Test Code: CS (Short answer type) 2012

M.Tech. in Computer Science

The selection test for M.Tech. in Computer Science will consist of two parts:

- Test MIII (objective type) in the forenoon session, and
- Test CS (short answer type) in the afternoon session.

The CS test will have two groups as follows:

- **Group A** : A test for all candidates in analytical ability and mathematics primarily at the B.Sc. (pass) level, carrying 30 marks.
- **Group B**: A test, divided into several sections, carrying equal marks of 70 in Mathematics, Statistics, and Physics primarily at the B. Sc. (Hons.) level, and in Computer Science, and Engineering and Technology primarily at the B.Tech. level. A candidate has to answer questions from only one of these sections according to his/her choice.

The syllabus and sample questions for the MIII test are available separately. The syllabus and sample questions for the CS test are given below.

Note:

1. Not all questions in the sample set are of equal difficulty. They may not carry equal marks in the test.
2. Each of the tests, MIII and CS, will have individual qualifying marks.

SYLLABUS for **Test CS**

Group A

Elements of set theory. Permutations and combinations. Functions and relations. Theory of equations. Inequalities.
Limits, continuity, sequences and series, differentiation and integration with applications, maxima-minima.
Elementary Euclidean geometry and trigonometry.
Elementary number theory, divisibility, congruences, primality.
Determinants, matrices, solutions of linear equations, vector spaces, linear independence, dimension, rank and inverse.

Group B

Mathematics

In addition to the syllabus for Mathematics in Group A, the syllabus includes:

Calculus and real analysis - real numbers, basic properties, convergence of sequences and series, limits, continuity, uniform continuity of functions, differentiability of functions of one or more variables and applications, indefinite integral, fundamental theorem of Calculus, Riemann integration,

improper integrals, double and multiple integrals and applications, sequences and series of functions, uniform convergence.
Linear algebra - vector spaces and linear transformations, matrices and systems of linear equations, characteristic roots and characteristic vectors, Cayley-Hamilton theorem, canonical forms, quadratic forms.
Graph Theory - connectedness, trees, vertex coloring, planar graphs, Eulerian graphs, Hamiltonian graphs, digraphs and tournaments.
Abstract algebra - groups, subgroups, cosets, Lagrange's theorem, normal subgroups and quotient groups, permutation groups, rings, subrings, ideals, integral domains, fields, characteristics of a field, polynomial rings, unique factorization domains, field extensions, finite fields.
Differential equations - solutions of ordinary and partial differential equations and applications.

Statistics

Notions of sample space and probability, combinatorial probability, conditional probability, Bayes' theorem and independence.
Random variable and expectation, moments, standard univariate discrete and continuous distributions, sampling distribution of statistics based on normal samples, central limit theorem, approximation of binomial to normal, Poisson law.
Multinomial, bivariate normal and multivariate normal distributions.
Descriptive statistical measures, product-moment correlation, partial and multiple correlation.
Regression - simple and multiple.
Elementary theory and methods of estimation - unbiasedness, minimum variance, sufficiency, maximum likelihood method, method of moments, least squares methods.
Tests of hypotheses - basic concepts and simple applications of Neyman-Pearson lemma, confidence intervals.
Tests of regression, elements of non-parametric inference, contingency tables and Chi-square, ANOVA, basic designs (CRD/RBD/LSD) and their analyses, elements of factorial designs.
Conventional sampling techniques, ratio and regression methods of estimation.

Physics

General properties of matter - elasticity, surface tension, viscosity.
Classical dynamics - Lagrangian and Hamiltonian formulation, symmetries and conservation laws, motion in central field of force, planetary motion, collision and scattering, mechanics of system of particles, small oscillation and normal modes, wave motion, special theory of relativity.
Electrodynamics - electrostatics, magnetostatics, electromagnetic induction, self and mutual inductance, capacitance, Maxwell's equation in free space and linear isotropic media, boundary conditions of fields at interfaces. Nonrelativistic quantum mechanics - Planck's law, photoelectric effect, Compton effect, wave-particle duality, Heisenberg's uncertainty principle, quantum mechanics, Schrodinger's equation, and some applications.
Thermodynamics and statistical Physics - laws of thermodynamics and their consequences, thermodynamic potentials and Maxwell's relations, chemical potential, phase equilibrium, phase space, microstates and macrostates, partition function free energy, classical and quantum statistics.
Atomic and molecular physics - quantum states of an electron in an atom, Hydrogen atom spectrum, electron spin, spin-orbit coupling, fine structure, Zeeman effect, lasers.
Condensed matter physics - crystal classes, 2D and 3D lattice, reciprocal lattice, bonding, diffraction and structure factor, point defects and dislocations, lattice vibration, free electron theory, electron motion in periodic potential, energy bands in metals, insulators and semiconductors, Hall effect, thermoelectric power, electron transport in semiconductors, dielectrics, Claussius Mossotti equation, Piezo, pyro and ferro electricity.

Nuclear and particle physics - Basics of nuclear properties, nuclear forces, nuclear structures, nuclear reactions, interaction of charged particles and e-m rays with matter, theoretical understanding of radioactive decay, particle physics at the elementary level.
Electronics - semiconductor physics, diode as a circuit element, clipping, clamping, rectification, Zener regulated power supply, transistor as a circuit element, CC CB CE configuration, transistor as a switch, OR and NOT gates feedback in amplifiers.
Operational Amplifier and its applications - inverting, noninverting amplifiers, adder, integrator, differentiator, waveform generator comparator and Schmidt trigger.
Digital integrated circuits - NAND, NOR gates as building blocks, XOR gates, combinational circuits, half and full adder.

Computer Science

Data structures - array, stack, queue, linked list, binary tree, heap, AVL tree, B-tree.
Programming languages - Fundamental concepts abstract data types, procedure call and parameter passing, languages like C and C++.
Design and analysis of algorithms Asymptotic notation, sorting, selection, searching.
Computer organization and architecture - Number representation, computer arithmetic, memory organization, I/O organization, microprogramming, pipelining, instruction level parallelism.
Operating systems - Memory management, processor management, critical section problem, deadlocks, device management, file systems.
Formal languages and automata theory - Finite automata and regular expressions, pushdown automata, context-free grammars, Turing machines, elements of undecidability.
Principles of Compiler Construction - Lexical analyzer, parser, syntax-directed translation, intermediate code generation.
Database management systems - Relational model, relational algebra, relational calculus, functional dependency, normalization (up to 3rd normal form).
Computer networks - OSI, LAN technology - Bus/tree, Ring, Star; MAC protocols; WAN technology - circuit switching, packet switching; data communications - data encoding, routing, flow control, error detection/correction, Internetworking, TCP/IP networking including IPv4.
Switching Theory and Logic Design - Boolean algebra, minimization of Boolean functions, combinational and sequential circuits synthesis and design.

Engineering and Technology

Moments of inertia, motion of a particle in two dimensions, elasticity, friction, strength of materials, surface tension, viscosity and gravitation.
Laws of thermodynamics and heat engines.
Electrostatics, magnetostatics and electromagnetic induction.
Magnetic properties of matter - dia, para and ferromagnetism.
Laws of electrical circuits - RC, RL and RLC circuits, measurement of current, voltage and resistance.
D.C. generators, D.C. motors, induction motors, alternators, transformers.
p-n junction, bipolar & FET devices, transistor amplifier, oscillator, multi-vibrator, operational amplifier.
Digital circuits - logic gates, multiplexer, de-multiplexer, counter, A/D and D/A converters.
Boolean algebra, minimization of switching functions, combinational and sequential circuits.
C Programming language.

SAMPLE QUESTIONS

Group A

A1. Imagine a cubic array made up of an $n \times n \times n$ arrangement of unit cubes: the cubic array is n cubes wide, n cubes high and n cubes deep. A special case is a $3 \times 3 \times 3$ Rubik's cube, which you may be familiar with. How many unit cubes are there on the surface of the $n \times n \times n$ cubic array?

A2. The integers $1, 2, \ldots, 10$ are circularly arranged in an arbitrary order. Show that there are always three successive integers in this arrangement, whose sum is at least 17.

A3. A piece of wire 16 $inches$ long is cut into two pieces. One piece is bent to form a square and the other is bent to form a circle. Where should the cut be made in order to minimize the total area of the square and the circle?

A4. If n is a positive integer, prove that

$$\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n-1} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{2n-1}$$

A5. Prove that the positive integers that cannot be written as sums of two or more consecutive integers are precisely the powers of 2.

Group B

Mathematics

1. Let $f : [0, 1] \to [0, 1]$ be a continuous function.

   (a) Show that $\exists\, x_0 \in (0, 1)$ such that
   $$\frac{f(x_0)}{3} = \int_0^1 f(x)x^2\,dx.$$

   (b) If $(2x-1)(f(x)-x) \geq 0,\ \forall\, x \in [0,1]$, show that x_0 as in part 1(a) cannot be less than or equal to $\frac{1}{2}$.

   (c) Define x_1 such that
   $$\int_0^{x_1} f(x)x^2\,dx = \frac{1}{3}\int_0^1 f(x)x^2\,dx.$$
   If f is monotonically increasing, show that x_1 cannot be less than $\frac{2}{3}$.

2. (a) Let
   $$A = \left\{ \sum_{i=1}^{\infty} \frac{x_i}{3^i} : x_i = 0 \text{ } or \text{ } 2,\ \forall i = 1, 2, \cdots \right\}.$$
   Show that $\frac{3}{4} \in A$.

   (b) For two real numbers a, b (both greater than 1), evaluate
   $$\lim_{n\to\infty} \left(\frac{a^{\frac{1}{n}} + b^{\frac{1}{n}}}{2} \right)^n.$$

3. (a) For $0 \leq x_1 \leq x_2 \leq 1$, let
   $$f(x_1, x_2) = \frac{x_1^2 + (x_2 - x_1)^2 + (1 - x_2)^2}{\max\{x_1, x_2 - x_1, 1 - x_2\}}.$$
   Determine the maximum value of $f(x_1, x_2)$ and give all possible values of the pair (x_1, x_2) for which this maximum value is achieved.

   (b) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a monotonic function such that
   $$\lim_{x\to\infty} \frac{f(2x)}{f(x)} = 1.$$
   Show that
   $$\lim_{x\to\infty} \frac{f(cx)}{f(x)} = 1$$
   for any $c > 0$.

4. (a) Let G be a simple graph with 23 vertices. The degree of every vertex is at least 11.

   (i) Can G be a regular graph of degree 11? Justify your answer.

   (ii) Argue whether G can be disconnected.

(b) Consider a simple connected planar graph with 10 vertices and 17 edges. Show that, using two colours, its vertices cannot be properly coloured, *i.e.*, no two neighbouring vertices have the same colour.

(c) In how many ways can one distribute 8 identical chocolates among 5 children - A, B, C, D and E?

5. (a) Given three matrices $A_{m\times n}$, $B_{n\times k}$, $C_{k\times p}$, the product $A*B*C$ can be computed in two ways:

   (i) $(A*B)*C$, and

   (ii) $A*(B*C)$.

   Establish the conditions on m, n, k and p under which (i) requires fewer arithmetic operations (additions and multiplications) than (ii).

   (b) Let V be the linear space of all functions of the form

$$f(x) = a\cos x + b\sin x.$$

   Consider the linear transformation $T(f) = f'' - 2f' - 3f$ from V to V. [f' and f'' are the first and second order derivatives of f respectively].

   (i) Find the matrix of the linear transformation T that transforms the basis $(\cos x, \sin x)$ to $(T(\cos x), T(\sin x))$.

   (ii) Is T an isomorphism?

   (iii) How many solutions f in V does the following differential equation have?

$$f''(x) - 2f'(x) - 3f(x) = \cos x$$

6. (a) Is the polynomial $x^{10}+x^5+1$ irreducible over $\mathbb{Q}$ (the field of rational numbers)? Justify your answer.

   (b) (i) Consider the additive group $\mathbb{Z}_{24}$ (the set of integers modulo 24). What are the orders of the elements 4, 12 and 16 in $\mathbb{Z}_{24}$?

   (ii) Let $(G,*)$ be a finite abelian group and $\circ(g)$ denote the order of the element g in G. Consider $u, v \in G$ such that $\circ(u) = m$ and $\circ(v) = n$. If the greatest common divisor $gcd(m,n) = 1$, then derive the order of $u*v$.

   (iii) For the group G in (ii) above, if $gcd(m,n) = d > 1$, then find an element in G whose order is $lcm(m,n)$. Justify your answer. *lcm* denotes the least common multiple.

Statistics

1. (a) A passenger airline company has found from past experience that 20% of the customers who buy tickets for a flight do not show up for the journey. The company wishes to ensure that a particular flight is at least 95% full with a probability of 0.9. How many tickets should it sell if the capacity of the flight is 300?

   (b) 18 boys and 2 girls are made to stand in a line in a random order. Let X be the number of boys standing between the girls. Find

   (i) $P(X = 5)$,

   (ii) $E(X)$.

2. (a) The mean μ of a normal population is unknown, but its standard deviation σ is known. The length of a $100(1-2\alpha)\%$ confidence interval for μ based on a random sample of size n from the population is found to be equal to L. By what factor should n be changed to ensure that a $100(1-\alpha)\%$ confidence interval for μ will be of length $L/2$?

   (b) If $X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables from the exponential distribution with mean $\theta > 0$, find the most powerful test based on them for testing $H_0 : \theta = 2$ against $H_1 : \theta = 1$. Find the power of the test.

3. (a) X_1 and X_2 are independent and identically distributed random variables from a Bernoulli distribution with parameter θ. Is the statistic $X_1 + \frac{X_2}{2}$ sufficient for θ? Justify your answer.

   (b) Let $X_1, X_2, \ldots, X_n$ be independent random variables, identically distributed as

$$g(x) = \begin{cases} \frac{1}{3\theta} & \text{if } -\theta \leq x \leq 2\theta, \\ 0 & \text{otherwise.} \end{cases}$$

   Find the maximum likelihood estimator of θ based on X_i, $i = 1, 2, \ldots, n$.

4. (a) The average height and weight of a group of students were found to be $1.7\ metres$ and $65\ kilograms$ respectively, while the correlation between heights and weights was 0.6. Using the regression equation for predicting weight from height, the estimated weight of a student whose height is $1.8\ metres$ was calculated to be $80\ kilograms$. Predict the height of a student whose weight is $60\ kilograms$.

   (b) Let Y_1, Y_2, Y_3 and Y_4 be uncorrelated random variables with

$$E(Y_k) = k\beta \text{ and } \mathrm{Var}(Y_k) = k^2\sigma^2,\ k = 1, 2, 3, 4,$$

   where β and σ are unknown parameters. Find the values of c_1, c_2, c_3 and c_4 for which

$$\sum_{k=1}^{4} c_k Y_k$$

   is unbiased for β and has the smallest variance among all linear unbiased estimators for β.

5. (a) X is a random variable having a normal distribution with mean 0 and variance 25. Let Y be another random variable, independent of X, taking values -1 and $+1$ with equal probability. Define

$$Z = XY + \frac{X}{Y} \text{ and } W = XY - \frac{X}{Y}.$$

   Find the probability distributions of (i) Z and (ii) $\left(\frac{Z+W}{10}\right)^2$.

   (b) Let X_1 and X_2 be independent samples from the uniform distribution over $(0, 1)$. Find the probability distribution of the geometric mean of X_1 and X_2.

6. (a) A researcher wishes to conduct an experiment to compare the effects of 4 different treatments. He is given 20 experimental units for this purpose, which are not entirely homogeneous. Assuming that there is only one significant source of heterogeneity among the units, suggest a suitable experimental design, with proper justification. Also give the analysis of variance for the design.

   (b) A population contains 10 units, labeled $U_1, U_2, \cdots, U_{10}$. For U_i, the value of a character Y under study, is Y_i, $1 \leq i \leq 10$. In order to estimate the population mean, $\bar{Y}$, a sample of size 4 is drawn in two steps as follows:

(I) A simple random sample of size 2 is drawn without replacement from the units $U_2, U_3, \ldots, U_9$;

(II) The sample drawn in step (I) is augmented by the units U_1 and U_{10}.

Based on this sample, suggest an unbiased estimator of $\bar{Y}$ and obtain its variance.

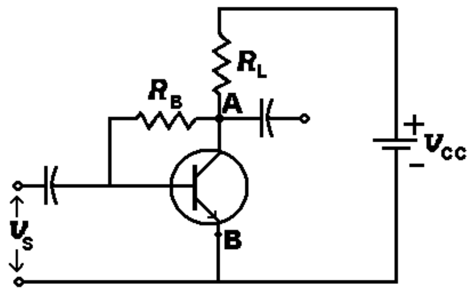
Physics

1. (a) An npn transistor shown in the figure below, is used in common-emitter mode with $\beta = 49$, $V_{cc} = 10\ V$, and $R_L = 2\ K\Omega$. A $100\ K\Omega$ resistor R_B is connected between the collector and the base of the transistor. Calculate

   (i) the quiescent collector current, and

   (ii) the collector to emitter voltage drop between points A and B.

   Assume base to emitter voltage drop is $0.7\ V$.



   (b) The set of Boolean functions $\{OR, NOT\}$ is functionally complete, *i.e.*, the set can implement any Boolean function. Prove algebraically that the set $\{XOR, AND\}$ also forms a functionally complete set of operations.

2. (a) Find the amount of pressure that is to be applied to change the boiling point of water by $3°K$, using the information given below:
   Latent heat of vaporization = $539\ cal/gm$; specific volume of
   vapor= $1677\ cc/gm$; specific volume of water = $1\ cc/gm$;
   T = $371°K$.

   (b) Sketch the isotherms for a gas obeying Van der Waal's equations of state and discuss the phase transition.

   (c) A classical system of N distinguishable non-interacting particles, each of mass m, is placed in a three-dimensional harmonic well having potential energy

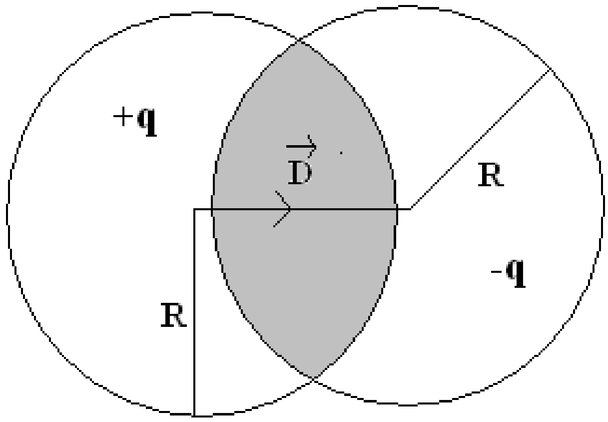
$$U = \frac{x^2 + y^2 + z^2}{2V^{2/3}},$$

   where V is a parameter. Find the partition function and Helmholtz free energy.

3. (a) Show that the rotational frequency spectrum of a diatomic molecule consists of equally spaced lines separated by an amount $\Delta r = \frac{h}{4\pi^2 I}$, where I is the moment of inertia of the molecule.

   (b) Express $\overrightarrow{L} \cdot \overrightarrow{S}$ in terms of J, L, and S, where the symbols carry their usual meaning. Hence, for $L = 1, S = \frac{1}{2}$, obtain the possible values of $\overrightarrow{L} \cdot \overrightarrow{S}$.

4. (a) Two spheres, each of radius R, carry uniform charge densities $+q$ and $-q$ respectively. The spheres are placed such that they partially overlap each other (see figure). $\overrightarrow{D}$ denotes the vector from the centre of the positive charged sphere to the centre of the negative charged sphere. Derive the electrostatic field at any point in the *shaded* (overlapping) region.



(b) The potential $V_0(\theta) = \kappa \ \sin^2(\theta/2)$ (κ is a constant), is specified on the surface of a hollow sphere of radius R. Find the potential *inside* the sphere.

(c) The electric field of an electromagnetic wave in vacuum is given by:

$$E_x = 0, \ \ E_y = 30 \ \cos(2\pi \ 10^8 t - \frac{2\pi}{3} x), \ \ E_z = 0.$$

Here E is in $Volts/metre$, t is in $seconds$ and x is in $metres$. Determine

(i) the frequency of the wave, and

ii) the direction of propagation of the wave.

5. (a) From the reaction $\Pi^- + p \rightarrow n + \gamma$, determine the possible values of the spin of a Π^- meson.

(b) Explain why the reaction $\Sigma^0 \rightarrow \Lambda^0 + \gamma$ is observed but not the decays $\Sigma^0 \rightarrow p + \Pi^-$ or $\Sigma^0 \rightarrow n + \Pi^0$.

(c) Determine the mass difference between two mirror nuclei which have N and Z differing by one unit.

(All the symbols carry their usual meaning in the above cases).

6. (a) Let $\overrightarrow{F} = \frac{k \overrightarrow{r}}{|\overrightarrow{r}|^3}$ be a central force per unit mass.

(i) Show that for a particle of mass m moving under this force F, the angular momentum is conserved.

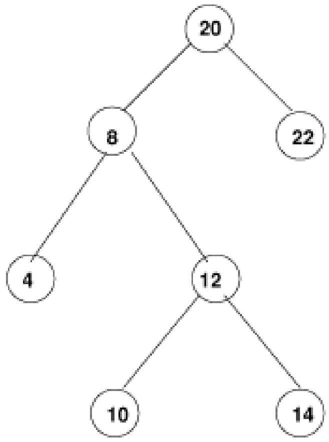
(ii) Write the Lagrangian for the above particle.

(b) The mean distance of Mars from the Sun is 1.5 times that of the Earth from the Sun. Find the time of revolution of Mars about the Sun with respect to that of the Earth about the Sun.

(c) A rocket when moving parallel to a long platform measures the length of the platform to be $\frac{9L}{10}$, where L is the length measured by a stationary observer. Find the time taken by the rocket to cross the platform.

Computer Science

1. (a) Given the values of two nodes in a binary search tree, write an algorithm to find their least (nearest) common ancestor. For example, in the figure below, the least common ancestor of the nodes 4 and 14 is 8. Your algorithm should take care of boundary conditions.



(b) Given a singly-linked list, devise a time and space efficient algorithm to find the m^{th} element from the *end* of the list.
If $m = 0$, then your algorithm should return the last element of the list. It should also take care of boundary conditions. Analyze the time complexity of your algorithm.

[NOTE: An algorithm that has minimum additional storage overhead and does not make redundant passes over the list will score full credit. Can you do it with constant additional space?]

2. (a) What is the output of the following program?

```
#include<stdio.h>

#define MUL(a,b) a*b
#define Pow(a) a*a

int main()
{
    int a=3;
    int b=2;
    printf("Ans: %d\n", MUL (MUL(a+1,b), Pow(b+1)));
    return 0;
}
```

(b) Consider the declaration below:

```
typedef char *Str_typed;
#define Str_defined char*
Str_typed s1, s2;
Str_defined s3, s4;
```

Which one of the following statements is correct?

I. `s1, s2, s3` and `s4` are character pointers

II. `s1, s2, s3` and `s4` are characters

III. `s1, s2, s3` are character pointers while `s4` is a character

IV. None of the above

(c) Consider the following list of parameter passing conventions in any programming language.

I. Call by name

II. Call by value

III. Call by reference

If the following piece of code in a certain programming language printed `16` for `j`, which of the above parameter passing conventions may have been used? Justify your answer.

```
program test (input, output);
var i, j;

procedure calc (p1, p2: integer);
    p2 = p2 * p2;
    p1 = p1 - p2;
    p2 = p2 - p1;
end;

begin (main)
    i = 2;
    j = 3;
    calc (i, j);
    print (j);
end (main)
```

Assume that the program is syntactically correct.

(d) Write an efficient algorithm to find the first non-repeated character in a string defined over the English alphabet set [a-z, A-Z].
For example, the first non-repeated character in `teeter` is `r`.
Analyze the time complexity of your algorithm.

[NOTE: You will get partial credit if the time complexity of your algorithm is quadratic or more in the length of the string.]

3. (a) A processor chip is used for applications in which 30% of execution time is spent on floating point additions, 20% on floating point multiplications, and 15% on floating point divisions. To enhance the performance of the processor, a design team examines the following three options, each costing about the same in design effort and manufacturing.

I. The floating point adder is made four times faster.

II. The floating point multiplier is made three times faster.

III. The floating point divider is made twenty times faster.

If only two of the above options can be implemented, which one should be discarded and why?

(b) Consider the following grammar with two missing productions:

$$\begin{array}{lcl} \text{S} & \rightarrow & \text{aS} \mid \ldots\ \textbf{(1)} \\ \text{A} & \rightarrow & \ldots\ \textbf{(2)} \mid \epsilon \\ \text{X} & \rightarrow & \text{cS} \mid \epsilon \\ \text{Y} & \rightarrow & \text{dS} \mid \epsilon \\ \text{Z} & \rightarrow & \text{eS} \end{array}$$

Reconstruct the grammar by filling in the missing productions **(1)** and **(2)**, using the $First$ and $Follow$ sets for this grammar given below:

	First	Follow
S	{a,b,c,d,e}	{\$} $\cup$ $Follow$(X) $\cup$ $Follow$(Y) $\cup$ $Follow$(Z)
A	{c,d,e,ϵ}	{b}
X	{c,ϵ}	($First$(Y) $\setminus$ ϵ) $\cup$ $First$(Z)
Y	{d,ϵ}	$First$(Z)
Z	{e}	$Follow$(A)
a	{a}	$First$(S)
b	{b}	$Follow$(S)
c	{c}	$First$(S)
d	{d}	$First$(S)
e	{e}	$First$(S)

Note that you can define only two productions. (Recall that X $\rightarrow$ A $\mid$ B represents two productions). Justify your answer.

4. (a) Consider the languages $L_1, L_2 \subseteq \Sigma^*$, where $\Sigma = \{a, b, c\}$. Define

$$L_1/L_2 = \{\ x : \exists\ y \in L_2 \text{ such that } xy \in L_1\ \}.$$

Let $L_1 = \{a^n b^n c^{2n} : n \geq 0\}$ and $L_2 = \{b^n c^{2n} : n \geq 0\}$.
Justify whether L_1 and L_1/L_2 are regular.

(b) Consider the following program fragment:

```
1. i = 1; sum=0;
2. while (i <= n)  do
     begin
3.        sum = sum + a[i];
4.        i = i + 1;
     end
```

Let

- A represent the initialization in line 1
- B represent the action within the loop in line 3
- I represent the increment in line 4
- T represent the test implied by line 2

Which of the following regular expressions represents all possible sequences of steps taken by this program? Justify your answer.

I. $A(TBI)^*$
II. $AT^+B^*I^*$
III. $AT(BIT)^+$

IV. $AT(BIT)^*$
V. $A(TBI)^+$

5. (a) Consider a relation $R(ABCD)$ as follows:

A	B	C	D
A_1	B_1	C_1	D_1
A_1	B_2	C_1	D_2
A_2	B_1	C_2	D_1
A_3	B_3	C_1	D_3
A_2	B_3	C_2	D_3
A_4	B_2	C_3	D_2
A_3	B_4	C_1	D_3
A_5	B_1	C_1	D_1

Assume that the four attributes A, B, C, and D are atomic and (A, B) is the key of R.

(i) Show that the relation is not in $3NF$.

(ii) Provide a lossless decomposition of R, so that the decomposed relations are in $3NF$, and determine their primary keys.

(iii) Write the following query in SQL: find the distinct values of A and the count of such values, for each value of C.

(b) Determine the minimum number of elementary logic gates (AND / OR / NOT) required to implement the Boolean expression
$AB \vee A\bar{B} \vee \bar{A}C$, where $\bar{x}$ denotes the complement of x.

(c) A logic circuit has three Boolean inputs `X`, `Y` and `Z`. Its output is `F(X,Y,Z)` such that:

```
F(X,Y,Z) = 1 if aX + bY +cZ > d
         = 0 otherwise.
```

`a,b,c,d` are real constants.

For which of the following values of `a,b,c,d` does this circuit represent an implementation of a three-input NAND gate with inputs `X`, `Y` and `Z`?

I. `a = b = c = 1; d = 2.5`
II. `a = b = c = 1; d = 1.5`
III. `a = b = c = -1; d = 0`
IV. None of the above

6. (a) Consider a channel with a capacity of 1 *MHz* and an *SNR* of 63.

(i) What is the upper limit on the data rate that the channel can carry?

(ii) How many signal levels are needed to achieve the data rate of $\frac{2}{3}$ of the upper limit obtained in 6a(i) above?

(b) Consider the use of 1000-bit frames on a 1 $Mbps$ satellite channel whose propagation time from the earth to the satellite is 270 $msec$. Assume that headers and acknowledgement frames are of negligible length. Calculate the maximum achievable channel utilization for the following two cases:

(i) Stop-and-wait protocol where $P = 10^{-3}$ is the probability that a single frame is in error. Assume that acknowledgement frames are never in error.

(ii) Error-free operation of stop-and-wait protocol where acknowledgements are always piggybacked onto the data frames.

(c) A virtual memory system is able to support virtual address space of $256\ GB$. An entry in the page table is $4\ bytes$ long.

(i) Calculate the minimum page size required for a three-level paging scheme.

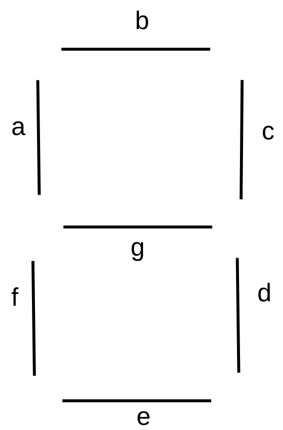
(ii) Draw a diagram indicating how the bits of a virtual address will be interpreted by the address translation mechanism. Indicate which bits (and how many) are used to index the page tables at each level, and which bits form the page offset for the case above.

Engineering and Technology

1. (a) A BCD seven segment decoder takes four inputs A,B,C,D and has seven outputs. The inputs represent the binary equivalent of an integer between 0 and 9. Each of the seven outputs a, b, c, d, e, f, and g corresponds to one of the seven LEDs in a seven-segment display as shown below.

(i) Give the truth table for segment g.

(ii) Draw its K-map and write minimum SOP form.



(b) Prove algebraically that the set of XOR and AND operations form a functionally complete set of Boolean operations.

2. (a) A perfectly spherical ice ball at rest at the top of a plane of length l, inclined at an angle α with the horizontal, is set to roll freely. If the rate of decrease of its volume V_t at any instance t is proportional to its surface at that instant, prove that $V_t = (V_0^{1/3} - k\sqrt{l})^3$, where V_0 is the initial volume of the ice ball and k is a constant. Assume that the ice ball remains spherical all through its downward journey and it suffers no friction.

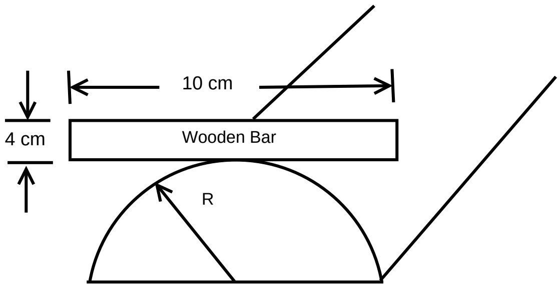
(b) A particle of mass m slides down from rest at the highest point of a smooth plane. The plane is inclined at an elevation θ fixed in an elevator which is going up with an acceleration a_0. The base of the inclined plane has a length b. Find the time taken by the particle to reach the bottom.

3. (a) A uniform rod of mass M and length l lies on a smooth horizontal plane. A particle of mass m moving on the plane with a speed v perpendicular to the length of the rod, strikes it at a distance $l/4$ from the centre and stops after the collision. Find

(i) the velocity of the centre of the rod, and

(ii) the angular velocity of the rod about its centre just after the collision.

(b) A homogeneous wooden bar of length $10\ cm$, thickness $4\ cm$ and weight $1\ Kg$ is balanced on the top of a semicircular cylinder of radius R as shown below. Calculate the minimum radius of the semicircular cylinder if the wooden bar is at stable equilibrium.

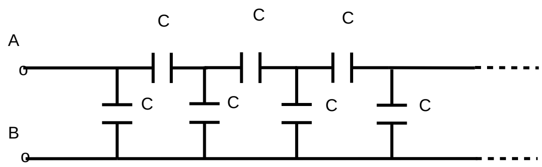


4. (a) The mean distance of Mars from the Sun is 1.5 times that of the Earth from the Sun. Find the time of revolution of Mars about the Sun with respect to that of the Earth about the Sun.

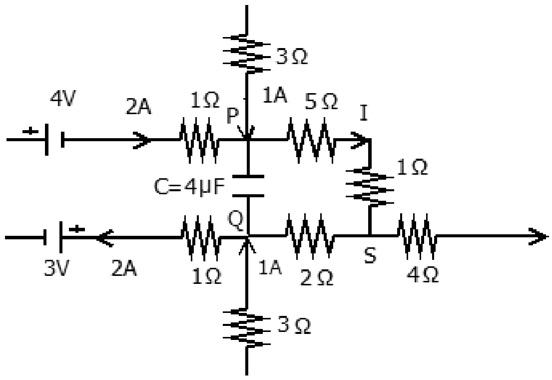
(b) A cyclic heat engine receives $300\ KJ$ from an energy reservoir at $900°K$. It rejects $100\ KJ$ to an energy reservoir at $300°K$. The machine produces $250\ KJ$ of work as output. Argue whether this cycle is reversible, irreversible or impossible.

(c) A heat engine has a solar collector receiving $0.2\ KW/m^2$, inside which a material is heated to $450°K$. The collected energy powers a heat engine which rejects heat at $52°C$. If the heat engine is supposed to deliver $2.5\ KW$, compute the minimum area of the solar collector.

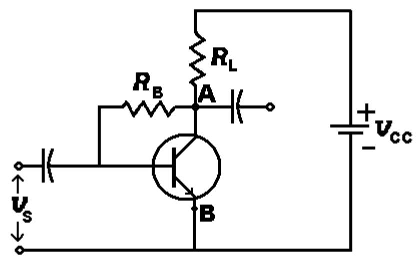
5. (a) Find the capacitance across terminals A and B of the infinite ladder shown below.



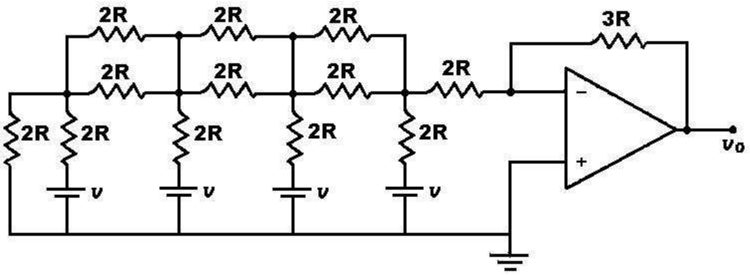
(b) Consider the portion of a circuit shown in the figure in steady state with the currents flowing in the branches having resistances as indicated. Calculate the energy stored in the capacitor C.



6. (a) An npn transistor shown in the following figure is used in common-emitter mode with $\beta = 49$, $V_{cc} = 10\ V$, and $R_L = 2\ K\Omega$. A 100 $K\Omega$ resistor R_B is connected between the collector and the base of the transistor. Calculate the quiescent collector current. Assume that the base to emitter voltage drop is $0.7\ V$.



(b) Consider the circuit shown in the following figure. Derive the output voltage v_0 for the circuit in terms of v and R.



7. A 7.92 KW, 220 V, 1000 $r.p.m.$ shunt motor has a full-load efficiency of 90%, an armature resistance of 0.1 Ω and a shunt field resistance of 110 Ω. The speed of this motor is reduced to 500 $r.p.m.$ by inserting an external resistance in series with the armature keeping the load torque constant.

   (i) Calculate the value of the external resistance, and the corresponding efficiency of the motor.

   (ii) Assuming that the constant loss is proportional to the armature current, re-calculate the efficiency of the motor.

8. Consider the following C function (recall that '`&`' denotes bit-wise AND, and '`>>n`' denotes right-shift by `n` bits):

```
int mystery(int x)
{
        int y = 0;

        while (x != 0)
        {
                if (x & 1) y++;
                x = x >> 1;
        }
        return y;
}
```

   (i) What does the function `mystery` compute when invoked on a positive integer `x`?

   (ii) What is the value of `y` returned by `mystery` when it is called as `mystery(65)`?

   (iii) How many times (denoted by k) is the `while` loop executed when called as `mystery(65)`?

   (iv) Can you modify the code to reduce k, for example to 2 for `x=65`? If yes, present the code for the new `mystery` function. If not, give reasons.