Test Code : QR (Short answer type) 2007

M.Tech. in Quality, Reliability and Operations Research

The candidates applying for M.Tech. in Quality, Reliability and Operations Research will have to take two tests : **Test MIII** (objective type) in the forenoon session and **Test QR** (short answer type) in the afternoon session.

For **Test MIII**, see a different Booklet. For **Test QR**, refer to this Booklet **ONLY.**

If you are from **Statistics / Mathematics Stream,** you will be required to **ANSWER PART I.**
If you are from **Engineering Stream**, you will be required to **ANSWER PART II.**

In **PART I**, a **TOTAL of TEN [10]** questions, are divided into **TWO Groups** : **S1**: **Statistics and S2: Probability – each group carrying FIVE [5]** questions. You will be required to answer a **TOTAL of SIX [6]** questions, taking **AT LEAST TWO [2]** from **each group**.

In **PART II**, there will be **SIX Groups: E1-E6**. **E1** will contain **THREE [3]** questions from **Engineering Mathematics** and each other group will contain **TWO [2]** questions from **Engineering and Technology**. You will be required to answer a total of **SIX [6]** questions taking **AT LEAST TWO [2]** from group **E1**.

Syllabus

PART I : STATISTICS / MATHEMATICS STREAM

Statistics (S1)

Descriptive statistics for univariate, bivariate and multivariate data.
Standard univariate probability distributions [Binomial, Poisson, Normal] and their fittings, properties of distributions. Sampling distributions.
Theory of estimation and tests of statistical hypotheses.
Multiple linear regression and linear statistical models, ANOVA.
Principles of experimental designs and basic designs [CRD, RBD & LSD].
Elements of non-parametric inference.
Elements of sequential tests.
Sample surveys – simple random sampling with and without replacement, stratified and cluster sampling.

Probability (S2)

Classical definition of probability and standard results on operations with events, conditional probability and independence.
Distributions of discrete type [Bernoulli, Binomial, Multinomial, Hypergeometric, Poisson, Geometric and Negative Binomial] and continuous type [Uniform, Exponential, Normal, Gamma, Beta] random variables and their moments.
Bivariate distributions (with special emphasis on bivariate normal), marginal and conditional distributions, correlation and regression.
Multivariate distributions, marginal and conditional distributions, regression, independence, partial and multiple correlations.
Order statistics [including distributions of extreme values and of sample range for uniform and exponential distributions].
Distributions of functions of random variables.
Multivariate normal distribution [density, marginal and conditional distributions, regression].
Weak law of large numbers, central limit theorem.
Basics of Markov chains and Poisson processes.

Syllabus

PART II : ENGINEERING STREAM

Mathematics (E1)

Elementary theory of equations, inequalities.

Elementary set theory, functions and relations, matrices, determinants, solutions of linear equations.

Trigonometry [multiple and sub-multiple angles, inverse circular functions, identities, solutions of equations, properties of triangles].

Coordinate geometry (two dimensions) [straight line, circle, parabola, ellipse and hyperbola], plane geometry, Mensuration.

Sequences, series and their convergence and divergence, power series, limit and continuity of functions of one or more variables, differentiation and its applications, maxima and minima, integration, definite integrals areas using integrals, ordinary and partial differential equations (upto second order), complex numbers and De Moivre's theorem.

Engineering Mechanics (E2)

Forces in plane and space, analysis of trusses, beams, columns, friction, principles of strength of materials, work-energy principle, moment of inertia, plane motion of rigid bodies, belt drivers, gearing.

Electrical and Electronics Engineering (E3)

D.C. circuits, AC circuits (1-ϕ), energy and power relationships, Transformer, DC and AC machines, concepts of control theory and applications.

Network analysis, 2 port network, transmission lines, elementary electronics (including amplifiers, oscillators, op-amp circuits), analog and digital electronic circuits.

Theromodynamics (E4)

Laws of thermodynamics, internal energy, work and heat changes, reversible changes, adiabatic changes, heat of formation, combustion, reaction, solution and dilution, entropy and free energy and maximum work function, reversible cycle and its efficiency, principles of internal combustion engines. Principles of refrigeration.

Engineering Properties of Metals (E5)

Structures of metals, tensile and torsional properties, hardness, impact properties, fatigue, creep, different mechanism of deformation.

Engineering Drawing (E6)

Concept of projection, point projection, line projection, plan, elevation, sectional view (1^{st} angle/3^{rd} angle) of simple mechanical objects, isometric view, dimensioning, sketch of machine parts.
(Use of set square, compass and diagonal scale should suffice).

SAMPLE QUESTIONS

PART I : STATISTICS / MATHEMATICS STREAM

GROUP S–1 : Statistics

1. Denote by $\phi(z)$ and $\Phi(z)$ the standard normal pdf and cdf respectively. Let Z be a random variable defined over the real line with pdf

   $f_\lambda(z) = c\,\phi(z)\,\Phi(\lambda z)$ where λ is a fixed constant, $-\infty < \lambda < \infty$.

   (a) Show that c = 2.
   (b) Show that $|Z|$ is CHI with 1 df.
   (c) Show that $E(Z) = \sqrt{[2/\pi]}\,\psi(\lambda)$ where $\psi(\lambda) = \lambda / \sqrt{[1+\lambda]^2}$.
   (d) Find the mode of the distribution of Z.

2. Let $[\{x_i ; i = 1, 2, \ldots, p\}; \{y_j ; j = 1,2,\ldots, q\}; \{z_k ; k = 1, 2, \ldots,r\}]$ represent random samples from $N(\alpha + \beta, \sigma^2)$, $N(\beta + \gamma, \sigma^2)$ and $N(\gamma + \alpha, \sigma^2)$ populations respectively.The populations are to be treated as independent.

   (a) Display the set of complete sufficient statistics for the parameters $(\alpha, \beta, \gamma, \sigma^2)$.
   (b) Find unbiased estimator for β based on the sample means only. Is it unique?
   (c) Show that the estimator in (b) is uncorrelated with all error functions.
   (d) Suggest an unbiased estimator for σ^2 with maximum d.f.
   (e) Suggest a test for $H_0 : \beta = \beta_0$.

3. Consider the linear regression model : $y = \alpha + \beta x + e$ where e's are iid $N(0, \sigma^2)$.

   (a) Based on n pairs of observations on x and y, write down the least squares estimates for α and β.
   (b) Work out exact expression for $Cov(\hat{\alpha}, \hat{\beta})$.
   (c) For a given y_0 as the "predicted" value, determine the corresponding predictand "x_0" and suggest an estimator "$\hat{x}_0$" for it.

4. A town has *N* taxis numbered 1 through *N*. A person standing on roadside notices the taxi numbers on *n* taxis that pass by. Let M_n be the largest number observed. Assuming independence of the taxi numbers and sampling with replacement, show that

   $$\hat{N} = (n + 1)\, M_n / n$$

   is an approximately unbiased estimator of *N* for large *N*.

5.(a) Let $x_1, x_2, \ldots, x_n$ be a random sample from the rectangular population with density

$$f(x) = \begin{cases} 1/\theta, & 0 < x < \theta \\ 0 & \text{otherwise} \end{cases}$$

Consider the critical region $x_{(n)} > 0.8$ for testing the hypothesis $H_0 : \theta = 1$, where $x_{(n)}$ is the largest of $x_1, x_2, \ldots, x_n$. What is the associated probability of error I and what is the power function?

(b) Let $x_1, x_2, \ldots, x_n$ be a random sample from a population having p.d.f.

$$f(x, \theta) = \begin{cases} \dfrac{\theta^3}{\Gamma(3)} e^{-\theta x} x^2, & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Obtain the maximum likelihood estimate of θ and also obtain the Cramer Rao lower bound to the variance of an unbiased estimator of θ.

6.(a) Give an example of a Latin Square Design of order 4 involving 4 rows, 4 columns and 4 treatments. Give the general definition of "treatment connectedness" in the context of a Latin Square Design and show that the Latin Square Design that you have given is indeed treatment connected.

(b) In a CRD set-up involving 5 treatments, the following computations were made:

$n = 105$, Grand Mean = 23.5, SSB = 280.00, SSW = 3055.00

(i) Compute the value of the F-ratio and examine the validity of the null hypothesis.

(ii) It was subsequently pointed out that there was one additional treatment that was somehow missed out! For this treatment, we are given sample size = 20, Sum = 500 and Sum of Squares (corrected) = 560.00.Compute **revised** value of F-ratio and draw your conclusions.

7. If X_1, X_2, X_3 constitute a random sample from a Bernoulli population with mean p, show why $[X_1 + 2X_2 + 3X_3] / 6$ is *not* a sufficient statistic for p.

8. If X and Y follow a trinomial distribution with parameters n, θ_1 and θ_2, show

that

(a) $E(Y/X=x)=\dfrac{(n-x)\theta_2}{1-\theta_1}$,

(b) $V(Y/X=x)=\dfrac{(n-x)\theta_2(1-\theta_1-\theta_2)}{(1-\theta_1)^2}$

Further show, in standard notations,

(c) $V_1E_2=\dfrac{n\theta_1\theta_2^2}{1-\theta_1}$, (d) $E_1V_2=\dfrac{n\theta_2(1-\theta_1-\theta_2)}{1-\theta_1}$,

(e) $V(Y)=n\theta_2(1-\theta_2)$

9. Life distributions of two independent components of a machine are known to be exponential with means μ and λ respectively. The machine fails if at least one of the components fails. Compute the chance that the machine will fail due to the second component. Out of *n* independent prototypes of the machine *m* of them fail due to the second component. Show that $m/(n-m)$ approximately estimates the odds ratio $\theta=\lambda/\mu$.

GROUP S–2 : Probability

1. A boy goes to his school either by bus or on foot. If one day he goes to the school by bus, then the probability that he goes by bus the next day is 7/10. If one day he walks to the school, then the probability that he goes by bus the next day is 2/5.

   (a) Given that he walks to the school on a particular Tuesday, find the probability that he will go to the school by bus on Thursday of that week.

   (b) Given that the boy walks to the school on both Tuesday and Thursday of that week, find the probability that he will also walk to the school on Wednesday.

   [You may assume that the boy will not be absent from the school on Wednesday or Thursday of that week.]

2. Suppose a young man is waiting for a young lady who is late. To amuse himself while waiting, he decides to take a random walk under the following set of rules:

He tosses an imperfect coin for which the probability of getting a head is 0.55. For every head turned up, he walks 10 yards to the north and for every tail turned up, he walks 10 yards to the south.

That way he has walked 100 yards.

(a) What is the probability that he will be back to his starting position?

(b) What is the probability that he will be 20 yards away from his starting position?

3. (a) A coin is tossed an odd number of times. If the probability of getting more heads than tails in these tosses is equal to the probability of getting more tails than heads then show that the coin is unbiased.

(b) For successful operation of a machine, we need at least three components (out of five) to be in working phase. Their respective chances of failure are 7%, 4%, 2%, 8% and 12%. To start with, all the components are in working phase and the operation is initiated. Later it is observed that the machine has stopped but the first component is found to be in working phase. What is the likelihood that the second component is also in working phase?

(c) A lot contains 20 items in which there are 2 or 3 defective items with probabilities 0.4 and 0.6 respectively. Items are tested one by one from the lot unless all the defective items are tested. What is the probability that the testing procedure will continue up to the twelfth attempt ?

4.(a) Let S and T be distributed independently as exponential with means $1/\lambda$ and $1/\mu$ respectively. Let $U = \min\{S,T\}$ and $V = \max\{S,T\}$. Find $E(U)$ and $E(U+V)$.

(b) Let X be a random variable with U(0,1) distribution. Find the p.d.f. of the random variable $Y = (X / (1 + X))$.

5.(a) Let U and V be independent and uniformly distributed random variables on [0,1] and let θ_1 and θ_2 (both greater than 0) be constants.

Define $X = (-1/\theta_1) \ln U$ and $Y = (-1/\theta_2) \ln V$. Let $S = \min\{X,Y\}$, $T=\max\{X,Y\}$ and $R = T - S$.

(i) Find P[S=X].
(ii) Show that S and R are independent.

(b) A sequence of random variables $\{X_n \mid n = 1, 2, \ldots\}$ is called a ***martingale*** if

(i) $E(|X_n|) < \infty$
(ii) $E(X_{n+1} \mid X_1, X_2, \ldots, X_n) = X_n$ for all n = 1, 2, …

Let $\{Z_n \mid n = 1, 2, \ldots\}$ be a sequence of iid random variables with $P[Z_n = 1] = p$ and $P[Z_n = -1] = q = 1 - p$, $0 < p < 1$. Let $X_n = Z_1 + Z_2 + \ldots + Z_n$ for n = 1, 2, …
Show that $\{X_n \mid n = 1, 2, \ldots\}$, so defined, is a martingale if and only if $p = q = ½$..

6.(a) Let X be a random variable with density

$$f_X(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

For the minimum $X_{(1)}$ of n iid random observations $X_1, X_2, \ldots, X_n$ from the above distribution, show that $n^{1/4} X_{(n)}$ converges in distribution to a random variable Y with density

$$f_Y(y) = \begin{cases} 4\, e^{-y^4} y^3, & y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

(b) A random sample of size n is taken from the exponential distribution having p.d.f.

$$f(x) = \begin{cases} e^{-x}, & 0 \le x < \infty \\ 0 & \text{otherwise.} \end{cases}$$

Find the p.d.f. of the sample range.

7.(a) In a recent study, a set of n randomly selected items is tested for presence of colour defect. Let A denote the event colour defect is present" and B denote the event "test reveals the presence of colour defect". Suppose $P(A) = \alpha$, $P(B \mid A) = 1-\beta$ and $P(\text{Not } B \mid \text{Not } A) = 1-\delta$, where $0 < \alpha, \beta, \delta < 1$. Let X be the number of items in the set with colour defects and Y be the number of items in the set detected as having colour defects.

(i) Find $E(X \mid Y)$.

(ii) If the colour defect is very rare and the test is a very sophisticated one such that $\alpha = \beta = \delta = 10^{-9}$, then find the probability that an item detected as having colour defect is actually free from it.

(b) Consider the following bivariate density function

$$f(x,y) = \begin{cases} c \cdot xy, & x > 0, y > 0, x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

i) Find c.
ii) Find the conditional expectation, $E(Y \mid X = x)$, for $0 < x < 1$.

8. Suppose in a big hotel there are *N* rooms with single occupancy and also suppose that there are *N* boarders. In a dinner party to celebrate the marriage anniversary of one of the boarders they start drinking alcohol to their hearts' content and as a consequence they become unable to identify their own rooms. What is the probability that at the end of the dinner party none of the boarders occupies the room originally assigned to them? What is the limiting value of this probability as $N \to \infty$?

9. (a) Consider a Markov Chain with state space I = {1,2,3,4,5,6} and transition probability matrix P given by

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/4 & 3/4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1/8 & 7/8 & 0 & 0 \\ 1/4 & 0 & 1/2 & 0 & 1/8 & 1/8 \\ 0 & 0 & 1/4 & 3/4 & 0 & 0 \end{bmatrix}$$

Find the various classes of this chain and classify them as recurrent or transient.

(b) Pulses arrive at a Geiger Counter according to a Poisson Process with parameter $\lambda > 0$. The counter is held open only a random length of time T (independent of the arrival time of the pulses), where T is exponentially distributed with parameter $\beta > 0$. Find the distribution of N = Total number of pulses registered by the counter

PART II : ENGINEERING STREAM

GROUP E-1 : Engineering Mathematics

1(a) Let *f(x)* be a polynomial in *x* and let *a*, *b* be two real numbers where $a \neq b$. Show that if *f(x)* is divided by *(x - a)(x - b)* then the reminder is

$$\frac{(x-a)f(b)-(x-b)f(a)}{b-a}.$$

(b) Find $\frac{dy}{dx}$ if $x^{Cosy} + y^{Cosx} = 1$.

2.(a) Let A be the fixed point (0,4) and B be a moving point (2t, 0). Let M be the mid-point of AB and let the perpendicular bisector of AB meets the y-axis at R. Find the equation of the locus of the mid-point P of MR.

(b) Inside a square ABCD with sides of length 12 cm, segment AE is drawn where E is the point on DC such that DE = 5 cm.
The perpendicular bisector of AE is drawn and it intersects AE, AD and BC at the points M, P and Q respectively.
Find the ratio PM : MQ.

3.(a) Evaluate the value of $3.9^{1/2}.27^{1/4}.81^{1/8}$. ...up to infinity.

(b) Let f be a twice differentiable function such that

$$f''(x) = -f(x);\ f'(x) = g(x) \text{ and } h(x) = f^2(x) + g^2(x).$$

Given that h(5)=11, find h(10).

4.(a) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\cdots(upto\ [n/2]\ terms)\right]=\frac{1}{2}.$$

(b) Test the convergence of the series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\cdots\infty$.

Assume $x > 0$ and examine **all** possibilities.

5.(a) Find the limit of the following function as $x \to 0$.

$$\frac{|x|}{\sqrt{(x^4+4x^2+7)}} \operatorname{Sin}\left(\frac{1}{3\sqrt{x}}\right)$$

(b) If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$ then show that $a \cdot b < 0$.

6.(a) If ω is a complex cube root of unity then show that
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$.

(b) Show that $\left[\sum_{r>s} x^r / r!\right] / \left[\sum_{r>s} y^r / r!\right] > x^s / y^s$ whenever $x > y > 0$.

7.(a) Cable of a suspension bridge hangs in the form of a parabola and is attached to the supporting pillars 200 m apart. The lowest point of the cable is 40 m below the point of suspension. Find the angle between the cable and the supporting pillars. State all the assumptions involved.

(b) Let A, B and C be the angles of a triangle with angle C as the smallest of them.

Show that

(i) $\operatorname{Sin}(C/2) \le 1/2$.

(ii) Hence, or otherwise, show that $\operatorname{Sin}(A/2)\operatorname{Sin}(B/2)\operatorname{Sin}(C/2) < 1/4$.

8(a) Evaluate the following integrals directly and compare them.

$$\iint_{ax^2+by^2\le 1} dxdy \quad \text{and} \quad \iint_{\sqrt{a}\,|x|\le 1,\ \sqrt{b}\,|y|\le 1} dxdy$$

(b) Determine x, y and z so that the 3 x 3 matrix with the following row vectors is orthogonal : $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, $(1/\sqrt{2}, -1/\sqrt{2}, 0)$, (x, y, z).

GROUP E-2 : Engineering Mechanics

1.(a) The simple planar truss in the given Fig.1 consists of two straight two-force members AB and BC that are pinned together at B. The truss is loaded by a downward force of P=12 KN acting on the pin at B. Determine the internal axial forces F_1 and F_2 in members AB and BC respectively. (Neglect the weight of the truss members).

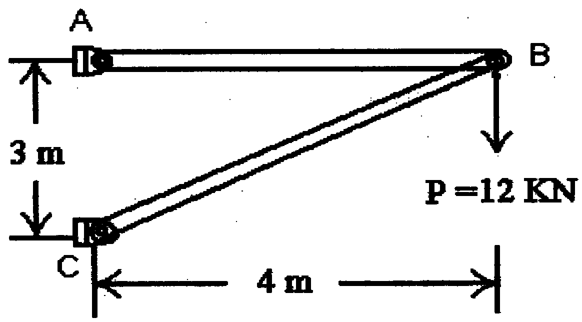


Fig. 1

(b) Derive the expression for moment of inertia I_{YY} of the shaded hollow rectangular section (Fig. 2).

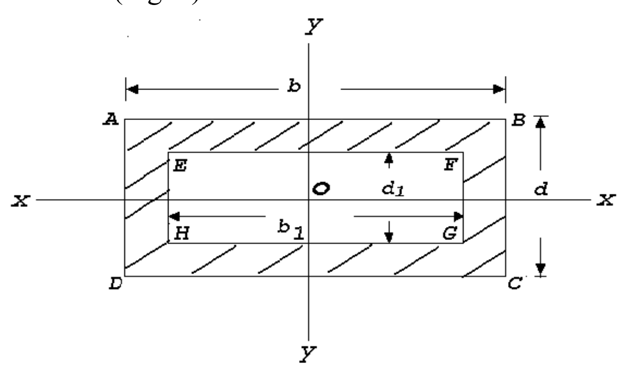


Fig. 2

2.(a) A turbine rotor weighs 20 tonnes and has a radius of gyration of 1.75 meter when running at 200 rpm. It is suddenly relieved of part of its load and its speed rises to 205 rpm in 1 sec. Find the unbalanced uniform turning moment.

(b) An Aluminium thin-walled tube (radius/thickness = 20) is closed at each end and pressurized by 6 MPa to cause plastic deformation. Neglect the elastic strain and find the plastic strain in the circumferential (hoop) direction of the tube. The plastic stress-strain curve is given by $\bar{\sigma} = 170$ (strain rate)$^{0.25}$.

3.(a) A uniform ladder 5 m long and 14 kg mass is placed against a vertical wall at an angle 50^0 to the horizontal ground. The co-efficient of friction between ladder and wall is 0.2 and between ladder and ground in 0.5. Calculate how far up the ladder a man of 63 kg. can climb before the ladder shifts.

(b) Determine the diameter of a steel shaft rotating at an angular velocity of 300 rpm transmitting 500 HP. The allowable stress = 800 kg/cm^2. The allowable angle of twist = 0.5^0 per m, G = 8×10^5 kg/cm^2 . What would be the savings if a hollow shaft is used to transmit the same power under the same condition, the ratio of diameters being 0.9?

4.(a) For the beam and loading shown in Fig.3, determine the equation defining the shear and bending moment at any point and at point D.

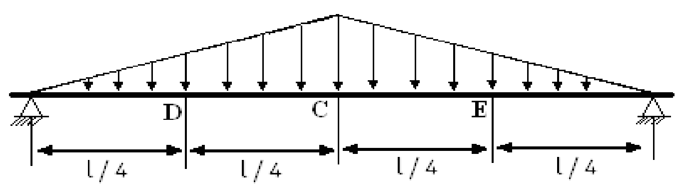


Fig. 3

(b) As illustrated in the given Fig.4 a metal punch (similar in principle to a paper punch) is used to punch holes in thin steel sheet that will be used to make a metal cabinet. To punch a 60 mm diameter disk or "slug" out of the sheet metal that is 10 mm thick requires a punch force of P=500 kg. Determine the average shear stress in the sheet metal resulting from the punching operation.

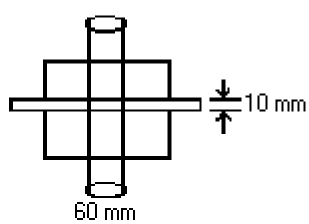


Fig. 4

5. (a) A tie rod in the suspension of a car is to be constructed from a grade of steel which has 0.1% proof stress equal to 250 MN/m^2. The tie rod is to be

constructed as a solid round bar of length 350 mm long. If the tie rod is subjected to a maximum axial force of 10 KN,

i) Determine the minimum diameter of the tie rod
ii) The extension of the tie rod under load (E= 2094 GN/m^2)
iii) The minimum diameter of the tie rod if a factor of safety of 2.5 is applied to the proof stress

(b) Find the width of the belt necessary to transmit 11.25 KW power to a pulley of diameter 300mm when the pulley makes 1600 rpm. Assume the co-efficient of friction between the belt and the pulley is 0.22 and angle of contact is 210^o. Maximum tension in the belt will not exceed 10N/mm width.

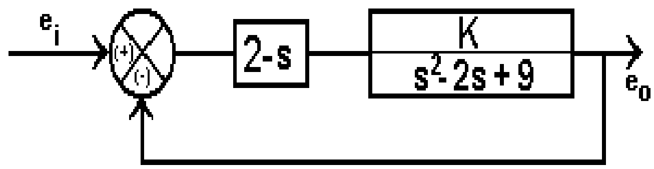
GROUP E-3 : Electrical and Electronics Engineering

1.(a) A centrifugal pump, which is gear-driven by a DC motor, delivers 810 kg of water per minute to a tank of height 11 meter above the level of the pump. Draw the block diagram of the overall arrangement. Determine input power across the gearing and current taken by the motor operated at 220 volt provided the efficiency of the pump, gearing and motor respectively be 70%, 70% and 90% only. (Take $g = 9.8\ ms^{-2}$).

(b) The rms value of a sinusoidal alternating voltage at a frequency of 50 Hz is 155volt. If at t = 0 it crosses the zero axis in a positive direction, determine the time taken to attain the first instantaneous value of 155 volt. How much time it takes to fall from the maximum peak value to its half? Explain with suitable waveform .

2.(a) On full-load unity power factor test, a meter having specification of 235 V and 5A makes 60 revolutions in 6 minutes, but its normal speed is 520 revolution/KWh. Does the meter has any inaccuracy? If so, find the percentage error.

(b) Write down the transfer function of the given system (as shown in Fig. 2) and find the values of K for which the system will be stable but underdamped.



3 (a) By intelligent selection of loop currents write down the mesh equations of the given circuit (as shown in Fig. 5) and determine the current flowing through that branch of the circuit containing capacitor. (All resistances/reactances are in ohms).

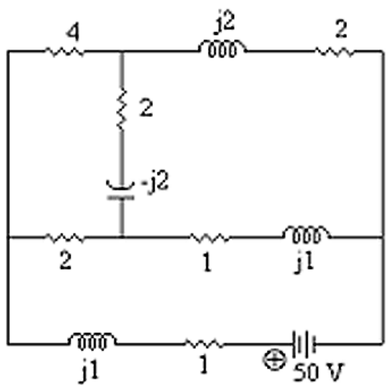


Fig. 5

(b)

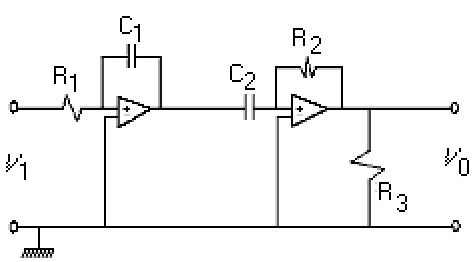


Fig. 6

Refer Fig. 6. Find the expression for V_0. What would be the nature of V_0 When $R_1 = R_2$ and $C_1 = C_2$?
(Consider the Op-amps to be identical)

4. (a) A series ac circuit that resonates at 48 Hz consists of a coil (having R = 30 Ω and L = 500 mH) and a capacitor. If the supply voltage is 100 volt determine the value of the capacitor.

(b) Calculate the value of a capacitor which when connected across the circuit (as of Q. 4 (a) above), enhances the resonant frequency to 60 Hz. Compare the value of the source current in both the cases.

5 (a) A 200/400 - V, 10KVA, 50Hz single phase transformer has, at full load, a Cu loss of 120W. If it has an efficiency of 98% at full load unity power factor, determine the iron losses. What would be the efficiency of the transformer at half load 0.8 power factor lagging?

(b) In the 2-port network given below, the parameters at two parts are related by the equations,

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

i) Find expressions for A,B,C and D
ii) Show that AD - BC = 1
iii) What are the physical interpretations of the above coefficients?

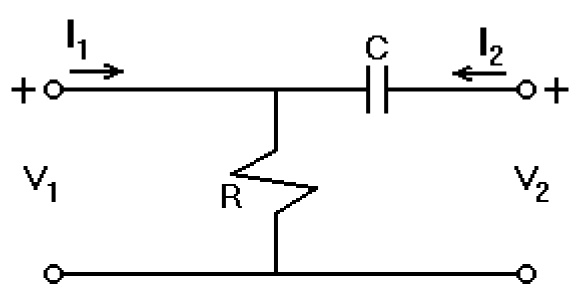


Fig 7

GROUP E-4 ; Thermodynamics

1.(a) In a thermodynamic system of a perfect gas, let U = f (V,T) where U, V and T refer to internal energy, volume of a gram-molecule of the substance and temperature (in absolute scale) respectively. An amount of heat δQ is added so that the volume expands by δV against a pressure P. Prove that:

$$C_p - C_V = \left[P + \left(\frac{\delta U}{\delta V} \right)_T \right] \left(\frac{\delta V}{\delta T} \right)_P$$

where C_p and C_V stand for specific heat at constant pressure and specific heat at constant volume respectively.

(b) 0.15 cu.m. of air at a pressure of 1.06 kg/cm^2 is compressed to a volume of 0.008 cu.m. at 361 kg/cm^2. Calculate (i) the quantity of heat rejected, (ii) change in internal energy if the process of compression is a) Adiabatic b) Polytropic with n = 1.3.

2.(a) A compression ignition engine has a stroke of 28 cm and a cylinder diameter of 18 cm. The clearance volume is 475 cm^3. The fuel injection takes place at constant pressure for 4.5% of the stroke. Find the air standard efficiency of the engine assuming that it works on diesel cycle. If the fuel injection takes place at 10% of the stroke, find the loss in air standard efficiency.

(b) A diesel engine has a compression ratio 14 to 1 and the fuel supply is cut off at 0.08 of the stroke. If the relative efficiency is 0.52, estimate the weight of fuel of a calorific value 10400 k.cal per kg that would be required per horsepower.

3.(a) Calculate the change in entropy of saturated steam at a given pressure such that the boiling point = 152.6 ^{o}C and the latent heat at this temperature = 503.6 cal/gm. [Use Log_e 1.56 = 0.445.]

(b) Draw the pv and T–Φ diagrams for a diesel cycle in which 1 kg of air at 1 kg / cm^2 and 90 ^{0}C is compressed through a ratio of 14 to 1. Heat is then added until the volume is 1.7 times the volume at the end of compression, after which the air expands adiabatically to its original volume. Take $C_v = 0.169$ and $\gamma = 1.41$.

4.(a) The approximated equation for adiabatic flow of super heated steam through a nozzle is given by pv^n = constant. Show that

$$p_2 / p_1 = (2 / (n+1))^{n / (n-1)}$$

where p_1 = pressure of steam at entry ; p_2 = pressure of steam at throat and p_2 / p_1 is the critical pressure ratio.

(b) The dry saturated steam is expanded in a nozzle from pressure of 10 bar to pressure of 4 bar. If the expansion is super saturated, find the degree of under cooling.

5. Three rods, one made of glass (k = 1.09 W/m-°C), one of pure Aluminium (k = 228 W/m-°C) and one of wrought iron (k = 57 W/m-°C), all have diameters of 1.25 cm, lengths of 30 cm, and are heated to 120 °C at one end. The rods extend into air at 20 °C, and the heat transfer coefficient on the surface is 9.0 W/m^2-°C. Find

(a) the distribution of temperature in the rods if the heat loss from the ends is neglected,

(b) the total heat flow from the rods neglecting the end heat loss,

(c) the heat flow from the rods if the end heat loss is not neglected, and heat transfer coefficient at the ends is also 9.0 W/m^2- °C

GROUP E-5 : Engineering Properties of Metals

1. (a) Distinguish between modulus of rigidity and modulus of rupture. Give an expression for the modulus of rigidity in terms of the specimen geometry, torque, and angle of twist. Is the expression valid beyond the yield strength (torsion)?

(b) A steel bar is subjected to a fluctuating axial load that varies from a maximum of 340 kN to a minimum of 120 kN compression. The mechanical properties of the steel are σ_u = 1090 MPa, σ_0 = 1010 MPa and σ_e = 510 MPa. Determine the bar diameter to give infinite fatigue life based on a safety factor of 2.5

2 (a) A cylindrical bar is subjected to a torsional moment M_T at one end. The twisting moment is resisted by shear stress μ set up in the cross section of the bar. The shear stress is zero at the centre of the bar and increases linearly with the radius. Find the maximum shear stress at the surface of the bar.

Given J = $\frac{\pi D^4}{32}$ (assuming that the torsional deformation is restricted within the zone of elasticity)

where, J : Polar moment of inertia
D : Diameter of cylinder.

(b) Consider a flat plane containing a crack of elliptical cross-section. The length of the crack is $2c$ and stress is perpendicular to the major axis of the ellipse. Show that

$$\sigma = \sqrt{\frac{2\gamma E}{\pi c}}$$

σ : stress
γ : surface energy
E : Young's modulus of elasticity

3. (a) Consider a tension specimen, which is subjected to a total strain ε at an elevated temperature where creep can occur. The total strain remains constant and the elastic strain decreases. Show that

$$\frac{1}{\sigma^{n-1}} = \frac{1}{\sigma_o^{n-1}} + BE(n-1)t$$

where,

$\varepsilon = \varepsilon_e + \varepsilon_p$	ε_e : elastic strain
$\varepsilon_e = \sigma / E$	ε_p : plastic strain
$\frac{d\varepsilon_p}{dt} = B\sigma^n$	t : time
$\sigma = \sigma_o$ at t = 0.	

(b) Distinguish between slip and twinning with diagrams.

4. (a) Suppose a crystalline material has *fcc* structure with atomic radius of 1.278Å. Determine the density of the crystalline material. Assume number of atoms per unit cell and molecular weight are n and M gm respectively.

   (b) Suppose there is an electron in an electric field of intensity 3200 volts/m. Estimate the force experienced by the electron. If it moves through a potential difference of 100 volts, find the kinetic energy acquired by the electron.

GROUP E-6 : Engineering Drawing

1.(a) A hollow cube of 5cm side is lying on H.P. and one of its vertical face is touching V.P. A slim rod, to be taken as its solid diagonal, is placed within it. Draw top and front / side views of solid diagonal and, from the drawn figure determine its true length.

   (b) Two balls are vertically erected to 18cm and 30 cm respectively above the flat ground. These balls are away from a 3 cm thick wall (on the ground) by 12 cm and 21 cm respectively but on either side of the wall. The distance between the balls, measured along the ground and parallel to the wall is 27 cm. Determine their approximate distance .

2. (a) Sketch the profile of a square thread, knuckle thread and a white-worth thread showing all relevant dimensions in terms of the pitch.

   (b) Sketch:

      i) single riveted lap joint,
      ii) double riveted lap joint chain-riveting,
      iii) double riveted lap joint zigzag-riveting, and
      iv) single cover single riveted butt joint.

3.(a) Draw the isometric view of an octahedron erected vertically up on one of its vertices. (Distinct free hand sketch only.)

(b) You are given two square prisms of same height of 10cm. Prism A has side 7cm and prism B has side of 5cm respectively. Longer face of B is lying on H.P. with its base perpendicular to V.P. Base of A is lying on H.P. but equally inclined to V.P. You are instructed to remove by cutting a portion of bottom base of A so that within the cavity maximum of B may be placed accordingly. Note that vertical face of B may be parallel to V.P. but just touch the central axis of A. Draw the sectional view of the combination and determine the volume of material to be removed from A.

4. A parallelepiped of dimension $100 \times 60 \times 80$ is truncated by a plane which passes through 85, 45 and 65 unit distance on the associated edges from the nearest top point of the object. Draw the isometric view of the truncated solid object. In third angle projection method, draw its plan. (All dimensions are in mm).

Note : A copy of one of the previous year's QR Test Question paper is appended in the following pages to give the candidates a rough idea.

BOOKLET No. **TEST CODE : QR**

Afternoon

Time : 2 hours

Group	Questions: Total	Questions: To be answered	Maximum marks
Part I (for Statistics/Mathematics Stream)			
S1 (Statistics)	5	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM EACH GROUP.	120
S2 (Probability)	5		
Part II (for Engineering Stream)			
E1 (Mathematics)	3	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM E1	120
E2 (Engineering Mechanics)	2		
E3 (Electrical and Electronics Engineering)	2		
E4 (Thermodynamics)	2		
E5 (Engineering Properties of Metals)	2		
E6 (Engineering Drawing)	2		

On the answer-booklet write your Name, Registration Number, Test Code, Number of this booklet, etc. in the appropriate places.

There are two parts in this booklet as detailed above. Candidates having Statistics background are required to answer questions from Part I as per instructions given. Those having Engineering background are required to answer questions from Part II as per instructions given.

USE OF CALCULATORS IS NOT ALLOWED. SLIDE RULE MAY BE USED

STOP ! WAIT FOR THE SIGNAL TO START

PART I (FOR STATISTICS/MATHEMATICS STREAM)

ATTENTION : ANSWER A TOTAL OF SIX [6] QUESTIONS, TAKING AT LEAST TWO [2] FROM EACH GROUP.

GROUP S-1
Statistics

1. (a) Determine the straight line of y on x which best (in the least squares sense) fits the following four points: (0,2), (1,1), (-4,3), (5,-2).

   (b) Out of the following two regression lines, find the line of regression of X on Y :

   $$2x + 3y = 7 \text{ and } 5x + 4y = 9.$$

2. Derive the following expression for estimating missing value ($Y_{ijk} = x$) in case of a $p \times p$ Latin Square Design:

   $$x = \frac{p[Y'_{i\bullet\bullet} + Y'_{\bullet j \bullet} + Y'_{\bullet\bullet k}] - 2Y'_{\bullet\bullet\bullet}}{(p-1)(p-2)},$$

   where the primes indicate totals for the row, column and treatment excluding the missing value and $Y'_{\bullet\bullet\bullet}$ is the grand total excluding the missing value.

3. (a) Show that the sample variance s^2 obeys the relation

$$(n-1)s^2 = \sum_{i=2}^{n} i(x_i - \bar{x}_i)^2 / (i-1)$$

where $\bar{x}_i = (x_1 + x_2 + ... + x_i)/i, \quad i = 2,3,\cdots,n.$

(b) If all the total (order zero) correlation coefficients in a set of p-variates are equal to ρ, show that the multiple correlation coefficient R of a variate with other $(p-1)$ variates is given by

$$1 - R^2 = (1-\rho)\frac{[1+(p-1)\rho]}{[1+(p-2)\rho]}$$

4. (a) In a population of N units, NP units possess a certain characteristics and $N(1-P)$ units lack it. A simple random sample of size n is drawn, without replacement, from the N units. Next a simple random sub-sample of size n_1 units is drawn from the n units of the sample without replacement and added to the original sample of n units. Is the proportion of units having the characteristic, amongst the $(n+n_1)$ units in the combined sample, an unbiased estimator of the population proportion? Justify.

(b) If $x_i (i = 1,2,...,n)$ are a random sample of size n from $N(\mu, \sigma^2)$, show that

$$\sqrt{\frac{n}{n-1}}\ (x_1 - \bar{x})\ /\ \sqrt{\left[(n-1)s^2 - \frac{n}{n-1}(x_1 - \bar{x})^2\right]\ /\ (n-2)}$$

follows a t-distribution with $(n-2)$ d.f.

5. An experienced inspector claims that he has the ability to predict if an item is defective or not without testing the item. Five items were selected at random from an assembly line. It turned out that the inspector could correctly predict four times out of five. Was the inspector guessing? You may accept inspector's claim at most 5% of the time when he is really guessing.

GROUP S–2
Probability

6. Let $x_1, \ldots, x_n$ be a random sample from uniform distribution over $[\theta, \theta+1]$.
   i) Find the density of $x_{(n)}$, the largest of the x_i's.
   ii) Find its expectation.

7. (a) In adding *n* real numbers in a computer, each is rounded off to the nearest integer. Assuming that the round-off error is a continuous random variable with constant density on the interval (-0.5, 0.5), determine the probability that the error in the sum is no greater than 0.98 $\sqrt{n}$ in magnitude, for large *n*.

   (b) In Nainital, boats leave Tallital for Mallital at 10 minute intervals, starting at 8 AM. A man arrives at Tallital X (a random number) minutes after 8 AM, where X has the distribution function.

$$P(X \le x) = \begin{cases} 0 & \text{if } x < 0, \\ x/60 & \text{if } 0 \le x \le 60. \\ 1 & \text{if } x > 60 \end{cases}$$

   What is the probability that his steamer leaves within four minutes of his arrival?

]

8. (a) Suppose that the times of successive failures of a machine form a Poisson process on $[0, \infty)$ with parameter $\lambda > 0$.
   (i) What is the probability of at least one failure during the time period *(t ,t + h)* (where *h > 0*)?
   (ii) What is the conditional probability of at least one failure by time *t* + *h*, given that there is no failure by time *t*?

   (b) Let $\{N_1(t) \;|\; t \in [0, \infty)\}$ and $\{N_2(t) \;|\; t \in [0, \infty)\}$ be independent Poisson processes with parameters λ_1 and λ_2 respectively $(\lambda_1, \lambda_2 > 0)$. Show that the conditional distribution of $N_1(t)$, given that $N_1(t) + N_2(t) = n$, is binomial for any fixed *t*.

9. The joint density function of X and Y is given by

$$f(x,y)=\begin{cases} c(1-x-y), & \text{for } 0<x<1-y,\ 0<y<1 \\ 0 & \text{, elsewhere.} \end{cases}$$

(i) Find c.
(ii) Find the conditional density of Y given $X=x$.
(iii) Compute the conditional expectation of Y given $X = 1/4$.

10. Let the distribution of the random variable X be $N(\mu,1)$ and let $Y=\dfrac{1-\Phi(X)}{\phi(X)}$ where Φ and ϕ denote the distribution function and the density function of $N(0,1)$ respectively. Find $E(Y)$.

PART II (FOR ENGINEERING STREAM)

ATTENTION : **ANSWER A TOTAL OF SIX [6] QUESTIONS TAKING AT LEAST TWO [2] FROM E1.**

GROUP E-1
Mathematics

1. (a) Let A = $\begin{bmatrix} a^2+\lambda & ab & ac & ad \\ ab & b^2+\lambda & bc & bd \\ ac & bc & c^2+\lambda & cd \\ ad & bd & cd & d^2+\lambda \end{bmatrix}$

   Show that $\det(\mathrm{A}) = \lambda^3\left(a^2+b^2+c^2+d^2+\lambda\right)$

   (b) Prove that $\lim_{x\to 0} x \sin\frac{1}{x} = 0$

2. (a) Solve the equation $1 + y'^2 = y\,y''$

   (b) Sum the series

   $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7} - \ldots\ldots\ldots$

3. (a) Evaluate $\int_0^1 \frac{1-x}{1+x}\,dx$.

   (b) Find the common roots of the equation $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{1985} + z^{100} + 1 = 0$

GROUP E-2
Engineering Mechanics

4. (a) Two vertical rods one of steel and the other of bronze are suspended from a horizontal ceiling, the horizontal distance between them being 80 cm . Each rod is 250 cm long and 12.5 mm in diameter. A horizontal crosspiece connects the lower ends of the bars. Where should a load of 1000 kg be placed on the cross piece so that it remains horizontal after being loaded ? Neglect the bending of the cross piece. [$E_b = 1 \times 10^6$ Kg/cm^2 ; $E_s = 2 E_b$]

   (b) A tramcar weighs 12 tons. The tract resistance on the level being 5 kg per ton, what horse power will be required to propel the car at a uniform speed of 18 kilometers an hour up an incline of 1 in 300, on the level and down an incline of 1 in 300? Take efficiency of the motors and drive as 70%.

5. (a) A simply supported beam of 10m is loaded as shown in Figure 1 below. Draw the corresponding shear force and the bending moment diagram.

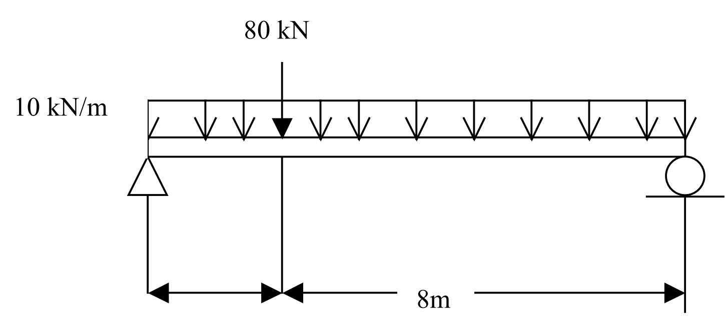
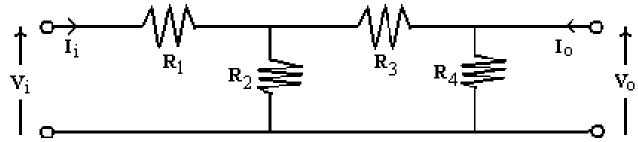


Figure 1

   (b) A sphere, a solid cylinder and a hollow thin cylinder shell each of same radius and made of same material roll down the same inclined plane without slip. Which one will reach the bottom of the plane first? What conditions need to be satisfied for rolling without slip?

GROUP E-3
Electrical & Electronics Engineering

6. (a) Derive the open circuit impedance parameters and short circuit admittance parameters of the given circuit as shown in Figure 2.



$R_1 = 2\Omega$, $R_2 = 1\Omega$, $R_3 = 2\Omega$, $R_4 = 2\Omega$

Figure 2

(b) Derive the transmission parameters considering the above formulation.

(c) If this section (as shown in the figure) is now cascaded with another such section determine its overall transmission parameters. Verify the result by direct calculation using KVL/KCL.

7. (a) Using ideal Op-Amps, design a circuit to solve the differential equation :

$$K_1 \frac{d^2 V}{dt^2} + K_2 \frac{dV}{dt} = K_3 \quad \text{where, } K_1, K_2, K_3 > 0.$$

(b) Draw the circuit diagram of an RC phase shift oscillator using FET. Calculate the feedback factor β using KVL/KCL method when $R_1 = R_2 = R_3 = R$ and $C_1 = C_2 = C_3 = C$ and all the free terminals of resistances are grounded. Determine the frequency of oscillation. Given $\alpha^{-1} = \omega CR$.

GROUP E-4
Thermodynamics

8. (a) Two Carnot engines I and II operate in series between a high temperature reservoir at 1027^0C and a low temperature reservoir at 27^0C. The engine I absorbs energy from the high temperature reservoir and rejects energy to a reservoir at temperature T. The engine II receives energy from the reservoir at T and rejects energy to the low temperature reservoir. The amount of energy absorbed by engine II from the reservoir at T is the same as that rejected by engine I to the reservoir at T. If engines I and II are found to have the same efficiency, determine the temperature T. If engine I receives 100 kJ energy as heat from the high temperature reservoir, calculate the work delivered by engine I and engine II.

   (b) The COP of a Carnot refrigerator can be increased either by decreasing the temperature of the high temperature reservoir, while the low temperature reservoir is held at constant temperature or by increasing the temperature of the low temperature reservoir while the high temperature reservoir is held at constant temperature. Determine which of the above two possibilities is more effective.

9. (a) Two identical bodies of constant heat capacity are at the same initial temperature T_i. A refrigerator operates between these two bodies until one body is cooled to temperature T_2. If the bodies remain at constant pressure and undergo no change of phase, show that the minimum amount of work needed to do this is

$$W\ (\mathrm{Min}) = C_P \left(\frac{T_i^2}{T_2} + T_2 - 2T_i \right)$$

   (b) A stationary mass of gas is compressed without friction from an initial state of 0.3 m^3 and 0.105 MPa to a final state of 0.15 m^3 and 0.105 MPa, the pressure remaining constant during the process. There is a transfer of 37.6 kJ of heat from the gas during the process. How much does the internal energy of the gas change?

GROUP E-5
Engineering Properties of Metals

10. (a) Explain how x-ray diffraction can be used to determine the lattice dimension of a metal.

    (b) The Bragg angle corresponding to the first reflection from (1,1,1) planes in a crystal is 30^0 when X-rays of wave length 1.75 $\overset{o}{A}$ are used. Draw conclusion about the inter spacing of the crystal.

11. (a) Construct a schematic diagram indicating recovery, recrystallization and grain growth and the properties namely internal strain, strength, ductility and grain size.

    (b) Consider a cylindrical bar which is subjected to a torsional moment at one end. The twisting moment is resisted by shear stresses set up in the cross section of the bar. The shear stress is zero at the center of the bar and increases linearly with the radius.

    Show the shear stress and shear strain on the outer surface are

$$\tau = \frac{16 M_T D_1}{\pi (D_1^4 - D_2^4)} \quad \text{and} \quad \gamma = \frac{r\theta}{L}$$

Where
- D_1 : outside diameter of the tube
- D_2 : inside diameter of the tube
- M_T : torsional moment
- θ : angle of twist
- L : Length of the specimen
- r : radial distance measured from center of bar

GROUP E-6
Engineering Drawing

12. (a) You are given a rectangular parallelopiped of size 4x3x12 cubic cm. The base of the object is just touching V.P. and making equal angle to the H.P. The longest edge is lying on the H.P. Draw its side view. If the object is now slightly rotated such that its base diagonal is now perpendicular to the H.P., ensuring that the above mentioned longest edge is still lying on the H.P. Again draw its side view.

    (b) From the last position, the object is now allowed to tilt to an angle of $\tan^{-1} 5/12$ ensuring that its solid diagonal is now resting on HP. Draw its orthogonal view roughly.

13. Draw the rough sketches (mentioning technical details in each case) of
    (i) Acme thread.
    (ii) Locking plate.
    (iii) Hexagonal headed bolt with hexagonal nut and washers.
    (iv) Worm gear.