Test Code: QR (Short answer type) 2012

M.Tech. in Quality, Reliability and Operations Research

The candidates applying for M. Tech. in Quality, Reliability and Operations Research will have to take two tests: **Test MIII** (objective type) in the forenoon session and **Test QR** (short answer type) in the afternoon session.

For Test **MIII**, see a different Booklet. For Test **QR**, refer to this Booklet **ONLY.**

If you are from **Statistics / Mathematics Stream,** you will be required to **ANSWER PART I.**

If you are from **Engineering Stream**, you will be required to **ANSWER PART II.**

In **PART I**, a **TOTAL of TEN [10]** questions, are divided into **TWO Groups**: **S1**: **Statistics and S2: Probability** – **each group carrying FIVE [5]** questions. You will be required to answer a **TOTAL of SIX [6]** questions, taking **AT LEAST TWO [2]** from **each group**.

In **PART II**, there will be **SIX Groups: E1 to E6**. **E1** will contain **THREE [3]** questions from **Mathematics** and each other group will contain **TWO [2]** questions from **Engineering and Technology**. You will be required to answer a total of **SIX [6]** questions taking **AT LEAST TWO [2]** from group **E1**.

Syllabus

PART I: STATISTICS / MATHEMATICS STREAM

Statistics (S1)

- Descriptive statistics for univariate, bivariate and multivariate data.
- Standard univariate probability distributions [Binomial, Poisson, Normal] and their fittings, properties of distributions. Sampling distributions.
- Theory of estimation and tests of statistical hypotheses.
- Multiple linear regression and linear statistical models, ANOVA.
- Principles of experimental designs and basic designs [CRD, RBD & LSD].
- Elements of non-parametric inference.
- Elements of sequential tests.
- Sample surveys – simple random sampling with and without replacement, stratified and cluster sampling.

Probability (S2)

- Classical definition of probability and standard results on operations with events, conditional probability and independence.
- Distributions of discrete type [Bernoulli, Binomial, Multinomial, Hypergeometric, Poisson, Geometric and Negative Binomial] and continuous type [Uniform, Exponential, Normal, Gamma, Beta] random variables and their moments.
- Bivariate distributions (with special emphasis on bivariate normal), marginal and conditional distributions, correlation and regression.
- Multivariate distributions, marginal and conditional distributions, regression, independence, partial and multiple correlations.
- Order statistics [including distributions of extreme values and of sample range for uniform and exponential distributions].
- Distributions of functions of random variables.
- Multivariate normal distribution [density, marginal and conditional distributions, regression].
- Weak law of large numbers, central limit theorem.
- Basics of Markov chains and Poisson processes.

PART II: ENGINEERING STREAM

Mathematics (E1)

- Elementary theory of equations, inequalities.
- Elementary set theory, functions and relations, matrices, determinants, solutions of linear equations.
- Trigonometry [multiple and sub-multiple angles, inverse circular functions, identities, solutions of equations, properties of triangles].
- Coordinate geometry (two dimensions) [straight line, circle, parabola, ellipse and hyperbola], plane geometry, Mensuration.
- Sequences, series and their convergence and divergence, power series, limit and continuity of functions of one or more variables, differentiation and its applications, maxima and minima, integration, definite integrals areas using integrals, ordinary and partial differential equations (upto second order), complex numbers and De Moivre's theorem.

Engineering Mechanics (E2)

- Forces in plane and space, analysis of trusses, beams, columns, friction, principles of strength of materials, work-energy principle, moment of inertia, plane motion of rigid bodies, belt drivers, gearing.

Electrical and Electronics Engineering (E3)

- DC circuits, AC circuits (1-ϕ), energy and power relationships, Transformer, DC and AC machines, concepts of control theory and applications.
- Network analysis, 2 port network, transmission lines, elementary electronics (including amplifiers, oscillators, op-amp circuits), analog and digital electronic circuits.

Thermodynamics (E4)

- Laws of thermodynamics, internal energy, work and heat changes, reversible changes, adiabatic changes, heat of formation, combustion, reaction, solution and dilution, entropy and free energy and maximum work function, reversible cycle and its efficiency, principles of internal combustion engines. Principles of refrigeration.

Engineering Properties of Metals (E5)

- Structures of metals, tensile and torsional properties, hardness, impact properties, fatigue, creep, different mechanism of deformation.

Engineering Drawing (E6)

- Concept of projection, point projection, line projection, plan, elevation, sectional view (1^{st} angle / 3^{rd} angle) of simple mechanical objects, isometric view, dimensioning, sketch of machine parts.
  (Use of set square, compass and diagonal scale should suffice).

SAMPLE QUESTIONS

PART I: STATISTICS / MATHEMATICS STREAM

GROUP S1: Statistics

1. Let X_1 and X_2 be independent χ^2 variables, each with n degrees of freedom. Show that $\frac{\sqrt{n}\left(X_1 - X_2\right)}{2\sqrt{X_1 X_2}}$ has the t distribution with n degrees of freedom and is independent of $X_1 + X_2$.

2. Let [{x_i ; $i = 1, 2, \ldots, p$}; {y_j ; $j = 1,2,\ldots, q$}; {z_k ; $k = 1, 2, \ldots,r$}] represent random samples from $N(\alpha + \beta, \sigma^2)$, $N(\beta + \gamma, \sigma^2)$ and $N(\gamma + \alpha, \sigma^2)$ populations respectively. The populations are to be treated as independent.
   (a) Display the set of complete sufficient statistics for the parameters (α, β, γ, σ^2).
   (b) Find unbiased estimator for β based on the sample means only. Is it unique?
   (c) Show that the estimator in (b) is uncorrelated with all error functions.
   (d) Suggest an unbiased estimator for σ^2 with maximum d.f.
   (e) Suggest a test for H_0: $\beta = \beta_0$.

3. Consider the linear regression model: $y = \alpha + \beta x + e$ where e's are iid $N(0, \sigma^2)$.
   (a) Based on n pairs of observations on x and y, write down the least squares estimates for α and β.
   (b) Work out exact expression for Cov($\hat{\alpha}$, $\hat{\beta}$).
   (c) For a given y_0 as the "predicted" value, determine the corresponding predictand "x_0" and suggest an estimator "$\hat{x}_0$" for it.

4. A town has N taxis numbered 1 through N. A person standing on roadside notices the taxi numbers on n taxis that pass by. Let M_n be the largest number observed. Assuming independence of the taxi numbers and sampling with replacement, show that

   $$\hat{N} = (n + 1)\, M_n / n$$

   is an approximately unbiased estimator of N for large N.

5.(a) Let $x_1, x_2, \ldots, x_n$ be a random sample from the rectangular population with density

$$f(x) = \begin{cases} \dfrac{1}{\theta}, & 0 < x < \theta \\ 0, & otherwise \end{cases}$$

Consider the critical region $x_{(n)} > 0.8$ for testing the hypothesis $H_0 : \theta = 1$, where $x_{(n)}$ is the largest of $x_1, x_2, \ldots, x_n$. What is the associated probability of type I error and what is the power function?

(b) Let $x_1, x_2, \ldots, x_n$ be a random sample from a population having p.d.f.

$$f(x, \theta) = \begin{cases} \dfrac{\theta^3}{\Gamma(3)} e^{-\theta x} x^2, & 0 < x < \infty \\ 0, & otherwise \end{cases}$$

Obtain the maximum likelihood estimate of θ and also obtain the Cramer Rao lower bound to the variance of an unbiased estimator of θ.

6.(a) Give an example of a Latin Square Design of order 4 involving 4 rows, 4 columns and 4 treatments. Give the general definition of "treatment connectedness" in the context of a Latin Square Design and show that the Latin Square Design that you have given is indeed treatment connected.

(b) In a CRD set-up involving 5 treatments, the following computations were Made:

$n = 105$, Grand Mean = 23.5, SSB = 280.00, SSW = 3055.00

i. Compute the value of the F-ratio and examine the validity of the null hypothesis.

ii. It was subsequently pointed out that there was one additional treatment that was somehow missed out! For this treatment, we are given sample size = 20, Sum = 500 and Sum of Squares (corrected) = 560.00. Compute revised value of F-ratio and draw your conclusions.

7. If X_1, X_2, X_3 constitute a random sample from a Bernoulli population with mean p, show why $[X_1 + 2X2 + 3X_3] / 6$ is *not* a sufficient statistic for p.

8. If X and Y follow a trinomial distribution with parameters n, θ_1 and θ_2, show that

   (a) $E(Y / X = x) = \dfrac{(n-x)\theta_2}{1-\theta_1}$,

   (b) $V(Y / X = x) = \dfrac{(n-x)\theta_2(1-\theta_1-\theta_2)}{(1-\theta_1)^2}$

9. Life distributions of two independent components of a machine are known to be exponential with means μ and λ respectively. The machine fails if at least one of the components fails. Compute the chance that the machine will fail due to the second component. Out of n independent prototypes of the machine m of them fail due to the second component. Show that $m/(n-m)$ approximately estimates the odds ratio $\theta = \lambda/\mu$.

GROUP S2: Probability

1. A boy goes to his school either by bus or on foot. If one day he goes to the school by bus, then the probability that he goes by bus the next day is 7/10. If one day he walks to the school, then the probability that he goes by bus the next day is 2/5.

   (a) Given that he walks to the school on a particular Tuesday, find the probability that he will go to the school by bus on Thursday of that week.

   (b) Given that the boy walks to the school on both Tuesday and Thursday of that week, find the probability that he will also walk to the school on Wednesday.

   [You may assume that the boy will not be absent from the school on Wednesday or Thursday of that week.]

2. Suppose a young man is waiting for a young lady who is late. To amuse himself while waiting, he decides to take a random walk under the following set of rules:

   He tosses an imperfect coin for which the probability of getting a head is 0.55. For every head turned up, he walks 10 meters to the north and for every tail turned up, he walks 10 meters to the south.

   That way he has walked 100 meters.

   (a) What is the probability that he will be back to his starting position?

   (b) What is the probability that he will be 20 meters away from his starting position?

3. (a) A coin is tossed an odd number of times. If the probability of getting more heads than tails in these tosses is equal to the probability of getting more tails than heads then show that the coin is unbiased.

   (b) For successful operation of a machine, we need at least three components (out of five) to be in working phase. Their respective chances of failure are 7%, 4%, 2%, 8% and 12%. To start with, all the components are in working phase and the operation is initiated. Later it is observed that the machine has stopped but the first component is found to be in working phase. What is the likelihood that the second component is also in working phase?

   (c) A lot contains 20 items in which there are 2 or 3 defective items with probabilities 0.4 and 0.6 respectively. Items are tested one by one from the lot unless all the defective items are tested. What is the probability that the testing procedure will continue up to the twelfth attempt?

4.(a) Let S and T be distributed independently as exponential with means $1/\lambda$ and $1/\mu$ respectively. Let $U = \min\{S, T\}$ and $V = \max\{S, T\}$. Find $E(U)$ and $E(U+V)$.

   (b) Let X be a random variable with $U(0,1)$ distribution. Find the p.d.f. of the random variable $Y = X / (1 + X)$.

5. (a) Let U and V be independent and uniformly distributed random variables on [0, 1] and let θ_1 and θ_2 (both greater than 0) be constants.

Define $X = -\frac{1}{\theta_1}\ln U$ and $Y = -\frac{1}{\theta_2}\ln V$. Let $S = \min\{X, Y\}$, $T=max\{X ,Y\}$ and $R = T - S$.

(i) Find $P[S=X]$.

(ii) Show that S and R are independent.

(b) A sequence of random variables $\{X_n \mid n = 1, 2, \ldots\}$ is called a *martingale* if

(i) $E(|X_n|) < \infty$

(ii) $E(X_{n+1} \mid X_1, X_2, \ldots, X_n) = X_n$ for all $n = 1, 2, \ldots$

Let $\{Z_n \mid n = 1, 2, \ldots\}$ be a sequence of iid random variables with $P[Z_n = 1] = p$ and $P[Z_n = -1] = q = 1-p, 0 < p < 1$. Let $X_n = Z_1 + Z_2 + \ldots + Z_n$ for $n = 1, 2, \ldots$. Show that $\{X_n \mid n = 1, 2, \ldots\}$, is a martingale if and only if $p = q = ½$.

6. (a) Let X be a random variable with density

$$f_X(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

For the minimum $X_{(1)}$ of n iid random observations $X_1, X_2, \ldots, X_n$ from the above distribution, show that $n^{1/4} X_{(n)}$ converges in distribution to a random variable Y with density

$$f_Y(y) = \begin{cases} 4e^{-y^4} y^3, & y > 0 \\ 0, & otherwise \end{cases}$$

(b) A random sample of size n is taken from the exponential distribution having p.d.f.

$$f(x) = \begin{cases} e^{-x}, & 0 \le x < \infty \\ 0, & otherwise. \end{cases}$$

Find the p.d.f. of the sample range.

7. (a) In a recent study, a set of n randomly selected items is tested for presence of colour defect. Let A denote the event "colour defect is present" and B denote the event "test reveals the presence of colour defect". Suppose $P(A) = \alpha$, $P(B|A) = 1-\beta$ and $P(\text{Not } B | \text{ Not } A) = 1-\delta$, where $0 < \alpha, \beta, \delta < 1$. Let X be the number of items in the set with colour defects and Y be the number of items in the set detected having colour defects.

(i) Find $E(X | Y)$.

(ii) If the colour defect is very rare and the test is a very sophisticated one such that $\alpha = \beta = \delta = 10^{-9}$, then find the probability that an item detected as having colour defect is actually free from it.

(b) Consider the following bivariate density function

$$f(x, y) = \begin{cases} c \cdot xy, & x > 0, y > 0, x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(i) Find c.

(ii) Find the conditional expectation, $E(Y | X = x)$, for $0 < x < 1$.

8. Suppose in a big hotel there are N rooms with single occupancy and also suppose that there are N boarders. In a dinner party to celebrate the marriage anniversary of one of the boarders they start drinking alcohol to their hearts' content and as a consequence they become unable to identify their own rooms. What is the probability that at the end of the dinner party none of the boarders occupies the room originally assigned to them? What is the limiting value of this probability as $N \to \infty$?

9. (a) Consider a Markov Chain with state space $I = \{1, 2, 3, 4, 5, 6\}$ and transition probability matrix P given by

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/4 & 3/4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1/8 & 7/8 & 0 & 0 \\ 1/4 & 0 & 1/2 & 0 & 1/8 & 1/8 \\ 0 & 0 & 1/4 & 3/4 & 0 & 0 \end{bmatrix}$$

Find the various classes of this chain and classify them as recurrent or transient.

(B) Pulses arrive at a Geiger counter according to a Poisson Process with parameter $\lambda > 0$. The counter is held open only a random length of time T (independent of the arrival time of the pulses), where T is exponentially distributed with parameter $\beta > 0$. Find the distribution of N = Total number of pulses registered by the counter.

PART II: ENGINEERING STREAM

GROUP E1: Mathematics

1. (a) Let $f(x)$ be a polynomial in x and let a, b be two real numbers where $a \neq b$. Show that if $f(x)$ is divided by $(x - a)(x - b)$ then the remainder is

$$\frac{(x-a)f(b) - (x-b)f(a)}{b-a}.$$

(b) Find $\frac{dy}{dx}$ if $x^{\cos y} + y^{\sin x} = 1$.

2.(a) Let A be the fixed point (0,4) and B be a moving point ($2t$, 0). Let M be the mid-point of AB and let the perpendicular bisector of AB meets the y-axis at R. Find the equation of the locus of the mid-point P of MR.

(b) Inside a square ABCD with sides of length 12 cm, segment AE is drawn where E is the point on DC such that DE = 5 cm. The perpendicular bisector of AE is drawn and it intersects AE, AD and BC at the points M, P and Q respectively. Find the ratio PM : MQ.

3.(a) Evaluate the value of $3.9^{1/2}.27^{1/4}.81^{1/8}\ldots\ldots\infty$.

(b) Let *f* be a twice differentiable function such that

$$f''(x) = -f(x);\ f'(x) = g(x) \text{ and } h(x) = f^2(x) + g^2(x).$$

Given that $h(5) = 11$, find $h(10)$.

4.(a) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\cdots(upto\,[n/2]\,terms)\right]=\frac{1}{2}.$$

(c) Test the convergence of the series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\cdots\infty$.

Assume $x > 0$ and examine all possibilities.

5.(a) Find the limit of the following function as $x\to 0$.

$$\frac{|x|}{\sqrt{\left(x^4+4x^2+7\right)}}\sin\left(\frac{1}{3\sqrt{x}}\right).$$

(b) If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$ then show that $a \cdot b < 0$.

6.(a) If ω is a complex cube root of unity then show that

$$a^3+b^3+c^3-3abc=(a+b+c)\left(a+b\omega+c\omega^2\right)\left(a+b\omega^2+c\omega\right).$$

(b) Show that $\left[\frac{\sum_{r>s} x^r}{r!}\right]\div\left[\frac{\sum_{r>s} y^r}{r!}\right]>\frac{x^s}{y^s}$, whenever $x > y > 0$.

7.(a) Cable of a suspension bridge hangs in the form of a parabola and is attached to the supporting pillars 200 m apart. The lowest point of the cable is 40 m below the point of suspension. Find the angle between the cable and the supporting pillars. State all the assumptions involved.

(b) Let A, B and C be the angles of a triangle with angle C as the smallest of them. Show that

(i) $\sin\left(\frac{C}{2}\right) \le \frac{1}{2}$

(ii) Hence, or otherwise, show that $\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) < \frac{1}{4}$.

8. (a) Evaluate the following two integrals directly and compare them.

$$\iint_{ax^2+by^2\le 1} dxdy \qquad \text{and} \qquad \iint_{\sqrt{a}|x|\le 1,\sqrt{b}|y|\le 1} dxdy.$$

(b) Determine *x*, *y* and *z* so that the 3×3 matrix with the following row vectors is orthogonal : (1 / √3, 1 / √3, 1 / √3), (1/ √2, -1/ √2, 0), (*x, y, z*).

GROUP E2: Engineering Mechanics

1.(a) The simple planar truss in the given Fig.1 consists of two straight two-force members AB and BC that are pinned together at B. The truss is loaded by a downward force of P=12 KN acting on the pin at B. Determine the internal axial forces F_1 and F_2 in members AB and BC respectively. (Neglect the weight of the truss members).

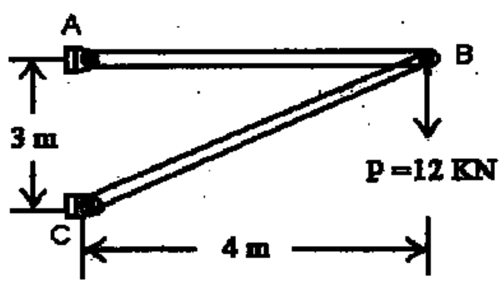


Fig. 1

(b) Derive the expression for moment of inertia I_{YY} of the shaded hollow rectangular section (Fig. 2).

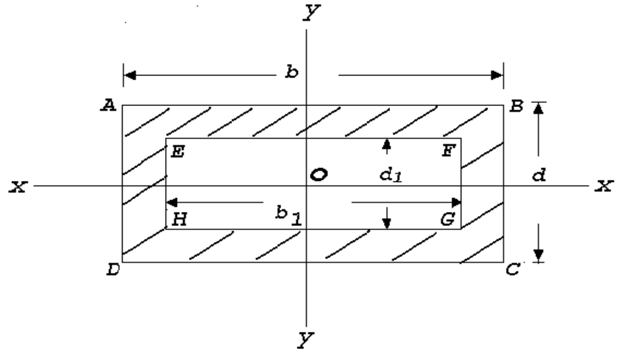


Fig. 2

2.(a) A turbine rotor weighs 20 tonnes and has a radius of gyration of 1.75 meter when running at 200 rpm. It is suddenly relieved of part of its load and its speed rises to 205 rpm in 1 sec. Find the unbalanced uniform turning moment.

(b) An Aluminium thin-walled tube (radius/thickness = 20) is closed at each end and pressurized by 6 MPa to cause plastic deformation. Neglect the elastic strain and find the plastic strain in the circumferential (hoop) direction of the tube. The plastic stress-strain curve is given by $\bar{\sigma} = 170(\textit{strain rate})^{0.25}$.

3.(a) A uniform ladder 5 m long and 14 kg mass is placed against a vertical wall at an angle 50^0 to the horizontal ground. The co-efficient of friction between ladder and wall is 0.2 and between ladder and ground in 0.5. Calculate how far up the ladder a man of 63 kg. can climb before the ladder shifts.

(b) Determine the diameter of a steel shaft rotating at an angular velocity of 300 rpm transmitting 500 HP. The allowable stress = 800 kg/cm^2. The allowable angle of twist = 0.5^0 per m, G = 8×10^5 kg/cm^2. What would be the savings if a hollow shaft is used to transmit the same power under the same condition, the ratio of diameters being 0.9?

4.(a) For the beam and loading shown in Fig.3, determine the equation defining the shear and bending moment at any point and at point D.

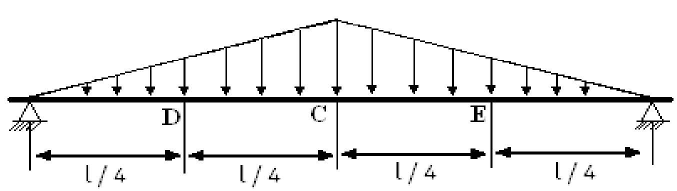


Fig. 3

(b) As illustrated in the given Fig.4 a metal punch (similar in principle to a paper punch) is used to punch holes in thin steel sheet that will be used to make a metal cabinet. To punch a 60 mm diameter disk or "slug" out of the sheet metal that is 10 mm thick requires a punch force of P=500 kg. Determine the average shear stress in the sheet metal resulting from the punching operation.

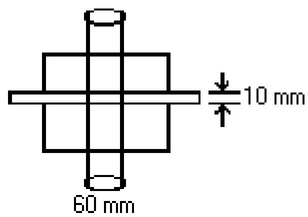


Fig. 4

5. (a) A tie rod in the suspension of a car is to be constructed from a grade of steel, which has 0.1% proof stress equal to 250 MN/m^2. The tie rod is to be constructed as a solid round bar of length 350 mm long. If the tie rod is subjected to a maximum axial force of 10 KN,

    (a) Determine the minimum diameter of the tie rod
    (b) The extension of the tie rod under load (E= 2094 GN/m^2)
    (c) The minimum diameter of the tie rod if a factor of safety of 2.5 is applied to the proof stress

   (b) Find the width of the belt necessary to transmit 11.25 KW power to a pulley of diameter 300mm when the pulley makes 1600 rpm. Assume the co-efficient of friction between the belt and the pulley is 0.22 and angle of contact is 210^0. Maximum tension in the belt will not exceed 10N/mm width.

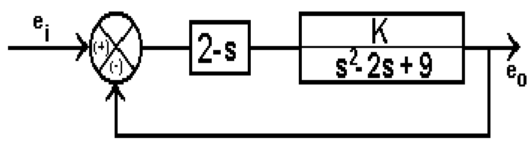
GROUP E3: Electrical and Electronics Engineering

1.(a) A centrifugal pump, which is gear-driven by a DC motor, delivers 810 kg of water per minute to a tank of height 11 meter above the level of the pump. Draw the block diagram of the overall arrangement. Determine input power across the gearing and current taken by the motor operated at 220 volt provided the efficiency of the pump, gearing and motor respectively be 70%, 70% and 90% only. (Take g = 9.8 ms^{-2}).

(b) The rms value of a sinusoidal alternating voltage at a frequency of 50 Hz is 155volt. If at t = 0 it crosses the zero axis in a positive direction, determine the time taken to attain the first instantaneous value of 155 volt. How much time it takes to fall from the maximum peak value to its half? Explain with suitable waveform.

2.(a) On full-load unity power factor test, a meter having specification of 235 V and 5A makes 60 revolutions in 6 minutes, but its normal speed is 520 revolution/KWh. Does the meter have any inaccuracy? If so, find the percentage error.

(b) Write down the transfer function of the given system (as shown in the following figure) and find the values of K for which the system will be stable but under damped.



3. (a) By intelligent selection of loop currents write down the mesh equations of the given circuit (as shown in Fig. 5) and determine the current flowing through that branch of the circuit containing capacitor. (All resistances/ reactance's are in ohms).

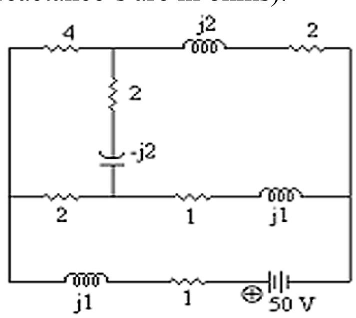


Fig. 5

(b)

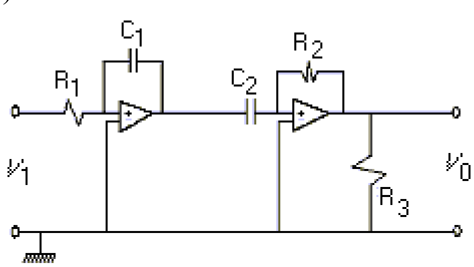


Fig. 6

Refer Fig. 6. Find the expression for V_0. What would be the nature of V_0 when $R_1 = R_2$ and $C_1 = C_2$? (Consider the Op-amps to be identical).

4. (a) A series ac circuit that resonates at 48 Hz consists of a coil (having R = 30 Ω and L = 500 mH) and a capacitor. If the supply voltage is 100 volt determine the value of the capacitor.

(b) Calculate the value of a capacitor which when connected across the circuit (as of Q. 4 (a) above), enhances the resonant frequency to 60 Hz. Compare the value of the source current in both the cases.

5. (a) A 200/400 - V, 10KVA, 50Hz single phase transformer has, at full load, a Cu loss of 120W. If it has an efficiency of 98% at full load unity power factor, determine the iron losses. What would be the efficiency of the transformer at half load 0.8 power factor lagging?

(b) In the 2-port network given below, the parameters at two parts are related by the equations,

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

(a) Find expressions for A, B, C and D
(b) Show that AD - BC = 1
(c) What are the physical interpretations of the above coefficients?

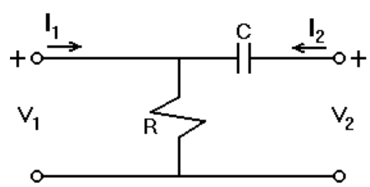


Fig 7

GROUP E4: Thermodynamics

1.(a) In a thermodynamic system of a perfect gas, let $U = f(V,T)$ where U, V and T refer to internal energy, volume of a gram-molecule of the substance and temperature (in absolute scale) respectively. An amount of heat δQ is added so that the volume expands by δV against a pressure P. Prove that:

$$C_P - C_V = \left[P + \left(\frac{\delta U}{\delta V} \right)_T \right] \left(\frac{\delta V}{\delta T} \right)_P$$

where C_p and C_V stand for specific heat at constant pressure and specific heat at constant volume respectively.

(b) 0.15 cu.m. of air at a pressure of 1.06 kg/cm^2 is compressed to a volume of 0.008 cu.m. at 361 kg/cm^2. Calculate (i) the quantity of heat rejected, (ii) change in internal energy if the process of compression is a) Adiabatic b) Polytropic with n = 1.3.

2.(a) A compression ignition engine has a stroke of 28 cm and a cylinder diameter of 18 cm. The clearance volume is 475 cm^3. The fuel injection takes place at constant pressure for 4.5% of the stroke. Find the air standard efficiency of the engine assuming that it works on diesel cycle. If the fuel injection takes place at 10% of the stroke, find the loss in air standard efficiency.

(b) A diesel engine has a compression ratio 14 to 1 and the fuel supply is cut off at 0.08 of the stroke. If the relative efficiency is 0.52, estimate the weight of fuel of a calorific value 10400 k.cal per kg that would be required per horsepower.

3.(a) Calculate the change in entropy of saturated steam at a given pressure such that the boiling point = 152.6 ^{o}C and the latent heat at this temperature = 503.6 cal/gm. [Use Log_e 1.56 = 0.445.]

(b) Draw the pv and $T-\Phi$ diagrams for a diesel cycle in which 1 kg of air at 1 kg / cm^2 and 90 ^{0}C is compressed through a ratio of 14 to 1. Heat is then added until the volume is 1.7 times the volume at the end of compression, after which the air expands adiabatically to its original volume. Take C_v = 0.169 and γ = 1.41.

4.(a) The approximated equation for adiabatic flow of super heated steam through a nozzle is given by $pv^n = \text{constant}.$ Show that

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{n/n+1}$$

where p_1 = pressure of steam at entry ; p_2 = pressure of steam at throat and p_2 / p_1 is the critical pressure ratio.

(b) The dry saturated steam is expanded in a nozzle from pressure of 10 bar to pressure of 4 bar. If the expansion is super saturated, find the degree of under cooling.

GROUP E5: Engineering Properties of Metals

1. (a) Distinguish between modulus of rigidity and modulus of rupture. Give an expression for the modulus of rigidity in terms of the specimen geometry, torque, and angle of twist. Is the expression valid beyond the yield strength (torsion)?

   (b) A steel bar is subjected to a fluctuating axial load that varies from a maximum of 340 kN to a minimum of 120 kN compression. The mechanical properties of the steel are σ_u = 1090 MPa, σ_0 = 1010 MPa and σ_e = 510 MPa. Determine the bar diameter to give infinite fatigue life based on a safety factor of 2.5

2 (a) A cylindrical bar is subjected to a torsional moment M_T at one end. The twisting moment is resisted by shear stress μ set up in the cross section of the bar. The shear stress is zero at the centre of the bar and increases linearly with the radius. Find the maximum shear stress at the surface of the bar.

Given $J = \dfrac{\pi D^4}{32}$ (assuming that the torsional deformation is restricted within the zone of elasticity)

where, J : Polar moment of inertia
D : Diameter of cylinder.

(b) Consider a flat plane containing a crack of elliptical cross-section. The length of the crack is $2c$ and stress is perpendicular to the major axis of the ellipse. Show that

$$\sigma = \sqrt{\frac{2\gamma E}{\pi c}}$$

σ : stress
γ : surface energy
E : Young's modulus of elasticity

3. (a) Consider a tension specimen, which is subjected to a total strain ε at an elevated temperature where creep can occur. The total strain remains constant and the elastic strain decreases. Show that

$$\frac{1}{\sigma^{n-1}} = \frac{1}{\sigma_o^{\,n-1}} + BE(n-1)t$$

where,

$\varepsilon = \varepsilon_e + \varepsilon_p$ — ε_e : elastic strain

$\varepsilon_e = \sigma / E$ — ε_p : plastic strain

$\frac{d\varepsilon_p}{dt} = B\sigma^n$ — t : time

$\sigma = \sigma_o$ at $t = 0$.

(b) Distinguish between slip and twinning with diagrams.

4. (a) Suppose a crystalline material has *fcc* structure with atomic radius of 1.278 Å Determine the density of the crystalline material. Assume number of atoms per unit cell and molecular weight are *n* and *M* gm respectively.

(b) Suppose there is an electron in an electric field of intensity 3200 volts/m. Estimate the force experienced by the electron. If it moves through a potential difference of 100 volts, find the kinetic energy acquired by the electron.

GROUP E6: Engineering Drawing

1.(a) A hollow cube of 5cm side is lying on HP and one of its vertical face is touching VP. A slim rod, to be taken as its solid diagonal, is placed within it. Draw top and front / side views of solid diagonal and, from the drawn figure determine its true length.

(b) Two balls are vertically erected to 18cm and 30 cm respectively above the flat ground. These balls are away from a 3 cm thick wall (on the ground) by 12 cm and 21 cm respectively but on either side of the wall. The distance between the balls, measured along the ground and parallel to the wall is 27 cm. Determine their approximate distance.

2. (a) Sketch the profile of a square thread, knuckle thread and a white-worth thread showing all relevant dimensions in terms of the pitch.

(b) Sketch:

(a) single riveted lap joint,
(b) double riveted lap joint chain-riveting,
(c) double riveted lap joint zigzag-riveting, and
(d) single cover single riveted butt joint.

3.(a) Draw the isometric view of an octahedron erected vertically up on one of its vertices. (Distinct free hand sketch only.)

(b) You are given two square prisms of same height of 10cm. Prism A has side 7cm and prism B has side of 5cm respectively. Longer face of B is lying on H.P. with its base perpendicular to V.P. Base of A is lying on H.P. but equally inclined to V.P. You are instructed to remove by cutting a portion of bottom base of A so that within the cavity maximum of B may be placed accordingly. Note that vertical face of B may be parallel to V.P. but just touch the central axis of A. Draw the sectional view of the combination and determine the volume of material to be removed from A.

4. A parallelepiped of dimension 100×60×80 is truncated by a plane which passes through 85, 45 and 65 unit distance on the associated edges from the nearest top point of the object. Draw the isometric view of the truncated solid object. In third angle projection method, draw its plan. (All dimensions are in mm).

Note: A copy of one of the previous year's TEST CODE: QR Question paper is appended in the following pages to give the candidate a rough idea.

BOOKLET No. TEST CODE: QR

Afternoon

Time: 2 hours

Group	Questions		Maximum marks
	Total	To be answered	
Part I (for Statistics/Mathematics Stream)			
S1 (Statistics)	5	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM EACH GROUP.	120
S2 (Probability)	5		
Part II (for Engineering Stream)			
E1 (Mathematics)	3	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM E1	120
E2 (Engineering Mechanics)	2		
E3 (Electrical and Electronics Engineering)	2		
E4 (Thermodynamics)	2		
E5 (Engineering Properties of Metals)	2		
E6 (Engineering Drawing)	2		

On the answer-booklet write your Name, Registration Number, Test Code, Number of this Booklet, etc. in the appropriate places.

There are two parts in this booklet as detailed above. Candidates having **statistics background** are required to answer questions from **Part I** only as per instructions given. Those having **engineering background** are required to answer questions from **Part II** only as per instructions given.

USE OF CALCULATORS IS NOT ALLOWED.

STOP! WAIT FOR THE SIGNAL TO START

PART I (FOR STATISTICS / MATHEMATICS STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS, TAKING AT LEAST TWO [2] FROM EACH GROUP.

GROUP S1

Statistics

1. a) A neighbourhood expresses frequent concerns about the dangers of a traffic intersection. Over the last two years there have been 16 accidents at this uncontrolled intersection. The municipality has finally responded and has put up stop signs on each of the four roads that enter the problematic intersection. After one year it was noted that there has been three accidents. From this data would you conclude that the stop signs have really helped in reducing the rate of accidents? Explain stating your assumptions clearly.

b) In a software development organization the project value (X) and productivity (Y) are known to be related. You have heard that the joint density of these two variables could probably be given by:

$$f(x, y) = \begin{cases} x\, exp(-x(1+y)) & x, y > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Note that the value of the project can be well approximated before execution. The organization wants to develop a regression equation of Y on X to get an idea about the productivity on the basis of the value of a project. The organization wanted to develop the equation empirically and accordingly paired observations on (x, y) were collected for a number of completed projects. Do you think that the values of X and / or Y needs to be transformed before fitting the equation? If yes, what is the transformation? Explain.

[10 +10=20]

2. Let X follow Poisson (λ). Suppose we have funds to collect only one observation. Find the uniformly minimum variance unbiased estimator of $h(\lambda) = e^{-\lambda}$. Does the UMVUE attain the Crammer-Rao lower bound?

[12+8=20]

3. In order to estimate the total employment in a group of N factories, m large factories are definitely selected. Let $y_1, y_2, \cdots, y_m$ represent the number of employees in these m factories. In addition, a simple random sample, *without replacement*, of $(n - m)$ factories is selected from the remaining $(N - m)$ factories and let y_i denote the number of employees in the ith factory, $i = m+1, m+2, \cdots, n$. The following two estimators of total employment are proposed:

$$t_1 = \frac{N}{n}\sum_{i=1}^{n} y_i$$
$$t_2 = \sum_{i=1}^{m} y_i + \frac{N-m}{n-m}\sum_{i=m+1}^{n} y_i.$$

Check whether t_1 and t_2 are unbiased for the population total Y. Also calculate the relative efficiency of t_1 as compared to that of t_2 as an estimator of Y.

[20]

4. Consider a completely randomized design on three treatments (coded 1, 2, 3). Let the linear model, under the usual Gauss Markoff setup, for analysis of the associated response be:

$$E(y_{ij}) = \mu + \tau_i \,;\; j = 1, 2, \cdots, n_i \,;\; i = 1, 2, 3.$$

a) Determine the condition of estimability of the parametric function $l_0\mu + \sum_{i=1}^{3} l_i\tau_i$, where l_0, l_1, l_2, l_3 are known constants. Hence, or otherwise, determine which of the following parametric functions are estimable:

i) $\tau_1 + 2\tau_2 - 3\tau_3$
ii) $\mu - \tau_2$
iii) $\mu - (\tau_1 + \tau_2 + \tau_3)/3$
iv) $\mu + (\tau_1 - \tau_3)/2 + \tau_2$

b) For those functions in part (a), that are estimable, obtain the best linear unbiased estimator.

c) Compute the variance of any one of the estimators that you may have obtained in part (b) above.

d) How do you propose to test the hypothesis H_0: $\tau_1 = \tau_2$? [Explicit derivation of the test statistic is not necessary; simply provide the outlines of your argument.]

[6+6+4+4=20]

5. a) There is a belief that the defect density of software exhibits an increasing trend as the size increases. In order to check this belief a software development company collected data on the size and defect density for 10 large software applications. The data are given below:

Sl. No.	Size	Defect Density	Sl. No.	Size	Defect Density
1	425	4.1	6	350	3.4
2	310	2.6	7	172	1.9
3	187	1.8	8	470	3.8
4	480	3.9	9	270	2.1
5	125	1.7	10	225	2.2

Do you think there is enough evidence of a trend of increasing defect density as size increases? Answer without fitting a regression line and without using any statistical table.

b) In an industry there is a belief that the brittleness of a particular product depends on the level of moisture of the raw material. In particular some technologists believe that when the moisture is beyond a threshold, say x_0, the chance of brittleness increases significantly; and in fact, they expect the brittleness to be generally higher than y_0. They also believe that when moisture is lower than x_0 brittleness is generally expected to be $\leq y_0$. In order to test this claim and also to estimate the chances of brittleness the technologists planned to carry out a study. They collected 100 items with brittleness higher than y_0 and another 100 items with brittleness $\leq y_0$ and the moisture content of the items were measured. The results are given below:

Moisture	Brittleness		Total
	$\leq y_0$	$> y_0$	
$\leq x_0$	60	20	80
$> x_0$	40	80	120
Total	100	100	200

From this data the technologists claim that the estimated chances of higher brittleness given higher and lower moistures are 0.67 and 0.25 respectively leading to an estimated relative risk of 2.67 of getting higher brittleness when the moisture increases. Do you agree with this conclusion? Would your conclusions, including the estimated relative risk, change if you knew that in the production process the estimated chances of moisture > x_0 is 0.1 and brittleness > y_0 is 0.40?

[8+12=20]

GROUP S2
Probability

6. a) Two fair dice are thrown independently. Three events *A*, *B* and *C* are defined as follows:

A: Odd face with first die
B: Odd face with second die
C: Sum of points on two dice is odd

Compute the probabilities of the events *A*, *B* and *C*. Hence, verify whether the events *A*, *B* and *C* are (*i*) Pairwise independent, (*ii*) Mutually independent.

b) Prove that the geometric mean *G* of the distribution

$$dF = 6\,(2 - x)\,(x - 1)\,dx,\ 1 \le x \le 2$$

is given by $6\,log(16\,G) = 19$.

[8+12=20]

7. a) Consider a sequence of Bernoulli trials with probability of success *p*, $(0 < p < 1)$ and of failure *q*, $(p + q = 1)$. Let p_n be the probability that an odd number of successes is obtained in a sequence of n such independent Bernoulli trials. Show that p_n satisfies the difference equation $p_n + (2p - 1)p_{n-1} = p$, and hence obtain an explicit expression for p_n.

b) A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes in these 600 throws.

[10+10=20]

8. The number of automobile accidents a driver will be involved in during a one-year period is modeled using a Poisson distribution with parameter $\lambda > 0$, where λ depends on the skill of the driver. Suppose that the skill of the driver can be adequately modeled by a gamma distribution with scale parameter β and shape parameter α. Assume that α is an integer.

a) Find the unconditional distribution of the number of accidents during a one-year period.
b) Show that the unconditional distribution obtained in (a) above is essentially a negative binomial distribution.
c) Hence, or otherwise, find the expected number of accidents during a one-year period.

[8+7+5=20]

9. A manufacturer sells a bottle of mineral water at a fixed price of `10. If the volume of water in the bottle is less than 800 ml then he is unable to sell it and it represents a total loss. The filled bottles have a normally distributed volume with mean $\mu$ ml and standard deviation 100 ml. The cost of filling per bottle is `c, where c = 0.002 μ + 1. Determine the mean volume μ which will maximize the expected profit of the manufacturer. [Use $\sqrt{-\ln(0.0003\pi)} = 2.447$.]

[20]

10. Let $\{N(t) \mid t \geq 0\}$ be a Poisson process with parameter $\lambda > 0$, where $N(t)$ is the number of events $\mathcal{E}$ that have occurred in a time interval of length t. Let $T_1, T_2, T_3, \cdots$ be the sequence of successive inter-event times. Define $T = T_1 + T_2 + \cdots + T_k$, where $k \in \{1, 2, ..., n\}$. Find the conditional distribution of T given $N(1) = n$.

[20]

PART II (FOR ENGINEERING STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS TAKING AT LEAST TWO [2] FROM E1.

GROUP E1
Mathematics

1. a) Prove that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & x \end{vmatrix}.$$

b) Solve the differential equation

$$y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}.$$

[10+10=20]

2. a) A point *F* is taken on the side *AD* of a square *ABCD*. At *C*, a perpendicular is drawn to *CF*, which meets extended *AB* at *E*. The area of the square *ABCD* is 256 square units and the area of the triangle *CEF* is 200 square units. Find the length of *BE*.

b) Suppose *P* is a point in the first quadrant of the two-dimensional Euclidean plane. Among all straight lines that pass through *P* and form a triangle with the positive semi-axes in the first quadrant, find the equation of the line for which the area of the triangle thus formed is least.

[10+10=20]

3. a) Given: $f(x,y) = \begin{cases} xy\,\dfrac{x^2-y^2}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0, & x^2+y^2 = 0 \end{cases}.$

Prove, using the definition of limit of a function, that the given function $f(x,y)$ is continuous at the origin (0,0).

b) Given: $(x+y)^{m+n} = x^m y^n$; prove that $\frac{dy}{dx}$ is independent of m and n.

c) Prove, with the help of first order derivative and using the monotonicity property of a function, that $\frac{2x}{(1-x^2)} > \log\frac{1+x}{1-x}$, where $0 < x < 1$.

[10+5+5=20]

GROUP E2
Engineering Mechanics

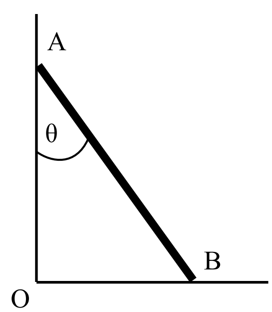
4. a) If I_{XX} and I_{YY} be the moments of inertia of a plane section about two perpendicular axes meeting at O, the moment of inertia I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by

$$I_{ZZ} = I_{XX} + I_{YY}$$

b) A screw jack has a thread of 12 mm pitch. What effort needs to be applied at the end of a handle of 450 mm to lift a load of 2.5 kN, if the corresponding efficiency is 50%?

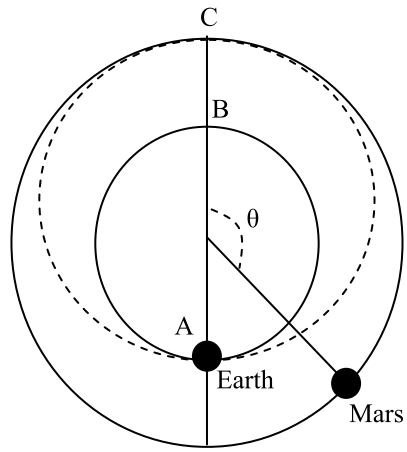
[10+10=20]

5. a) A ladder AB of length L and mass W leans against a wall as shown below.



Assume the coefficient of static friction μ_s is the same at both the surfaces of contact. Determine the smallest value of μ_s for which the equilibrium will be maintained.

b)



The figure above shows the orbits of the Earth, the Mars and of a spaceship launched from the Earth (the dotted circle). In reality all the orbits are elliptical. But for the sake of simplicity, these are assumed to be circular. The radius of the orbit and the orbital period of the earth are 1 AU (Astronomical Unit) and 1 year respectively. Assume that the radius and the period of the Mars's orbit are 1.52 AU and 1.88 year respectively.

Find the location of the Mars defined by θ at which the spaceship should be launched so that it meets the Mars at C. Use Kepler's third law, which states that for all objects orbiting the Sun we have $T^2 = r^3$, where T is the orbital period in years and r is the radius of the orbit in AU. Also state the reasons for not launching the spaceship from B when the Mars is at C, even though the distance between the Earth and the Mars is the least at such point.

[10+10=20]

GROUP E3
Electrical & Electronics Engineering

6. Three electronic gadgets, made in USA, are allowed to operate in series combination across the line voltage of 110 Volt at 60 Hz. In this condition, the individual impedances are (6+j6), (4-j5) and (2+j12) Ohms respectively. These three gadgets have been brought to India with an intention to operate in series across the line voltage of 225 Volt at 50 Hz.

a) Determine the equivalent impedance, the magnitude and direction of series current with respect to the supply voltage (225 Volt, 50 Hz).

b) To make the overall supply current 'purely real', a passive electronic component has to be attached to this combination. Discuss the most suitable mode of attachment with complete circuit diagram out of the different possible ways of attachments. Also determine the nature and value of the requisite electronic component to be used.

[8+12=20]

7. a) A six-pole armature is wound with 600 conductors. The magnetic flux and the speed are such that the average emf generated in each conductor is 2.3 V. Calculate the terminal voltage on no-load when the armature is (i) lap-connected and (ii) wave-connected.

b) Write the expression for efficiency of a transformer and hence establish the condition for maximum efficiency. Define all day efficiency of a transformer. What is its practical significance?

c) Write the torque equation for a three-phase induction motor and predict the nature of torque-slip characteristic of the motor. Show starting torque and slip at which torque is maximized on the T-s curve.

[6+8+6=20]

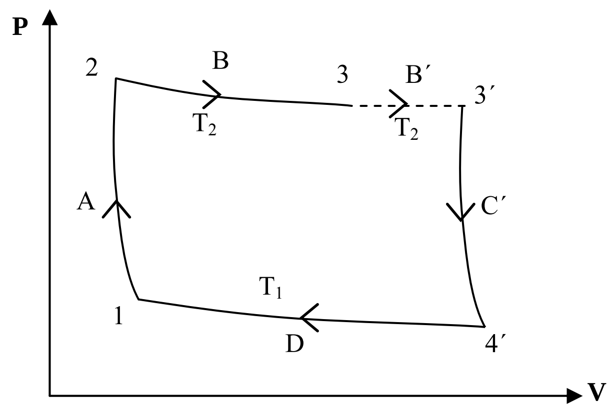
GROUP E4
Thermodynamics

8. a) From the first law of thermodynamics, derive the expressions for the change of entropy during isochoric process, isobaric process, isothermal process, and reversible adiabatic process.

b) A mass of m_1 kg of a certain perfect gas at a temperature $T_1{}^0K$ is mixed at constant pressure with m_2 kg of mass of the same gas at a temperature $T_2{}^0K$ ($T_1 > T_2$). The system is thermally insulated. Find the change in entropy of the mixture and deduce the same for equal masses of the gas. Show that the change in entropy for equal masses of the gas is necessarily positive.

[10+10=20]

9\. a) Consider the modified Carnot cycle with an irreversible step (isothermal free expansion) B′ as shown in the figure below.



Write the expressions for heat absorbed, work done and change in entropy at each of the five steps of the cycle and hence derive the expressions for the change in entropy of the universe and the efficiency of the cycle. Offer your comments on the results obtained.

b) Calculate the coefficient of thermal expansion α and the isothermal compressibility β for a van der Waals gas.

[12+8=20]

GROUP E5
Engineering Properties of Metals

10\. a) "Resilience of a material is equal to the area under the elastic part of the stress –strain curve, whereas, Toughness of a material is equal to the area under both elastic and plastic part of the stress-strain curve". If you agree, please find the expression for the above two areas for ductile materials. Describe an application area where these two properties of metals are highly relevant.

b) "Torsion test provides a more fundamental measure of the plasticity of a metal than the tension test". Is it correct? Justify.

c) “The basic equation of fracture toughness illustrates the design trade-off that is inherent in fracture mechanics design”. Explain the kind of trade-off that is possible giving the equation.

d) “Fatigue strength is seriously reduced by the introduction of a stress raiser such as, a hole or a notch.” How? How do you measure the notch sensitivity of a material in fatigue?

[4×5=20]

11. a) Find the condition of instability in (i) tension and (ii) torsion

b) Divide the entire fatigue process into the relevant stages based on the structural changes that take place when a metal is subjected to cyclic stress.

c) The transition temperature behavior of a wide spectrum of materials falls into three categories. Briefly describe these categories with figures.

[10+5+5=20]

GROUP E6
Engineering Drawing

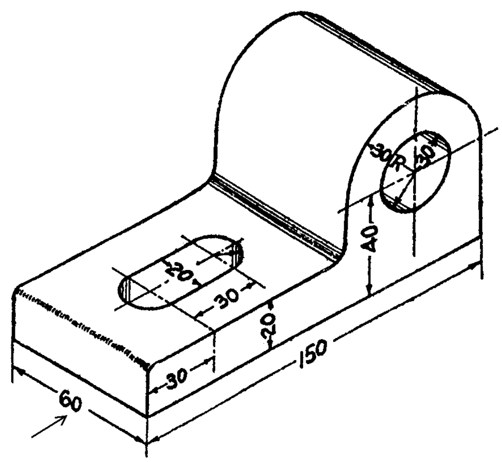
12. A cube of 10 cm is placed on H.P. and two of its adjoining side surfaces are equally inclined to V.P. There is a hollow cylindrical cavity centrally spaced along the axis of solid diagonal of the cube. The height and the radius of the cylindrical cavity are 10 cm and 1 cm respectively.

(a) Draw the front view, side view and top view of the composite cube mentioning the angle of projection.

(b) Draw the isometric view of the composite cube.

[12 + 8 = 20]

13. (a) Sketch the elevation, plan and left-side view of the given object mentioning the dimensions and angle of projection (First angle / Third angle projection).



All dimensions are in mm

(b) Draw a Knuckle Joint used for fastening together two or more rods subjected to tensile or compressive stress.

[12 + 8 = 20]