_1, \ldots, x_n$ from this distribution the quantity for which $\bar{x}$ is an unbiased estimator is

   $(A)$ $\lambda$ $(B)$ $\frac{1}{1-e^{-\lambda}}$ $(C)$ $\frac{\lambda}{1-e^{-\lambda}}$ $(D)$ $\frac{\lambda e^{-\lambda}}{1-e^{-\lambda}}$

2. In a simple linear regression problem, suppose $\bar{x} = 2, \bar{y} = 3$, and the sample correlation between $x$ and $y$ is 0.5. The predicted value of $y$ for given $x = 6$ is 7. The slope of the regression line for predicting $x$ from $y$ is

   $(A)$ 1 $(B)$ 0.5 $(C)$ 0.25 $(D)$ none of these

3. Based on a sample of $n$ observations from $N(\mu, \sigma^2)$ with $\sigma$ known, let the length of a 90% confidence interval for $\mu$ be $L$. Suppose $Z_\alpha$ denotes the $(1-\alpha)$-th percentile of a standard normal distribution. Then for constructing a 95% confidence interval for the same $\mu$ with the same length $L$, the required sample size will be
   $(A)$ $\frac{Z_{.025}}{Z_{.05}}n$ $(B)$ $\frac{Z^2_{.025}}{Z^2_{.05}}n$ $(C)$ $\frac{Z_{.025}}{Z_{.05}}n\sigma$ $(D)$ $\frac{Z^2_{.025}}{Z^2_{.05}}n\sigma^2$

## Short Answer Type

1. Suppose die A has 4 red faces and 2 green faces while die B has 2 red faces and 4 green faces. Assume that both the dice are unbiased. An experiment is started with the toss of an unbiased coin. If the toss results in a Head, then die A is rolled repeatedly while if the toss of the coin results in a Tail, then die B is rolled repeatedly. For $k = 1, 2, 3, \ldots,$, define $X_k$ to be the indicator random variable such that $X_k$ takes the value 1 if the $k$-th roll of the die results in a red face, and takes the value 0 otherwise.

   (a) Find the probability mass function of $X_k$.

(b) Calculate the correlation between $X_1$ and $X_7$.

2. Let $X_1$ and $X_2$ be independent normal random variables with common mean $\mu$ and variances $\sigma_1^2$ and $\sigma_2^2$ respectively. Let $\bar{x_1}$ and $\bar{x_2}$ be the sample means based on $n$ observations from the joint distribution of $(X_1, X_2)$. Define $T_1$ and $T_2$ such that

$$T_1 = \frac{\bar{x_1} + \bar{x_2}}{2} \qquad \text{and} \qquad T_2 = \frac{\bar{x_1}\sigma_1^{-2} + \bar{x_2}\sigma_2^{-2}}{\sigma_1^{-2} + \sigma_2^{-2}}$$

   (a) Show that $T_1$ and $T_2$ are both unbiased for $\mu$.

   (b) Find the variances of $T_1$ and $T_2$.

   (c) Show that $Var(T_1) \leq Var(T_2)$.

3. An experienced inspector claims that he has the ability to predict if an item is defective or not just by its appearance without any confirmatory inspection of the item. Five items were selected at random from an assembly line. It turned out that the inspector could correctly predict four times out of five. Was the inspector guessing randomly? You may accept inspector's claim at most 5% of the time when he is really guessing.

4. Fold over a $2^{5-2}$ design to construct a $2^{6-2}$ design. Write the complete defining relation of the resulting design. What is its resolution? Is the resulting design a minimal design?

*Sample Questions for Statistical Quality Control*

## Multiple Choice

1. If $C_p$ is high and $C_{pk}$ is low, then:
(A) Process mean needs centering, (B) Process variability needs to be reduced, (C) Both mean and variability are to be controlled, (D) No need for process change.

2. The AQL for a given sampling plan is 1.0% percent. This means that:
(A) The producer takes a small risk of rejecting product which is 1% defective or better, (B) all accepted lots are 1% defective or better, (C) The average quality level of the plan is 1%, (D) the average quality limit is 1%.

3. For a $3\sigma$ control chart for individual measurement the ARL of a controlled process and the ARL when the process mean shifts by $1\sigma$ are respectively:

   $(A)$ 370 and 44, $(B)$ 740 and 44, $(C)$ 370 and 723, $D)$ None of the above.

## Short-Answer Type

1. (a) For a single sampling plan with curtailed inspection associated with both acceptance and rejection, derive the expression for ASN as a function of product quality.

   (b) Derive the following properties of the binomial OC function of a single sampling plan with sample size $n$ and acceptance number $c$ at a process average $p$:

$$B(c+1, n+1, p) - B(c, n, p) = qb(c+1, n, p)$$

where

$$B(c, n, p) = \sum_{0}^{c} b(x, n, p) = \sum_{0}^{c} \binom{n}{x} p^x (1-p)^{n-x}$$

2. Show that if $\lambda = 2(w+1)$ for the EWMA control chart, the chart is equivalent to a $w$ period moving average control chart in the sense that the control limits are identical for large $t$.

3. (a) For a control chart for a normally distributed quality characteristic if all of the next seven points fall on the same side of the center line, we conclude that the process is out of control. (i)What is the $\alpha$ risk. (ii)If the mean shifts by one standard deviation and remains there during collection of the next seven samples what is the $\beta$ risk associated with this decision rule.

   (b) Compute the confidence intervale of $C_p$ computed as 1.9 using sample standard deviation obtained from a sample of size 26 as the estimate of the standard deviation. What assumptions have been made. $[\chi^2_{0.975,25} = 40.65; \quad \chi^2_{0.025,25} = 16.79]$.

4. The surface tension ($s$) of a chemical product, in dyne/cm against air at $20^oC$ , is given by the relationship $s = 20 + 0.1(3 + 0.5x)^2$, where $x$ is a component of the product with probability distribution $f(x) = \frac{160}{x^6}$, $x \geq 2$. Find the mean and variance of $s$. Would you recommend computation of the process capability index $C_p$ for $s$ using the standard expression $\frac{(USL-LSL)}{6\sigma}$, if the specification for $s$ is $21.8 \pm 1.0$? Justify.

Sample Questions for Quality Management and Quality Assurance Systems

**Multiple Choice**

1. In the statement below the key elements in the TQM philosophy are:

   (A) Customer focus, Radical process improvement, Quality control specialists and Human side of quality,
   (B) Incremental process improvement, Quality control specialists, Human side of quality and Measurement and analysis,
   (C) Customer focus,Incremental process improvement,Human side of quality and Measurement and analysis,
   (D) Customer focus, Radical process improvement,Human side of quality and Measurement and analysis.

2. The overall intentions and direction of an organisation with respect to Quality as formally expressed by its top management is:
   (A) Quality Plan,
   (B) Quality Objective,
   (C) Quality Policy,
   (D) Quality target.

3. Quality Planning is:

   (A) Identifying which quality standards are relevant to the project and determining how to satisfy them,
   (B) Monitoring specific project results to determine if they comply with relevant quality standards and identifying ways to eliminate the causes of unsatisfactory performances,
   (C) Evaluating overall project performance on a regular basis with a view to provide confidence that the project will satisfy the relevant quality standards,
   (D) Taking actions to increase the effectiveness and efficiency of the project so as to provide added benefits to both the performing organisation and the customer of the project.

**Short-Answer Type**

1. International Standards of the QMS and EMS promote the adoption of a process approach when developing, implementing and improving the effectiveness of a quality/environment management system, to enhance customer satisfaction by meeting customer requirements. What is meant by process approach and what are the generic requirements of the QMS in this context.

2. Explain the following concepts in the context of Environmental Management System (EMS) with an example in each case: a) environment b) environmental aspect c) environmental impact. Why and how we undertake an aspect impact analysis?

3. " Six Sigma initiative has given the organisations a well defined, disciplined problem solution framework" - justify the statement citing the commonly used templates of problem solving.

4. Explain in detail the following:

   (a) Cost of poor quality

   (b) Cost of non-quality

   Identify the cost elements that are common to both $(a)$ and $(b)$.

*Sample Questions for Operations Research*

## Multiple Choice

1. Consider the problem of solving a linear programming problem with simplex method. Find which of the following statements is false with respect to the simplex method.
   (A) The optimality conditions for the maximization and minimization problems are different in the simplex method, (B) The feasibility conditions for the maximization and minimization problems are different in the simplex method, (C) In a simplex method, the pivot element cannot be zero or negative, (D) In the simplex method, if the leaving variable does not correspond to the minimum ratio, at least one variable will definitely become negative in the next iteration.

2. Consider a balanced transportation problem. If $B$ is any basis then the determinant of $B$ is
   $(A)$ 1, $(B)$ $-1$, $(C)$ 2, $(D)$ either 1 or $-1$.

3. Consider a $(M/M/1):(GD/\infty/\infty)$ queue. Then the expected number of customers in the system (assuming that $\rho$ is the traffic intensity of the system) is
   $(A)$ $\frac{1}{1-\rho}$, $(B)$ $\frac{\rho}{1-\rho}$, $(C)$ $\frac{\rho^2}{1-\rho}$, $(D)$ $\frac{1}{\rho(1-\rho)}$.

## Short Answer Type

1. (a) Consider the following problem:
   minimize $c^t x$, subject to $Ax \geq b, x \geq 0, A \in R^{m\times n}, c \in R^n$, and $b \in R^m$.
      i. Show that the optimal solution to this linear programming problem is equivalent to solving a system of equations in non-negative variables.
      ii. If the optimal solution is not unique then show that there cannot be finitely many optimal solutions to the linear programming problem.

   (b) The set of all feasible solutions to a linear programming problem is a convex set.

2. (a) Consider the following problem

   minimize $Q(x) = c^t x + \frac{1}{2} x^t D x$, subject to $Ax \geq b, x \geq 0$, where $D$ is a symmetric matrix, Let $K$ be the set of all feasible solutions to the problem. $\bar{x}$ is an interior point of $K$. ( that is $A\bar{x} > b$ and $\bar{x} > 0$).

   i. What are the necessary conditions for $\bar{x}$ to be an optimum solution to the problem?

   ii. Show that if $D$ is positive semi definite, then $\bar{x}$ is a global optimum solution to the problem.

   (b) Consider the problem of minimizing $f(x)$ subject to $x \in X$ where $f$ is a convex function and $X \subseteq R^n$ is a covex set. Then derive a characterization of $f$ in terms of $epif$, where
   $epif = \{(x, y) : x \in X, y \in R, y \geq f(x)\}$

3. (a) If the $i^{th}$ row of the payoff matrix of an $m \times n$ rectangular game be stricly dominated by a convex combination of the other rows of the matrix, then show that the deletion of the $i^{th}$ row from the matrix does not affect the set of optimal strategies for the row player.

   (b) Consider the queuing model $(M \mid M \mid S) : (\infty \mid FCFS)$. Find the probability distribution of busy period and the waiting time distribution.

   (c) Show that a balanced transportation problem always has a feasible solution.

4. An aircraft company uses rivets at an approximately constant rate of 5000 Kg. per year. The rivets cost Rs. 20 per Kg. and the company personnel estimate that it costs Rs. 200 to place an order and the carrying cost of inventory is 10% per year.

   (a) How frequently should orders for rivets be placed and what quantities should be ordered for?

   (b) If the actual costs are Rs. 500 to place an order and 15% for carrying cost, the optimal policy should change. How much would the company loose every year because of imperfect cost information?

**Multiple Choice**

1. In a series system with two independent components, each having exponential strength with mean $1/\lambda$, is subject to common stress having exponential distribution with mean $1/\mu$. The reliability of the system is

$$(A)\ \frac{\mu}{\mu+\lambda},\ (B)\ \left(\frac{\mu}{\mu+\lambda}\right)^2,\ (C)\ \frac{\mu}{\mu+2\lambda},\ (D)\ \left(\frac{\mu}{\mu+2\lambda}\right)^2.$$

2. Consider the following functions:

$$(i)\ e^{-\lambda t},\ (ii)\ e^{\beta t},\ (iii)\ \gamma t,\ (iv)\ \delta t^{-2},$$

where $\alpha, \beta, \gamma, \delta$ are positive constants and $t \geq 0$. Which of these four functions are valid hazard rates?

$$(A)\ \text{All},\ (B)\ (i), (ii), (iii),\ (C)\ (ii), (iii), (iv), (D)\ \text{None}.$$

3. In a $k$-out-of-$n : G$(good) system, the number of minimal path sets and minimal cut sets are, respectively,

$(A)$ $\binom{n}{k}$, $\binom{n}{n-k}$

$(B)$ $\binom{n}{k}$, $\binom{n}{n-k-1}$

$(C)$ $\binom{n}{k}$, $\binom{n}{n-k+1}$

$(D)$ $\binom{n}{n-k+1}$, $\binom{n}{k}$

**Short-Answer Type**

1. (a) The UN security council consists of five permanent members having 7 points each and ten temporary members having 1 point each. In order to get a resolution passed, at least 39 points are needed in favour. Present this problem (of getting a resolution passed) as a coherent system with 15 components and obtain the structural importance of the components. Find a non-trivial modular decomposition of the system.

   (b) Prove that the min path sets of a coherent system are the min cut sets of its dual and vice versa.

2. (a) A total of $n$ independent light bulbs each having life distribution $F(t) = 1 - \exp[-(\lambda t)^3]$ are put to test. The observations of failure times are subject to random censoring from the right. Find an explicit expression for the maximum likelihood estimator of the mean life of the light bulbs.

   (b) Prove that the maximum likelihood estimator of the mean lifetime based on type II censored data from exponential life distribution is unbiased.

3. (a) The hazard rate for an item is given by

$$h(x) = \begin{cases} a, & \text{if } 0 < x \leq x_0 \\ a + b(x - x_0), & \text{if } x > x_0, \end{cases}$$

   where $a, b, x_0$ are positive constants. Derive the reliability function. For known $x_0$ and based on random right censored lifetime data, write down the likelihood function for estimating $a$ and $b$.

   (b) A manufacturer of 60W light bulbs claims that the survival probability of a bulb for 1000 hours is 0.99. A sample of 100 bulbs on test shows 93 survivors beyond 1000 hours. Is the claim of the manufacturer acceptable? You may use normal approximation.

4. (a) Suppose the $n$ components of a parallel system have independent exponential lifetimes with failure rates as $\lambda_1, \cdots, \lambda_n$. Compute the system reliability function and the mean system lifetime.

(b) Suppose that two independent systems, with identical and exponentially distributed lifetime with failure rate $3 \times 10^{-7}$ per hour, are either to be placed in active parallel or a stand-by configuration. For $t = 7$ years, derive the reliability gain of one over the other configuration.

## Multiple Choice

1.

$$\text{Consider the matrix} A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}. \text{ The rank of A is}$$

$(A)$ 1, $(B)$ 2, $(C)$ 3, $(D)$ 4.

2. Consider the following system of equations.

$$\begin{aligned} 2x + 3y + 5 &= 9 \\ 7x + 3y - 2z &= 8 \\ 2x + 3y + \lambda z &= \mu \end{aligned}$$

For what values of $\lambda$ & $\mu$ the system has infinitely many solutions.

$(A)$ $\lambda = 5, \mu \neq 9$, $(B)$ $\lambda \neq 5, \mu = 4$, $(C)$ $\lambda = 5, \mu = 9$, $(D)$ none.

3. The value of the following infinite series is

$$1 - \frac{1}{4} + \frac{1}{7} - \frac{1}{10} + \frac{1}{13} - \dots$$

$(A)$ $\frac{1}{\sqrt{3}}(log2+\pi/3)$, $(B)$ $\frac{1}{3}(log3+\pi/2)$, $(C)$ $\frac{1}{3}(log2+\pi/\sqrt{3})$, $(D)$ none of the above.

4. Let $f$ be the bilinear form on $\Re^2$ defined by

$$f((x_1, x_2), (y_1, y_2)) = 2x_1y_1 - 3x_1y_2 + x_2y_2$$

Then the matrix $A$ of $f$ in the basis $\{u_1 = (1, 0), u_2 = (1, 1)\}$ is equal to

$(A)$ $\begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}$, $(B)$ $\begin{pmatrix} 2 & 2 \\ -1 & 0 \end{pmatrix}$, $(C)$ $\begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$, $(D)$ none of the above.

5. Let $f$ be the linear function on $\Re^2$ defined by $\phi(x, y) = x - 2y$
Then $\{T^t(\phi)\}(x, y)$ for $T(x, y) = (x, 0)$ is

$(A)\ x,\ (B)\ 2x - 2y,\ (C)\ -2y,\ (D)$ none of the above.

6. If $A\&B$ are non-singular matrices of the same order such that $(A + B)$ and $(A + AB^{-1}A)$ are also non-singular, then
$(A + B)^{-1} + (A + AB^{-1}A)^{-1} =$

$(A)\ A^{-1},\ (B)\ B^{-1},\ (C)\ (A + B^{-1})^{-1},\ (D)$ none.

7. Find $1 + 4 + 18 + 96 + \cdots$ upto 50 terms.

$(A)\ 51! - 1,\ (B)\ 52!,\ (C)\ 50! - 1,\ (D)$ none of the above.

8. $\int_c^\infty xe^{-2x}dx =$

$(A)\ 1/4(1 + 2c)e^{-2c},\ (B)\ 1/4(1 + e^{-2c} + 2ce^{-2c}),\ (C)\ 1/4(1 - e^{-2c} + 2ce^{-2c}),\ (D)\ 1/4(1 - 2c)e^{-2c}$.

9. A boy walks on the $xy$ plane from point (2,1) to point (7,5). He moves 1 unit in a single step, in either $+x$ or $+y$ direction. In how many different ways can he complete his journey?

$(A)\ 2^9/(4! \cdot 5!),\ (B)\ 9!,\ (C)\ 2^9,\ (D)\ 9!/(4! \cdot 5!)$.

10. The number of real root(s) of the equation $x^3 + 2x^2 + 2x = 0$ is

$(A)\ 1,\ (B)\ 2,\ (C)\ 3,\ (D)\ 0$.

11. The inequality $| 6 - x^2 | < | x |$ is satisfied for

$(A)\ 1 < | x | < 2,\ (B)\ 2 < | x | < 3,\ (C)\ 2 < | x | < 4,\ (D)\ 1 < | x | < 3$.

12. Consider the sets $X = \{a, b, c\}$ and $Y = \{1, 2\}$. The number of onto functions from $X$ to $Y$ is

$(A)\ 3,\ (B)\ 6,\ (C)\ 8,\ (D)\ 4$.

13. The indefinite integral
$\int \frac{1}{1+\cos^2 x} dx$ equals

$(A)\ \log_e(1 + \cos^2 x) + C,\ (B)\ \tan^{-1}(\cos x) + C,\ (C)\ \frac{1}{\sqrt{2}}\tan^{-1}(\frac{\tan x}{\sqrt{2}}) + C,\ (D)\ -\frac{\log_e(1+\cos^2 x)}{\sin(2x)} + C$.

14. The function $f(x) = x^{\frac{5}{3}} - x^{\frac{2}{3}}, -\infty < x < \infty$, has

$(A)$ no local maximum, $(B)$ no local minimum,
$(C)$ no inflection point, $(D)$ none of the above.

15. The limit

$$\lim_{x\to\infty}(\frac{3x-1}{3x+1})^{4x}$$

equals

$(A)$ $e^{-\frac{8}{3}}$, $(B)$ 0, $(C)$ 1, $(D)$ $e^{\frac{4}{9}}$.

16. Consider the function

$$f(x) = \begin{cases} \sqrt{1-(x-1)^2}, & \text{if } 0 < x \leq 2 \\ \sqrt{1-(x-3)^2}, & \text{if } 2 < x < 4, \end{cases}$$

Then in the interval $(0, 4)$, $f(x)$ is

$(A)$ everywhere differentiable, $(B)$ not everywhere continuous,
$(C)$ everywhere continuous but not everywhere differentiable,
$(D)$ differentiable wherever continuous.

17. Let $g(x, y) = max\{12 - x, 8 - y\}$. Then the minimum value of $g(x, y)$ as $(x, y)$ varies over the line $x + y = 10$ is

$(A)$ 3, $(B)$ 7, $(C)$ 1, $(D)$ 5.

18. If

$$f(x) = \frac{x^2(2-x)^3e^x}{(1+4x)^5(2x^2-4x+1)}$$

then $f'(1)$ equals

$(A)$ $-4$, $(B)$ $-4f(1)$, $(C)$ 4, $(D)$ $4f(1)$.

**Short Answer Type**

1. (a) Let $f(x) = \sum_{k=0}^{n} a_k x^k$, where $a_k$'s satisfy $\sum_{k=0}^{n} \frac{a_k}{k+1} = 0$. Show that there exists a root of $f(x) = 0$ in the interval $(0, 1)$.

   (b) Find $\int_{-2}^{4} [x] dx$, where $[x]$ is the largest integer less than or equal to $x$.

2. (a) Consider a system $Ax = b$. Suppose (C:d) is obtained from (A:b) by a sequence of elementary row operations. Show that the system $Ax = b$ and $Cx = d$ have the same set of solutions.

   (b) Consider a symmetric matrix $A$ such that neither $A$ nor $-A$ is positive definite. Show that there exists a vector $x \neq 0$ such that $x'Ax = 0$.

3. (a) If $x = y^{y^{y^{\cdot^{\cdot^{\infty}}}}}$, then show that

$$\frac{dy}{dx} = \frac{y(1 - x \log y)}{x^2}.$$

   (b) Compute $I = \lim_{n\to\infty}[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{4n}]$.

4. Test the convergence of the series $\sum_{n=1}^{\infty} (\frac{n+1}{4n+1})^n (x-2)^{2n}$. If the series is convergent then find the domain of convergence.

5. A population of weasels is growing at rate of 3% per year. Let $w_n$ be the number of weasels $n$ years from now and suppose that there are currently 350 weasels.

   (a) Write a difference equation which describes how the population changes from year to year.

   (b) Solve the difference equation of part (a). If the population growth continues at the rate of 3%, how many weasels will there be 15 years from now?

(c) Plot $w_n$ versus $n$ for $n = 0, 1, 2, ..., 100$.

(d) How many years will it take for the population to double?

(e) Find $\lim_{n\to\infty} w_n$.

What does this say about the long-term size of the population? Will this really happen?

6. We have difference equation $y(k+1) = 2y(k) - 5$, what is the solution for initial value $y(0) = 1$ and what is the equilibrium value? How is the stability of this equilibrium value?