Test Code: QR (Short answer type) 2008

M.Tech. in Quality, Reliability and Operations Research

The candidates applying for M.Tech. in Quality, Reliability and Operations Research will have to take two tests : **Test MIII** (objective type) in the forenoon session and **Test QR** (short answer type) in the afternoon session.

For Test **MIII**, see a different Booklet. For Test **QR**, refer to this Booklet **ONLY.**

If you are from **Statistics / Mathematics Stream,** you will be required to **ANSWER PART I.**

If you are from **Engineering Stream**, you will be required to **ANSWER PART II.**

In **PART I**, a **TOTAL of TEN [10]** questions, are divided into **TWO Groups** : **S1**: **Statistics and S2: Probability** – **each group carrying FIVE [5]** questions. You will be required to answer a **TOTAL of SIX [6]** questions, taking **AT LEAST TWO [2]** from **each group**.

In **PART II**, there will be **SIX Groups: E1-E6**. **E1** will contain **THREE [3]** questions from **Engineering Mathematics** and each other group will contain **TWO [2]** questions from **Engineering and Technology**. You will be required to answer a total of **SIX [6]** questions taking **AT LEAST TWO [2]** from group **E1**.

Syllabus

PART I: STATISTICS / MATHEMATICS STREAM

Statistics (S1)

Descriptive statistics for univariate, bivariate and multivariate data.
Standard univariate probability distributions [Binomial, Poisson, Normal] and their fittings, properties of distributions. Sampling distributions.
Theory of estimation and tests of statistical hypotheses.
Multiple linear regression and linear statistical models, ANOVA.
Principles of experimental designs and basic designs [CRD, RBD & LSD].
Elements of non-parametric inference.
Elements of sequential tests.
Sample surveys – simple random sampling with and without replacement, stratified and cluster sampling.

Probability (S2)

Classical definition of probability and standard results on operations with events, conditional probability and independence.
Distributions of discrete type [Bernoulli, Binomial, Multinomial, Hypergeometric, Poisson, Geometric and Negative Binomial] and continuous type [Uniform, Exponential, Normal, Gamma, Beta] random variables and their moments.
Bivariate distributions (with special emphasis on bivariate normal), marginal and conditional distributions, correlation and regression.
Multivariate distributions, marginal and conditional distributions, regression, independence, partial and multiple correlations.
Order statistics [including distributions of extreme values and of sample range for uniform and exponential distributions].
Distributions of functions of random variables.
Multivariate normal distribution [density, marginal and conditional distributions, regression].

Syllabus

Weak law of large numbers, central limit theorem.
Basics of Markov chains and Poisson processes.

PART II : ENGINEERING STREAM

Mathematics (E1)

Elementary theory of equations, inequalities.
Elementary set theory, functions and relations, matrices, determinants, solutions of linear equations.
Trigonometry [multiple and sub-multiple angles, inverse circular functions, identities, solutions of equations, properties of triangles].
Coordinate geometry (two dimensions) [straight line, circle, parabola, ellipse and hyperbola], plane geometry, Mensuration.
Sequences, series and their convergence and divergence, power series, limit and continuity of functions of one or more variables, differentiation and its applications, maxima and minima, integration, definite integrals areas using integrals, ordinary and partial differential equations (upto second order), complex numbers and De Moivre's theorem.

Engineering Mechanics (E2)

Forces in plane and space, analysis of trusses, beams, columns, friction, principles of strength of materials, work-energy principle, moment of inertia, plane motion of rigid bodies, belt drivers, gearing.

Syllabus

Electrical and Electronics Engineering (E3)

D.C. circuits, AC circuits (1-ϕ), energy and power relationships, Transformer, DC and AC machines, concepts of control theory and applications.
Network analysis, 2 port network, transmission lines, elementary electronics (including amplifiers, oscillators, op-amp circuits), analog and digital electronic circuits.

Thermodynamics (E4)

Laws of thermodynamics, internal energy, work and heat changes, reversible changes, adiabatic changes, heat of formation, combustion, reaction, solution and dilution, entropy and free energy and maximum work function, reversible cycle and its efficiency, principles of internal combustion engines. Principles of refrigeration.

Engineering Properties of Metals (E5)

Structures of metals, tensile and torsional properties, hardness, impact properties, fatigue, creep, different mechanism of deformation.

Engineering Drawing (E6)

Concept of projection, point projection, line projection, plan, elevation, sectional view (1^{st} angle/3^{rd} angle) of simple mechanical objects, isometric view, dimensioning, sketch of machine parts.
(Use of set square, compass and diagonal scale should suffice).

SAMPLE QUESTIONS

PART I: STATISTICS / MATHEMATICS STREAM

GROUP S-1: Statistics

1. Let X_1 and X_2 be independent χ^2 variables, each with n degrees of freedom. Show that $\frac{\sqrt{n}\left(X_1 - X_2\right)}{2\sqrt{X_1 X_2}}$ has the t distribution with n degrees of freedom and is independent of $X_1 + X_2$.

2. Let $[\{x_i\ ;\ i = 1, 2, \ldots, p\};\ \{y_j\ ;\ j = 1,2,\ldots, q\};\ \{z_k\ ;\ k = 1, 2, \ldots,r\}]$ represent random samples from $N(\alpha + \beta, \sigma^2)$, $N(\beta + \gamma, \sigma^2)$ and $N(\gamma + \alpha, \sigma^2)$ populations respectively. The populations are to be treated as independent.
   (a) Display the set of complete sufficient statistics for the parameters $(\alpha, \beta, \gamma, \sigma^2)$.
   (b) Find unbiased estimator for β based on the sample means only. Is it unique?
   (c) Show that the estimator in (b) is uncorrelated with all error functions.
   (d) Suggest an unbiased estimator for σ^2 with maximum d.f.
   (e) Suggest a test for H_0: $\beta = \beta_0$.

3. Consider the linear regression model : $y = \alpha + \beta x + e$ where e's are iid $N(0, \sigma^2)$.
   (a) Based on n pairs of observations on x and y, write down the least squares estimates for α and β.
   (b) Work out exact expression for Cov($\hat{\alpha}$, $\hat{\beta}$).
   (c) For a given y_0 as the "predicted" value, determine the corresponding predictand "x_0" and suggest an estimator "$\hat{x}_0$" for it.

4. A town has N taxis numbered 1 through N. A person standing on roadside notices the taxi numbers on n taxis that pass by. Let Mn be the largest number observed. Assuming independence of the taxi numbers and sampling with replacement, show that

$$\hat{N} = (n + 1)\, M_n / n$$

is an approximately unbiased estimator of N for large N.

5.(a) Let $x_1, x_2, \ldots, x_n$ be a random sample from the rectangular population with density

$$f(x) = \begin{cases} \frac{1}{\theta}, & 0 < x < \theta \\ 0, & otherwise \end{cases}$$

Consider the critical region $x_{(n)} > 0.8$ for testing the hypothesis $H_0 : \theta = 1$, where $x_{(n)}$ is the largest of $x_1, x_2, \ldots, x_n$. What is the associated probability of error I and what is the power function?

(b) Let $x_1, x_2, \ldots, x_n$ be a random sample from a population having p.d.f.

$$f(x,\theta) = \begin{cases} \frac{\theta^3}{\Gamma(3)} e^{-\theta x} x^2, & 0 < x < \infty \\ 0, & otherwise \end{cases}$$

Obtain the maximum likelihood estimate of θ and also obtain the Cramer Rao lower bound to the variance of an unbiased estimator of θ.

6.(a) Give an example of a Latin Square Design of order 4 involving 4 rows, 4 columns and 4 treatments. Give the general definition of "treatment connectedness" in the context of a Latin Square Design and show that the Latin Square Design that you have given is indeed treatment connected.

(b) In a CRD set-up involving 5 treatments, the following computations were made:

$n = 105$, Grand Mean = 23.5, SSB = 280.00, SSW = 3055.00

(i) Compute the value of the F-ratio and examine the validity of the null hypothesis.

(ii) It was subsequently pointed out that there was one additional treatment that was somehow missed out! For this treatment, we are given sample size = 20, Sum = 500 and Sum of Squares (corrected) = 560.00. Compute **revised** value of F-ratio and draw your conclusions.

7. If X_1, X_2, X_3 constitute a random sample from a Bernoulli population with mean p, show why $[X_1 + 2X2 + 3X_3] / 6$ is *not* a sufficient statistic for p.

8. If X and Y follow a trinomial distribution with parameters n, θ_1 and θ_2, show

that

(a) $E(Y/X=x)=\dfrac{(n-x)\theta_2}{1-\theta_1}$,

(b) $V(Y/X=x)=\dfrac{(n-x)\theta_2(1-\theta_1-\theta_2)}{(1-\theta_1)^2}$

Further show, in standard notations,

(c) $V_1E_2=\dfrac{n\theta_1\theta_2^2}{1-\theta_1}$, (d) $E_1V_2=\dfrac{n\theta_2(1-\theta_1-\theta_2)}{1-\theta_1}$,

(e) $V(Y)=n\theta_2(1-\theta_2)$

9. Life distributions of two independent components of a machine are known to be exponential with means μ and λ respectively. The machine fails if at least one of the components fails. Compute the chance that the machine will fail due to the second component. Out of n independent prototypes of the machine m of them fail due to the second component. Show that $m/(n-m)$ approximately estimates the odds ratio $\theta=\lambda/\mu$.

GROUP S–2: Probability

1. A boy goes to his school either by bus or on foot. If one day he goes to the school by bus, then the probability that he goes by bus the next day is 7/10. If one day he walks to the school, then the probability that he goes by bus the next day is 2/5.

   (a) Given that he walks to the school on a particular Tuesday, find the probability that he will go to the school by bus on Thursday of that week.

   (b) Given that the boy walks to the school on both Tuesday and Thursday of that week, find the probability that he will also walk to the school on Wednesday.

   [You may assume that the boy will not be absent from the school on Wednesday or Thursday of that week.]

2. Suppose a young man is waiting for a young lady who is late. To amuse himself while waiting, he decides to take a random walk under the following set of rules:

   He tosses an imperfect coin for which the probability of getting a head is 0.55. For every head turned up, he walks 10 yards to the north and for every tail turned up, he walks 10 yards to the south.

   That way he has walked 100 yards.

   (a) What is the probability that he will be back to his starting position?

   (b) What is the probability that he will be 20 yards away from his starting position?

3. (a) A coin is tossed an odd number of times. If the probability of getting more heads than tails in these tosses is equal to the probability of getting more tails than heads then show that the coin is unbiased.

   (b) For successful operation of a machine, we need at least three components (out of five) to be in working phase. Their respective chances of failure are 7%, 4%, 2%, 8% and 12%. To start with, all the components are in working phase and the operation is initiated. Later it is observed that the machine has stopped but the first component is found to be in working phase. What is the likelihood that the second component is also in working phase?

   (c) A lot contains 20 items in which there are 2 or 3 defective items with probabilities 0.4 and 0.6 respectively. Items are tested one by one from the lot unless all the defective items are tested. What is the probability that the testing procedure will continue up to the twelfth attempt?

4.(a) Let S and T be distributed independently as exponential with means $1/\lambda$ and $1/\mu$ respectively. Let $U = \min\{S,T\}$ and $V = \max\{S,T\}$. Find $E(U)$ and $E(U+V)$.

   (b) Let X be a random variable with $U(0,1)$ distribution. Find the p.d.f. of the random variable $Y = (X / (1 + X))$.

5.(a) Let U and V be independent and uniformly distributed random variables on $[0,1]$ and let θ_1 and θ_2 (both greater than 0) be constants.

Define $X = -\frac{1}{\theta_1}\ln U$ and $Y = -\frac{1}{\theta_2}\ln V$. Let $S = \min\{X,Y\}$, $T=max\{X,Y\}$ and $R = T - S$.

(i) Find $P[S=X]$.
(ii) Show that S and R are independent.

(b) A sequence of random variables $\{X_n \mid n = 1, 2, \ldots\}$ is called a ***martingale*** if

(i) $E(\mid X_n \mid) < \infty$
(ii) $E(X_{n+1} \mid X_1, X_2, \ldots, X_n) = X_n$ for all $n = 1, 2, \ldots$

Let $\{Z_n \mid n = 1, 2, \ldots\}$ be a sequence of iid random variables with $P[Z_n = 1] = p$ and $P[Z_n = -1] = q = 1-p, 0 < p < 1$. Let $X_n = Z_1 + Z_2 + \ldots + Z_n$ for $n = 1, 2, \ldots$. Show that $\{X_n \mid n = 1, 2, \ldots\}$, so defined, is a martingale if and only if $p = q = ½$..

6.(a) Let X be a random variable with density

$$f_X(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

For the minimum $X_{(1)}$ of n iid random observations $X_1, X_2, \ldots, X_n$ from the above distribution, show that $n^{1/4} X_{(n)}$ converges in distribution to a random variable Y with density

$$f_Y(y) = \begin{cases} 4e^{-y^4} y^3, & y > 0 \\ 0, & otherwise \end{cases}$$

(b) A random sample of size n is taken from the exponential distribution having p.d.f.

$$f(x) = \begin{cases} e^{-x}, & 0 \le x < \infty \\ 0, & otherwise. \end{cases}$$

Find the p.d.f. of the sample range.

7.(a) In a recent study, a set of n randomly selected items is tested for presence of colour defect. Let A denote the event colour defect is present" and B denote the event "test reveals the presence of colour defect". Suppose $P(A) = \alpha$, $P(B \mid A) = 1-\beta$ and $P(\text{Not } B \mid \text{Not } A) = 1-\delta$, where $0 < \alpha, \beta, \delta < 1$. Let X be the number of items in the set with colour defects and Y be the number of items in the set detected having colour defects.

(i) Find $E(X \mid Y)$.

(ii) If the colour defect is very rare and the test is a very sophisticated one such that $\alpha = \beta = \delta = 10^{-9}$, then find the probability that an item detected as having colour defect is actually free from it.

(b) Consider the following bivariate density function

$$f(x, y) = \begin{cases} c \cdot xy, & x > 0, y > 0, x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

i) Find c.

ii) Find the conditional expectation, $E(Y \mid X = x)$, for $0 < x < 1$.

8. Suppose in a big hotel there are N rooms with single occupancy and also suppose that there are N boarders. In a dinner party to celebrate the marriage anniversary of one of the boarders they start drinking alcohol to their hearts' content and as a consequence they become unable to identify their own rooms. What is the probability that at the end of the dinner party none of the boarders occupies the room originally assigned to them? What is the limiting value of this probability as $N \rightarrow \infty$?

9. (a) Consider a Markov Chain with state space $I = \{1,2,3,4,5,6\}$ and transition probability matrix P given by

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/4 & 3/4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1/8 & 7/8 & 0 & 0 \\ 1/4 & 0 & 1/2 & 0 & 1/8 & 1/8 \\ 0 & 0 & 1/4 & 3/4 & 0 & 0 \end{bmatrix}$$

Find the various classes of this chain and classify them as recurrent or transient.

(b) Pulses arrive at a Geiger counter according to a Poisson Process with parameter $\lambda > 0$. The counter is held open only a random length of time T (independent of the arrival time of the pulses), where T is exponentially distributed with parameter $\beta>0$. Find the distribution of N = Total number of pulses registered by the counter

PART II: ENGINEERING STREAM

GROUP E-1: Engineering Mathematics

1(a) Let $f(x)$ be a polynomial in x and let a, b be two real numbers where $a \neq b$. Show that if $f(x)$ is divided by $(x - a)(x - b)$ then the reminder is

$$\frac{(x-a)f(b)-(x-b)f(a)}{b-a}.$$

(b) Find $\frac{dy}{dx}$ if $x^{Cosy} + y^{Cosx} = 1.$

2.(a) Let A be the fixed point (0,4) and B be a moving point ($2t$, 0). Let M be the mid-point of AB and let the perpendicular bisector of AB meets the y-axis at R. Find the equation of the locus of the mid-point P of MR.

(b) Inside a square ABCD with sides of length 12 cm, segment AE is drawn where E is the point on DC such that DE = 5 cm. The perpendicular bisector of AE is drawn and it intersects AE, AD and BC at the points M, P and Q respectively. Find the ratio PM: MQ.

3.(a) Evaluate the value of $3.9^{1/2}.27^{1/4}.81^{1/8}\ldots\ldots\infty$.

(b) Let f be a twice differentiable function such that

$$f''(x) = -f(x);\; f'(x) = g(x) \text{ and } h(x) = f^2(x) + g^2(x).$$

Given that $h(5)=11$, find $h(10)$.

4.(a) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\cdots(upto\,[n/2]\,terms)\right]=\frac{1}{2}.$$

(b) Test the convergence of the series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\cdots\infty$.

Assume $x > 0$ and examine **all** possibilities.

5.(a) Find the limit of the following function as $x \to 0$.

$$\frac{|x|}{\sqrt{(x^4+4x^2+7)}}\sin\left(\frac{1}{3\sqrt{x}}\right).$$

(b) If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$ then show that $a \cdot b < 0$.

6.(a) If ω is a complex cube root of unity then show that

$$a^3+b^3+c^3-3abc=(a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega).$$

(b) Show that $\left[\frac{\sum_{r>s} x^r}{r!}\right] \div \left[\frac{\sum_{r>s} y^r}{r!}\right] > \frac{x^s}{y^s}$, whenever $x > y > 0$.

7.(a) Cable of a suspension bridge hangs in the form of a parabola and is attached to the supporting pillars 200 m apart. The lowest point of the cable is 40 m below the point of suspension. Find the angle between the cable and the supporting pillars. State all the assumptions involved.

(b) Let A, B and C be the angles of a triangle with angle C as the smallest of them.

Show that

(i) $\sin\left(\frac{C}{2}\right) \leq \frac{1}{2}$

(ii) Hence, or otherwise, show that $\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) < \frac{1}{4}$.

8(a) Evaluate the following two integrals directly and compare them.

$$\iint\limits_{ax^2+by^2\leq 1} dxdy \qquad \text{and} \qquad \iint\limits_{\sqrt{a}|x|\leq 1,\sqrt{b}|y|\leq 1} dxdy.$$

(b) Determine x, y and z so that the 3×3 matrix with the following row vectors is orthogonal : $(1 / \surd 3, 1 / \surd 3, 1 / \surd 3)$, $(1/ \surd 2, -1/ \surd 2, 0)$, (x, y, z).

GROUP E-2: Engineering Mechanics

1.(a) The simple planar truss in the given Fig.1 consists of two straight two-force members AB and BC that are pinned together at B. The truss is loaded by a downward force of P=12 KN acting on the pin at B. Determine the internal axial forces F_1 and F_2 in members AB and BC respectively. (Neglect the weight of the truss members).

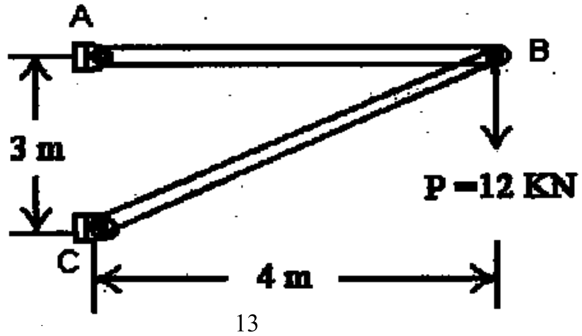


Fig. 1

(b) Derive the expression for moment of inertia I_{YY} of the shaded hollow rectangular section (Fig. 2).

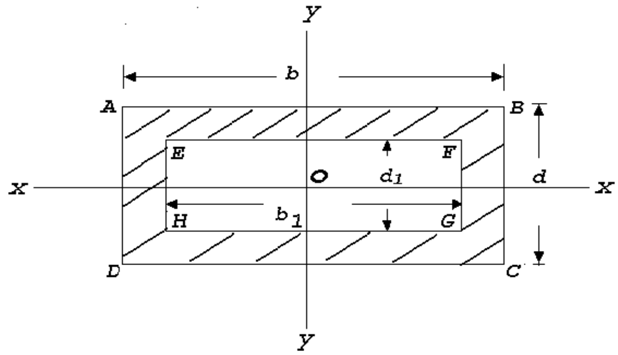


Fig. 2

2.(a) A turbine rotor weighs 20 tonnes and has a radius of gyration of 1.75 meter when running at 200 rpm. It is suddenly relieved of part of its load and its speed rises to 205 rpm in 1 sec. Find the unbalanced uniform turning moment.

(b) An Aluminium thin-walled tube (radius/thickness = 20) is closed at each end and pressurized by 6 MPa to cause plastic deformation. Neglect the elastic strain and find the plastic strain in the circumferential (hoop) direction of the tube. The plastic stress-strain curve is given by $\bar{c}$ = 170 (strain rate)$^{0.25}$.

3.(a) A uniform ladder 5 m long and 14 kg mass is placed against a vertical wall at an angle 50^0 to the horizontal ground. The co-efficient of friction between ladder and wall is 0.2 and between ladder and ground in 0.5. Calculate how far up the ladder a man of 63 kg. can climb before the ladder shifts.

(b) Determine the diameter of a steel shaft rotating at an angular velocity of 300 rpm transmitting 500 HP. The allowable stress = 800 kg/cm^2. The allowable angle of twist = 0.5^0 per m, G = 8×10^5 kg/cm^2 . What would be the savings if a hollow shaft is used to transmit the same power under the same condition, the ratio of diameters being 0.9?

4.(a) For the beam and loading shown in Fig.3, determine the equation defining the shear and bending moment at any point and at point D.

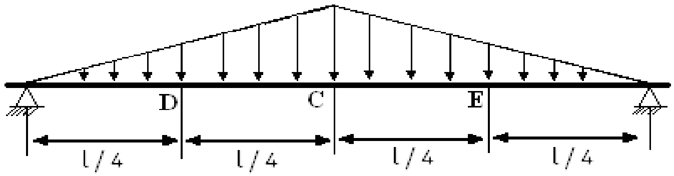


Fig. 3

(b) As illustrated in the given Fig.4 a metal punch (similar in principle to a paper punch) is used to punch holes in thin steel sheet that will be used to make a metal cabinet. To punch a 60 mm diameter disk or "slug" out of the sheet metal that is 10 mm thick requires a punch force of P=500 kg. Determine the average shear stress in the sheet metal resulting from the punching operation.

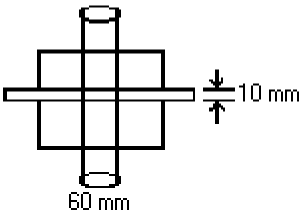


Fig. 4

5. (a) A tie rod in the suspension of a car is to be constructed from a grade of steel, which has 0.1% proof stress equal to 250 MN/m^2. The tie rod is to be constructed as a solid round bar of length 350 mm long. If the tie rod is subjected to a maximum axial force of 10 KN,

i) Determine the minimum diameter of the tie rod
ii) The extension of the tie rod under load (E= 2094 GN/m^2)
iii) The minimum diameter of the tie rod if a factor of safety of 2.5 is applied to the proof stress

(b) Find the width of the belt necessary to transmit 11.25 KW power to a pulley of diameter 300mm when the pulley makes 1600 rpm. Assume the co-efficient of friction between the belt and the pulley is 0.22 and angle of contact is 210^o. Maximum tension in the belt will not exceed 10N/mm width.

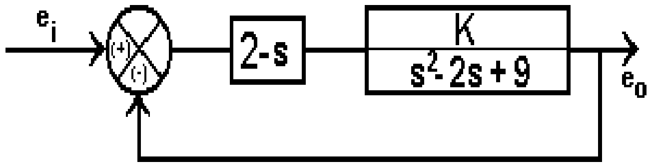
GROUP E-3: Electrical and Electronics Engineering

1.(a) A centrifugal pump, which is gear-driven by a DC motor, delivers 810 kg of water per minute to a tank of height 11 meter above the level of the pump. Draw the block diagram of the overall arrangement. Determine input power across the gearing and current taken by the motor operated at 220 volt provided the efficiency of the pump, gearing and motor respectively be 70%, 70% and 90% only. (Take g = 9.8 ms^{-2}).

(b) The rms value of a sinusoidal alternating voltage at a frequency of 50 Hz is 155volt. If at t = 0 it crosses the zero axis in a positive direction, determine the time taken to attain the first instantaneous value of 155 volt. How much time it takes to fall from the maximum peak value to its half? Explain with suitable waveform.

2.(a) On full-load unity power factor test, a meter having specification of 235 V and 5A makes 60 revolutions in 6 minutes, but its normal speed is 520 revolution/KWh. Does the meter has any inaccuracy? If so, find the percentage error.

(b) Write down the transfer function of the given system (as shown in the following figure) and find the values of K for which the system will be stable but underdamped.



3 (a) By intelligent selection of loop currents write down the mesh equations of the given circuit (as shown in Fig. 5) and determine the current flowing through that branch of the circuit containing capacitor. (All resistances/ reactances are in ohms).

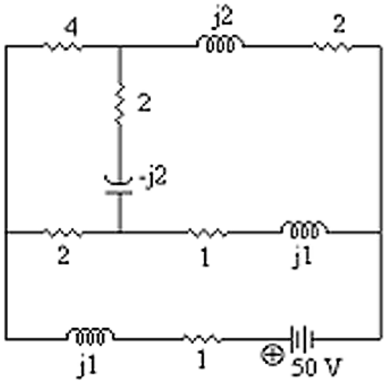


Fig. 5

(b)

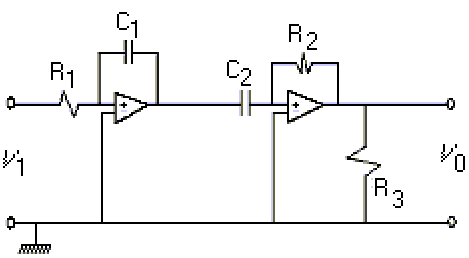


Fig. 6

Refer Fig. 6. Find the expression for V_0. What would be the nature of V_0 when $R_1 = R_2$ and $C_1 = C_2$? (Consider the Op-amps to be identical).

4. (a) A series ac circuit that resonates at 48 Hz consists of a coil (having R = 30 Ω and L = 500 mH) and a capacitor. If the supply voltage is 100 volt determine the value of the capacitor.

(b) Calculate the value of a capacitor which when connected across the circuit (as of Q. 4 (a) above), enhances the resonant frequency to 60 Hz. Compare the value of the source current in both the cases.

5 (a) A 200/400 - V, 10KVA, 50Hz single-phase transformer has, at full load, a Cu loss of 120W. If it has an efficiency of 98% at full load unity power factor, determine the iron losses. What would be the efficiency of the transformer at half load 0.8 power factor lagging?

(b) In the 2-port network given below, the parameters at two parts are related by the equations,

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

i) Find expressions for A, B, C and D

ii) Show that AD - BC = 1

iii) What are the physical interpretations of the above coefficients?

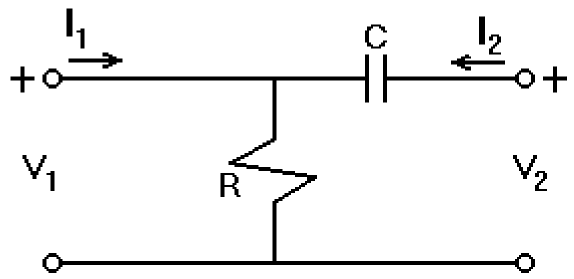


Fig 7

GROUP E-4: Thermodynamics

1 (a) In a thermodynamic system of a perfect gas, let $U = f(V,T)$ where U, V and T refer to internal energy, volume of a gram-molecule of the substance and temperature (in absolute scale) respectively. An amount of heat δQ is added so that the volume expands by δV against a pressure P. Prove that:

$$C_P - C_V = \left[P + \left(\frac{\delta U}{\delta V} \right)_T \right] \left(\frac{\delta V}{\delta T} \right)_P$$

where C_p and C_V stand for specific heat at constant pressure and specific heat at constant volume respectively.

(b) 0.15 cu.m. of air at a pressure of 1.06 kg/cm^2 is compressed to a volume of 0.008 cu.m. at 361 kg/cm^2. Calculate (i) the quantity of heat rejected, (ii) change in internal energy if the process of compression is a) Adiabatic b) Polytropic with $n = 1.3$.

2 (a) A compression ignition engine has a stroke of 28 cm and a cylinder diameter of 18 cm. The clearance volume is 475 cm^3. The fuel injection takes place at constant pressure for 4.5% of the stroke. Find the air standard efficiency of the engine assuming that it works on diesel cycle. If the fuel injection takes place at 10% of the stroke, find the loss in air standard efficiency.

(b) A diesel engine has a compression ratio 14 to 1 and the fuel supply is cut off at 0.08 of the stroke. If the relative efficiency is 0.52, estimate the weight of fuel of a calorific value 10400 k.cal per kg that would be required per horsepower.

3.(a) Calculate the change in entropy of saturated steam at a given pressure such that the boiling point = 152.6 ^{o}C and the latent heat at this temperature = 503.6 cal/gm. [Use $\text{Log}_e 1.56 = 0.445$.]

(b) Draw the pv and $T - \Phi$ diagrams for a diesel cycle in which 1 kg of air at 1 kg / cm^2 and 90 ^{0}C is compressed through a ratio of 14 to 1. Heat is then added until the volume is 1.7 times the volume at the end of compression, after which the air expands adiabatically to its original volume. Take $C_v = 0.169$ and $\gamma = 1.41$.

4.(a) The approximated equation for adiabatic flow of super heated steam through a nozzle is given by $pv^n = \text{constant}.$ Show that

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{n/n+1}$$

where $p_1 =$ pressure of steam at entry ; $p_2 =$ pressure of steam at throat and p_2 / p_1 is the critical pressure ratio.

(b) The dry saturated steam is expanded in a nozzle from pressure of 10 bar to pressure of 4 bar. If the expansion is super saturated, find the degree of under cooling.

GROUP E-5: Engineering Properties of Metals

1. (a) Distinguish between modulus of rigidity and modulus of rupture. Give an expression for the modulus of rigidity in terms of the specimen geometry, torque, and angle of twist. Is the expression valid beyond the yield strength (torsion)?

   (b) A steel bar is subjected to a fluctuating axial load that varies from a maximum of 340 kN to a minimum of 120 kN compression. The mechanical properties of the steel are $\sigma_u = 1090$ MPa, $\sigma_0 = 1010$ MPa and $\sigma_e = 510$ MPa. Determine the bar diameter to give infinite fatigue life based on a safety factor of 2.5

2 (a) A cylindrical bar is subjected to a torsional moment M_T at one end. The twisting moment is resisted by shear stress μ set up in the cross section of the bar. The shear stress is zero at the centre of the bar and increases linearly with the radius. Find the maximum shear stress at the surface of the bar.

Given $J = \dfrac{\pi D^4}{32}$ (assuming that the torsional deformation is restricted within the zone of elasticity)

where, J : Polar moment of inertia
D : Diameter of cylinder.

(b) Consider a flat plane containing a crack of elliptical cross-section. The length of the crack is $2c$ and stress is perpendicular to the major axis of the ellipse. Show that

$$\sigma = \sqrt{\frac{2\gamma E}{\pi c}}$$

c : stress
γ : surface energy

E: Young's modulus of elasticity

3. (a) Consider a tension specimen, which is subjected to a total strain ε at an elevated temperature where creep can occur. The total strain remains constant and the elastic strain decreases. Show that

$$\frac{1}{\sigma^{n-1}} = \frac{1}{\sigma_o^{\,n-1}} + BE(n-1)t$$

where,

$\varepsilon = \varepsilon_e + \varepsilon_p$ — ε_e : elastic strain
$\varepsilon_e = \sigma/E$ — ε_p : plastic strain
$\frac{d\varepsilon_p}{dt} = B\sigma^n$ — t : time
$\sigma = \sigma_o$ at $t = 0$.

(b) Distinguish between slip and twinning with diagrams.

4. (a) Suppose a crystalline material has *fcc* structure with atomic radius of 1.278 Å Determine the density of the crystalline material. Assume number of atoms per unit cell and molecular weight are n and M gm respectively.

(b) Suppose there is an electron in an electric field of intensity 3200 volts/m. Estimate the force experienced by the electron. If it moves through a potential difference of 100 volts, find the kinetic energy acquired by the electron.

GROUP E-6: Engineering Drawing

1.(a) A hollow cube of 5cm side is lying on H.P. and one of its vertical face is touching V.P. A slim rod, to be taken as its solid diagonal, is placed within it. Draw top and front / side views of solid diagonal and, from the drawn figure determine its true length.

(b) Two balls are vertically erected to 18cm and 30 cm respectively above the flat ground. These balls are away from a 3 cm thick wall (on the ground) by 12 cm and 21 cm respectively but on either side of the wall. The distance between the balls, measured along the ground and parallel to the wall is 27 cm. Determine their approximate distance.

2. (a) Sketch the profile of a square thread, knuckle thread and a white-worth thread showing all relevant dimensions in terms of the pitch.

(b) Sketch:

i) single riveted lap joint,
ii) double riveted lap joint chain-riveting,
iii) double riveted lap joint zigzag-riveting, and
iv) single cover single riveted butt joint.

3.(a) Draw the isometric view of an octahedron erected vertically up on one of its vertices. (Distinct free hand sketch only.)

(b) You are given two square prisms of same height of 10cm. Prism A has side 7cm and prism B has side of 5cm respectively. Longer face of B is lying on H.P. with its base perpendicular to V.P. Base of A is lying on H.P. but equally inclined to V.P. You are instructed to remove by cutting a portion of bottom base of A so that within the cavity maximum of B may be placed accordingly. Note that vertical face of B may be parallel to V.P. but just touch the central axis of A. Draw the sectional view of the combination and determine the volume of material to be removed from A.

4. A parallelepiped of dimension $100\times 60\times 80$ is truncated by a plane which passes through 85, 45 and 65 unit distance on the associated edges from the nearest top point of the object. Draw the isometric view of the truncated solid object. In third angle projection method, draw its plan. (All dimensions are in mm).

Note: A copy of one of the previous year's QR Test Question paper is appended in the following pages to give the candidate a rough idea.

BOOKLET No. TEST CODE: QR

<u>Afternoon</u>

Time: 2 hours

Group	Questions: Total	Questions: To be answered	Maximum marks
Part I (for Statistics/Mathematics Stream)			
S1 (Statistics)	5	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM EACH GROUP.	120
S2 (Probability)	5		
Part II (for Engineering Stream)			
E1 (Mathematics)	3	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM E1	120
E2 (Engineering Mechanics)	2		
E3 (Electrical and Electronics Engineering)	2		
E4 (Thermodynamics)	2		
E5 (Engineering Properties of Metals)	2		
E6 (Engineering Drawing)	2		

On the answer-booklet write your Name, Registration Number, Test Code, Number of this booklet, etc. in the appropriate places.

There are two parts in this booklet as detailed above. Candidates having Statistics background are required to answer questions from Part I as per instructions given. Those having engineering background are required to answer questions from Part II as per instructions given.

USE OF CALCULATORS IS NOT ALLOWED. SLIDE RULE MAY BE USED

STOP! WAIT FOR THE SIGNAL TO START

PART I (FOR STATISTICS/MATHEMATICS STREAM)

ATTENTION : ANSWER A TOTAL OF SIX [6] QUESTIONS, TAKING AT LEAST TWO [2] FROM EACH GROUP.

GROUP S-1
Statistics

1. A Spring Balance is used to estimate the weights of two objects, namely A and B. Objects themselves cannot be weighed in the balance. So a container is used to hold the objects during weighing. Let the unknown weights of the container, object A and object B be w_0, w_1 and w_2 respectively. First, object A is weighed after placing it in the container and the sum of their weights y_1, is recorded. Then the object is taken out of the container, placed back in it and reweighed. Let the weight be y_2. Similarly two observations on sum of the weights of container and object B are generated as y_3 and y_4. Assume that the errors of measurements are independently distributed with zero mean and constant variances.

   (a) Obtain least square estimates of w_0, w_1 and w_2. Are they unique? Justify.
   (b) Give the condition of estimability of the parametric function $l_0 w_0 + l_1 w_1 + l_2 w_2$, where l_0, l_1 and l_2 are known constants.
   (c) Obtain the best linear unbiased estimator of $w_0 + 3w_1 - 2w_2$.

(10 +6+4) = [20]

2. Let $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N$ be a random sample of size N from $N_p(\boldsymbol{\mu}\mathbf{1}, \Sigma)$, where

$$\mathbf{x}_\alpha = \left(x_{1\alpha}, \ldots, x_{p\alpha}\right)', \ \alpha = 1, 2, \ldots, N \ ; \ \mathbf{1}_{p\times 1} = (1, \ldots, 1)' \text{ and } \Sigma \text{ is known.}$$

   Two estimators are suggested for μ as follows:

$$T_1 = \frac{\mathbf{1}'\bar{\mathbf{x}}}{\mathbf{1}'\mathbf{1}} \text{ and } T_2 = \frac{\mathbf{1}'\Sigma^{-1}\bar{\mathbf{x}}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}},$$

   where, $\bar{\mathbf{x}} = \left(\bar{x}_1, \ldots, \bar{x}_p\right)'$, $\bar{x}_i = \dfrac{1}{N}\sum_{\alpha=1}^{N} x_{i\alpha}$, for $i = 1, 2, \ldots, p$.

   (a) Show that T_1 and T_2 are unbiased estimators of μ.

(b) Find the variances of T_1 and T_2.

(c) Show that $\text{var}(T_2) \leq \text{var}(T_1)$.

(4+6+10) = [20]

3. If $x_1, x_2, ..., x_n$ is a random sample of size *n* from the uniform distribution having the probability density function

$$f(x) = \begin{cases} 1 & \text{if } \alpha \leq x \leq \alpha + 1 \\ 0 & \text{elsewhere} \end{cases}$$

Show that $x_{(1)} - \frac{1}{n+1}$ is a consistent estimator of α where $x_{(1)}$ is the first order statistic.

[20]

4. Obtain the probabilities that a sample selected in 3 draws according to SRSWR sampling scheme will consist of one, two and three distinct units, the population size being *N*.

Let $\bar{y}_\nu$ be the sample mean based on ν distinct units. Show that

$$\text{Var}\left(\bar{y}_\nu\right) < \text{Var}\left(\bar{y}\right)$$

where $\bar{y}$ is the sample mean based on all the sampled units.

[20]

5. (a) A sample of 230 high-school students were asked about the type of person who affected their life most positively. The following table gives the number of students for three types of schools (namely; public, private and residential) and the type of person making the most positive impact.

Person, who influenced	Type of School		
	Public	Private	Residential
Parents (P)	30	25	33
Friends (F)	13	8	4
Tutor (T)	50	44	10
Sibling (S)	7	3	3
	100	80	50

i. Define the population under study.
ii. What are the variables of interest?
iii. For each type of school, draw a bar chart showing the relative frequency of persons making the most positive impact. Compare these three bar charts and comment on whether students' opinion is influenced by the type of school they attend.

(2 + 2+ 6) = [10]

(b) Suppose that X is a random variable whose probability distribution is $N\left(\mu,\sigma^2\right)$, where σ^2 is assumed to be known. To test $H_o : \mu = \mu_o$ versus $H_1 : \mu > \mu_o$ we propose the following:

Obtain a sample of size n, compute the sample mean $\overline{X}$ and reject H_o if $\overline{X} > c$, where c is a constant to be determined.

i. Write down the operating characteristic function $L(\mu)$ of this test.
ii. If n is known and assuming that the probability of type I error is equal to α, find the value of c.
iii. Consider the case where n is not known. Assume that the probability of type II error is equal to β for $\mu = \mu_1$. Find the values of n and c.

(2+3+5)=[10]

GROUP S–2
Probability

6. Suppose that in Darjeeling, the probability that a rainy day is followed by a rainy day is 0.80, and the probability that a sunny day is followed by a rainy day is 0.60. Find the probabilities that a rainy day is followed by

(a) a rainy day, a sunny day, and another rainy day;
(b) two sunny days and then a rainy day;
(c) two rainy days and then two sunny days;
(d) a rainy day two days later.

[20]

7. (a) Let $X_{1,} X_2, ..., X_n$ be independent random variables with distribution function *F(x)*. Let $Y_n = \max\ (X_1,..., X_n)$, and define the new random variable

$$Z_n = n - nF\left(Y_n\right)$$

Find the distribution of Z_n.

[10]

(b) Consider two independent series of events 'E' and 'F' occurring according to Poisson processes with rates λ and μ respectively $(\lambda, \mu > 0)$. Let *N* denote the number of occurrences of 'E' between two successive occurrences of 'F'. Show that *N* has a geometric distribution.

[10]

8. (a) Let $X_1, X_2, ..., X_n$ be independent and identically distributed normal random variables with mean μ and variance 1. Set $S_n = \sum_{i=1}^{n} X_i^2$. Let Y_ν be a χ^2 random variable with ν degrees of freedom and θ be an unknown constant. Set $T = \theta Y_\nu$. Find the constants θ and ν so that the random variables S_n and *T* have same means and same variances.

[14]

(b) The number 6.5 is divided into two nonnegative real numbers, uniformly at random. For instance, into 4.07 and 2.43 or $6.5 - \sqrt{23}$ and $\sqrt{23}$. Next each of these two parts is rounded to the nearest integer. For example, 4 and 2 in the first case and 2 and 5 in the second case. What is the probability that the two numbers will add up to 7.

[6]

9. (a) Let U be a random variable following a normal distribution with mean 0 and variance c^2. Let V be another random variable, independent of U and $P(V=1) = P(V=-1) = \frac{1}{2}$. Define $W = UV$.

(i) Show that U and W have the same distribution.
(ii) Find the correlation coefficient between U and W.
(iii) Check whether U and W are independent or not.

(10 + 6 + 4) = [20]

10. (a) In a multiple choice test, each question has four alternative answers which are graded as:

A : Fully Correct

B : Mostly Correct

C : Mostly Incorrect

D : Completely Incorrect

The "Knowledge" X of an examinee is a random variable with four possible values, namely; 1 (excellent), $\frac{2}{3}$ (good), $\frac{1}{3}$ (fair) and 0 (poor). An examinee with $X = 1$ chooses A with probability 1, with $X = 2/3$ chooses at random between A and B, with $X = 1/3$ chooses at random between A, B, and C and with $X = 0$ chooses at random between A, B, C and D.

What scores will you assign for the alternatives A, B, C and D, if it is required that the expected score of an examinee is equal to his knowledge X for each possible value of X?

[10]

(b) There are n coaches in a train going from New Delhi to Kolkata. Five friends traveling by the train, for some reasons, could not meet each other at New Delhi Station before getting aboard.

(i) What is the probability that the five friends will be in different coaches?
(ii) What is the probability that any three of the friends will be in one (unspecified) coach?

[10]

PART II (FOR ENGINEERING STREAM)

ATTENTION : ANSWER A TOTAL OF SIX [6] QUESTIONS TAKING AT LEAST TWO [2] FROM E1.

GROUP E-1
Mathematics

1. (a) Show that $1+\frac{\sqrt{2}-1}{2\sqrt{2}} + \frac{3-2\sqrt{2}}{12} + \frac{5\sqrt{2}-7}{24\sqrt{2}} + \frac{17-12\sqrt{2}}{80} + ...\infty$

$$= 2 + (\sqrt{2}+1)\ \log_e \frac{1}{\sqrt{2}}$$

(b) Prove that 15 divides $1^1 \cdot 1! + 2^2 \cdot 2! + ... + 50^{50} \cdot 50!$

(12+8) = [20]

2. (a) If the population of a country doubles in 50 years, show using differential equations that it will treble in $50 \log_2 3$ years, assuming that the rate of increase is proportional to the number of inhabitants?

(b) A rectangle ABCD is inscribed in a circle. Let PQ be the diameter of the circle parallel to the side AB. If the measure of $\angle$ BPC is 30°, find the ratio of the area of the rectangle and the area of the circle.

(10+10) = [20]

3. (a) If $f(x) = (x-1)\, e^{x} + 1$, show that $f(x)$ is positive for all $x > 0$.

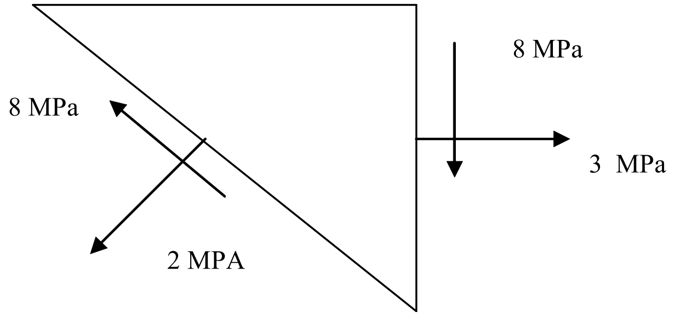
(b) Show that

$$\begin{vmatrix} a-b & a+b & c-d & c+d \\ -b & a & -d & c \\ -c-d & -c+d & a+b & a-b \\ -d & -c & b & a \end{vmatrix} = \left(a^2+b^2+c^2+d^2\right)^2$$

(10 + 10) = [20]

GROUP E-2
Engineering Mechanics

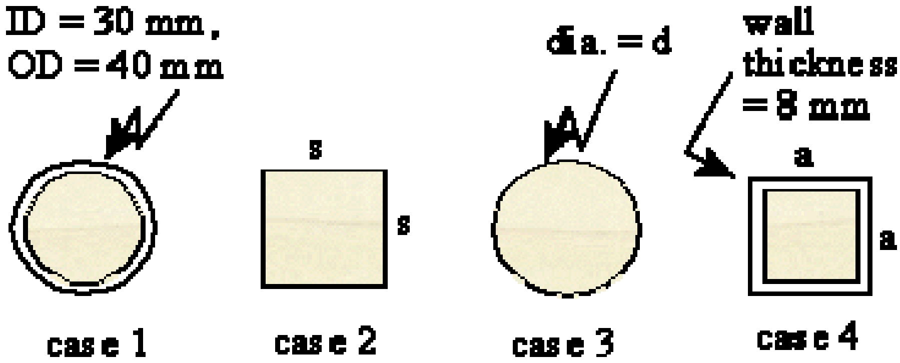
4. (a) At a point in a stressed body, the state of stress on two planes 45° apart is shown in figure below. Determine the two principal stresses.



(b) By how much will a 2m structural steel wire of diameter 2.2 mm stretch, if hanging under its own weight? The elastic modulus is 200 GPa and the density is 7860 kg/m^3.

(10+10) = [20]

5. (a) Four shapes are available for beam construction, a pipe (ID = 30 mm, OD = 40 mm), square cross section (solid), a solid rod, and square cross section (annular, with wall thickness 8 mm). If all shapes have equal cross sectional areas, which will best support a load (least maximum stress)?



(b) Show that the weight of a hollow shaft with the ratio of its internal diameter to its external diameter being 0.8 transmitting a moment of 600 Kgm and the and the allowable stress of 600 Kg/cm^2 is around 51% of the solid shaft of same strength.

(10+10)=[20]

GROUP E-3
Electrical & Electronics Engineering

6. (a) Show that for a stepdown autotransformer $Z_{in} = (1 + N_1 / N_2)^2 Z_l$ where N_1(up), N_2(down) are number of turns respectively from the intermediate tapping point, Z_{in} and Z_l are overall input impedance on primary side and output impedance on secondary side respectively. Draw the diagram showing all those.

(b) The output of an audio power amplifier is directly fed to a loud speaker having internal impedance of $8\,\Omega$. With the specification (from the Thevenin's equivalent of amplifier circuit): V_{th} = 16V and Z_{th} = $72\,\Omega$, determine the amount of power transferred to the loud speaker.

An ideal impedance matching transformer is placed in between the amplifier and loudspeaker to provide maximum power transfer to the loud speaker. Draw the complete circuit diagram. Find the required turns ratio of the transformer to achieve the maximum power transfer, the quantity of which you have to determine.
(Same Thevenin's equivalent of amplifier circuit may be considered here).

(7 + (4+9)) = [20]

7. (a) Given a positive number N in base r with an integer part of n digits, the r's complement of N is defined as $r^n - N$ for $N \neq 0$ and as 0 for $N = 0$. Now calculate 2's complement of 11101100. Justify whether there can be two different binary numbers whose 2's complements are same

(b) Design 2-input XOR gate using only NAND (/NOR) gates and draw its symbolic diagram. Let for each of the three set of XOR gate, A (binary value) be the first input. Determine individually, for what 3 set of second input (to XOR gate) you may have 3 set of output: $\overline{A}$, 0 and 1.

(9 + 11) = [20]

GROUP E-4
Thermodynamics

8. (a) Two reversible engines are operating between three thermal reservoirs at temperature T_1, T_2 and T_3, respectively. The first engine operates with the reservoirs at T_1 and T_2 while the second engine operates between the reservoirs at T_2 and T_3. The second engine absorbs energy from the reservoir at T_2 such that the reservoir is unaffected. Determine a relation among T_1, T_2 and T_3 if

i. the efficiencies of both the engines are same
ii. the work output of both the engines are same.

(b) Two identical bodies of mass m having specific heat C are available at temperature T_1. It is desired to operate a refrigerator between the two bodies to cool one of them to T_2. Determine the minimum amount of work to be done on the device and the final temperature of the second body.

(12 + 8) = [20]

9. (a) The efficiency of a Carnot heat engine can be increased either by increasing the source temperature while the sink temperature is held constant or by decreasing the sink temperature while the source temperature is held constant. Which of the above two possibilities is more effective?

(b) A four stroke diesel engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	90 mm
Stroke	80 mm
Speed	5000 rev/min
Fuel consumption rate	0.09 kg/min
Calorific value	44 MJ/kg
Net Brake load	60 N
Torque arm	0.5 m
MEP	280 kPa

Calculate the following:

i. The Mechanical Efficiency
ii. The Brake Thermal Efficiency
iii. The Indicated Thermal Efficiency.

(10+10) = [20]

GROUP E-5
Engineering Properties of Metals

9. (a) Examine whether the following statements are true or false.

i. Hooke’s law is applicable to describe the behavior of metal in plastic region of deformation.
ii. Ductility or, brittleness is not an absolute property of metal.
iii. Fatigue failure occurs in metal parts, which are subjected to tensile or torsional stresses.

iv. Yielding in metal is not accompanied with permanent change of shape and size.
v. Stress rupture test may be viewed as an Accelerated life test for Fatigue failures.
vi. The fracture Transition Plastic (FTP) is the temperature where the Crack Arrest Temperature (CAT) curve crosses the tensile – strength curve.
vii. Creep is an important manifestation of elastic behaviour.
viii. Low cycle fatigue conditions result from cyclic strain rather than from cyclic stress.
ix. The torsion test provides more fundamental measure of the plasticity of a metal than the tension test.
x. Hot hardness gives a good indication of the potential usefulness of an alloy for high temperature strength applications.
xi. The hardness measured in terms of the height of rebound of the indenter is called scratch hardness.
xii. Anisotropy in metals cheracterises the dependence of properties on preferred orientation.
xiii. Resilience of material describes the ability of the material to absorb energy when deformed plastically.

(b) Explain the phenomenon of strain hardening of metal in tension. What is its relation with neecking and when does necking occur?

(13 + 7) = [20]

10. (a) What is the difference between stress intensity factor and plane strain fracture toughness. Can you consider fracture toughness to be independent of crack length, crack geometry or, loading system? Explain.

(b) A 4031 steel bar is subjected to a fluctuating axial load that varies from maximum of 325 kN tension to a minimum of 115 kN compression. The mechanical properties of steel are:
$\sigma_u = 1090\ Mpa,\ \ \sigma_0 = 1010\ \ MPa,\ \sigma_e = 510\ MPa$.
(i) Find the mean stress and alternating stress levels.
(ii) Determine the bar diameter to give infinite fatigue life based an a safety factor of 2.5.

(13 + 7) = [20]

GROUP E-6

Engineering Drawing

12 (a) A hollow cube of 10 cm is placed on H.P. with two adjacent vertical faces making equal angle to V.P. In this position draw the plan and side view of the cube. Mark the solid diagonal of cube properly.

(b) Now the cube is hanged by a string fastened to top vertex of the solid diagonal and is allowed to move freely on solid diagonal axis. Draw the plan and front view of the cube on hanging position.

[In each case, mention whether you use first angle *or* third angle projection]

(7 + 13) = [20]

13. (a) Sketch a hexagonal nut and bolt. Your diagram should show a practical way of using them. Explain why a washer is sometimes used.

(b) Sketch the profile of a symmetric V-shaped thread. Indicate the height (H), pitch (P), major diameter (D) and the minor diameter (D_1). What is the relationship between the height (H) and the pitch (P) of the screw.

(12+8) = [20]